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Robust bayesian analysis of an autoregressive model with exponential innovations

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Abstract. In this work, robust Bayesian analysis of the Bayesian estimation of an autoregressive model with exponential innovations is performed. Using a Bayesian robustness methodology, we show that, using a suitable generalized quadratic loss, we obtain optimal Bayesian estimators of the parameters corresponding to the smallest oscillation of the posterior risks.

Résumé. Dans ce travail, nous considérons l'estimation Bayésienne du paramètre d'un processus auto-régressif d'ordre un avec erreurs exponentielles. En utilisant une méthodologie de robustesse Bayésienne appropriée et une fonction perte quadratique généralisée adéquate, nous montrons qu'on peut construire un estimateur Bayésien robuste correspondant à la plus petite oscillation du risque a posteriori..

Key words: Autoregressive process; Bayes; Estimation; Exponential; Loss function; Robustness.
AMS 2010 Mathematics Subject Classification : 62F15; 62F35.

1. Introduction

Autoregressive models with exponential innovations attract interest in various papers as, Davis and McCormick (1989) and Nielsen and Shephard (2003). These models are useful for modelling a wide range of phenomena which do not allow for negative values as in water quality analysis and hydrology modelling (see, for example, Gaver and Lewis (1980)). Cox (1981) provides a wide ranging discussion of many developments of these models. Bell and Smith (1986) studied estimating and testing problems on the first-order autoregressive processes.

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The robustness aspect of the estimators of the autoregressive parameter is studied by many authors. See, e.g., Kale and Sinha (1969) or Fellag and Ibazizen (2001). In Bayesian context, Turkman (1990) studied inference in AR(1) models with nonnegative innovations based on noninformative prior, showing that, for large samples, the Bayesian estimator is equivalent to some modified maximum likelihood estimator (MLE). Ibazizen and Fellag (2003) considered the same model using a more general prior distribution.

Our aim here is to perform a robust analysis of the Bayesian estimator of the nonnegative autoregressive parameter using the oscillation of the posterior risks (PR). Many works refer to the various approaches of Bayesian robustness in statistics. For reviews, see Berger (1990) and Rios Insua and Ruggeri (2000). We consider Bayesian estimation of the fist-order autoregressive parameter with exponential innovations, under basic and generalized quadratic loss functions. Since the autoregressive parameter is in $(0,1)$ and the innovation parameter is positive, the natural priors used in Baseline model will be beta distribution for the autoregressive parameter and gamma for the innovations's parameter. Using an exhaustive Monte carlo study and the methodology provided by Męczarski and Zieliński (1991), we study stability of Bayesian estimation of the two parameters. The paper is outlined as follows : in the section 2, we present the model and problematic. Robustness analysis of the Bayesian estimators is derived in Section 3 using basic and generalized quadratic loss function. Section 4 presents the results stability of the Bayesian estimation using exhaustive Monte carlo experiments. We end up with some discussion.

2. The model

Consider the first-order autoregressive process of the form

$$X_t = \rho X_{t-1} + \epsilon_t, \quad t = \dots, -1, 0, 1, ... \tag{1}$$

where the ϵ_t's are the innovations independently distributed according to an exponential distribution of parameter θ denoted Ex(θ), i.e. with density

$$f(y) = \theta e^{-\theta y} I_{(0,\infty)}(y), \quad \theta > 0$$

As in Turkman (1990), assume that $0 < \rho < 1$ and X_1 is distributed according to Ex$((1-\rho)\theta)$ such that the process is mean stationary. Suppose that all we observe is a segment of the process

$$X_1, X_2, ..., X_n \quad n \quad \text{fixed} \tag{2}$$

The likelihood function based on the observations $x = \{x_1, x_2, ..., x_n\}$ is

$$p(x|\rho,\theta) = (1-\rho)\theta^n e^{-\theta(n\overline{x} - \rho S)} I_A(x),$$

where

$$A = \{x : x_1 > 0, x_t - \rho x_{t-1} \geq 0, t = 2, ..., n\}$$

and $n\overline{x} = \sum_{t=1}^{n} x_t$, $S = n\overline{x} - (x_n - x_1)$. Let us define the modified maximum likelihood estimator (MLE) of ρ introduced by Anděl (1988, 1989) as follows

$$\rho_0 = \min\left(1, \frac{x_2}{x_1}, ..., \frac{x_n}{x_{n-1}}\right). \tag{3}$$

Since $\rho \in (0,1)$ and $x_t - \rho x_{t-1} \geq 0$ for all $t = 2, ..., n$, the parameter ρ lies in the interval $(0, \rho_0)$. Turkman (1990) considered the following priors for ρ and θ

$$\pi_0(\rho) \propto \frac{1}{1-\rho}\, I_{(0,1)}(\rho) \quad ; \quad \pi_1(\theta) \propto \frac{1}{\theta}\, I_{(0,\infty)}(\theta). \tag{4}$$

The posterior distributions of ρ and θ are then (see Turkman (1990))

$$p_0(\rho|x) = K_0\, \frac{1}{(n\bar{x} - \rho S)^n}\, I_{(0,\rho_0)}(\rho) \text{ and } p_1(\theta|x) = K_0^\star\, \theta^{n-2} e^{-\theta n\bar{x}}(e^{\theta\rho_0 S} - 1)\, I_{(0,\infty)}(\theta)$$

with

$$K_0 = \frac{(n-1)S\,(n\bar{x})^{n-1} r^{n-1}}{1 - r^{n-1}} \quad \text{and} \quad K_0^\star = \frac{(n\bar{x})^{n-1} r^{n-1}}{\Gamma(n-1)(1-r^{n-1})}. \tag{5}$$

Under the quadratic loss function, he derived the following Bayesian estimators of ρ and θ

$$\hat{\rho}_{Q,0} = \frac{\rho_0}{n-2}\left(\frac{n-1}{1-r^{n-1}} - \frac{1}{1-r}\right) \quad \text{and} \quad \hat{\theta}_{Q,0} = \frac{n-1}{n\bar{x}r}\left(\frac{1-r^n}{1-r^{n-1}}\right), \tag{6}$$

where $r = 1 - \rho_0(S/n\bar{x})$. Ibazizen and Fellag (2003) generalized this prior and obtained the estimator of ρ, under the quadratic loss function.

In the following, we propose to study the sensitivity of the Bayesian estimation of the autoregressive parameter to prior.

3. Bayesian robustness

In this section, we propose to investigate the properties of the Bayesian estimators using different priors. First of all, assume that $\pi_0(\rho)$ is a general prior of ρ. Using the noninformative prior of θ given above, we obtain the following joint posterior density of (ρ, θ)

$$p(\rho, \theta) \propto (1-\rho)\, \theta^n\, e^{-\theta(n\bar{x} - \rho S)}\, \pi_0(\rho)\, \pi_1(\theta)\, I_{(0,\rho_0)}(\rho)\, I_{(0,\infty)}(\theta)$$

3.1. Basic quadratic loss function

3.1.1. **Bayesian stability for** ρ

Assume that the prior of θ is

$$\pi_1(\theta) \propto \frac{1}{\theta}\, I_{(0,\infty)}(\theta)$$

The posterior density of ρ is then as follows

$$p(\rho|x) \propto (1-\rho)\,(n\bar{x} - \rho S)^{-n} \pi_0(\rho)\, I_{(0,\rho_0)}(\rho)$$

Under the quadratic loss function the Bayesian estimator $\hat{\rho}_Q$ and the corresponding posterior risk PRQ are given by the posterior mean and variance respectively.

Suppose that the prior $\pi_0(\rho)$ is a beta distribution $Beta(a_0, b_0)$ where a_0 and b_0 are fixed. Then, after easy computations, we obtain the following posterior density of ρ

$$p(\rho|x) = K(a_0, b_0)\, \rho^{a_0 - 1}\,(1-\rho)^{b_0}\,(n\bar{x} - \rho S)^{-n}\, I_{(0,\rho_0)}(\rho) \tag{7}$$

where

$$K(a_0, b_0) = \frac{a_0\ \rho_0\ F_1[a_0+1, n, -b_0, a_0+2, 1-r, \rho_0]}{(a_0+1)\ F_1[a_0, n, -b_0, a_0+1, 1-r, \rho_0]}. \tag{8}$$

Notice that the notation $F1[\alpha, \beta_1, \beta_2, \gamma, x, y]$ means $AppellF1[\alpha, \beta_1, \beta_2, \gamma, x, y]$, the well known Appell hypergeometric function of two variables x and y defined by the following infinite serie

$$AppellF1[\alpha, \beta_1, \beta_2, \gamma, x, y] = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} \frac{[\alpha]_{m+n}[\beta_1]_m[\beta_2]_n}{m!n![\gamma]_{m+n}} x^m y^n$$

where $[a]_0 = 0, [a]_m = a(a+1)\ldots(a+m-1),\ m \in N^{\star}$. The integral representation of this function is

$$AppellF_1[\alpha, \beta_1, \beta_2, \gamma, x, y] = \frac{\Gamma(\gamma)}{\Gamma(\alpha)\Gamma(\gamma-\alpha)} \int_0^1 u^{\alpha-1}(1-u)^{\gamma-\alpha-1}(1-ux)^{-\beta_1}(1-uy)^{-\beta_2} du$$

In order to simplify the notations, in all the following, we decide to write F_1 instead of $AppellF1$. The Bayesian estimator of ρ is given by

$$\hat{\rho}(a_0, b_0) = \frac{a_0\ \rho_0\ F_1[a_0+1, n, -b_0, a_0+2, 1-r, \rho_0]}{(a_0+1)\ F_1[a_0, n, -b_0, a_0+1, 1-r, \rho_0]}. \tag{9}$$

In order to perform the stability Bayesian analysis on ρ, suppose that the prior is not exactly specified. So, following the methodology of Męczarski and Zieliński (1991), we keep the value b_0 fixed and consider the parameter $a_1 \le a \le a_2$ instead of a_0 with a_1 and a_2 fixed. Then, we obtain the oscillation of the posterior risk of the Bayesian estimator of ρ denoted $PR1(a)$ when a varies from a_1 until a_2. We have

$$PR1(a) = E(\rho^2|x) - 2\hat{\rho}(a_0, b_0)E(\rho|x) + \hat{\rho}(a_0, b_0)^2 \tag{10}$$

where

$$\begin{aligned} E(\rho|x) &= \frac{a\ \rho_0\ F_1[a+1, n, -b_0, a+2, 1-r, \rho_0]}{(a+1)\ F_1[a, n, -b_0, a+1, 1-r, \rho_0]} \\ E(\rho^2|x) &= \frac{a\ \rho_0^2\ F_1[a+2, n, -b_0, a+3, 1-r, \rho_0]}{(a+2)\ F_1[a, n, -b_0, a+1, 1-r, \rho_0]} \end{aligned}$$

and $\hat{\rho}(a_0, b_0)$ is the Bayesian estimator of ρ given above. Naturally, analytical expressions of these formulas are not available. However, Using R software, we will perform all the computations using R packages for $AppellF1$ function. The oscillation of the posterior risk of the Bayesian estimator of ρ is then,

$$R1(0) = |\max_{a_1 \le a \le a_2} PR1(a) - \min_{a_1 \le a \le a_2} PR1(a)|. \tag{11}$$

3.1.2. Bayesian stability for θ

Now, the prior of ρ is

$$\pi_0(\rho) \propto \frac{1}{1-\rho} I_{(0,1)}(\rho)$$

and we suppose that the prior $\pi_1(\theta)$ is a gamma distribution $G(\alpha_0, \beta_0)$ where α_0 and β_0 fixed. Then, after easy computations, we obtain the following posterior density of θ

$$p(\theta|x) = C(\alpha_0, \beta_0) \left\{ \theta^{\alpha_0+n-2} e^{-\theta(n\bar{x}r+\beta_0)} - \theta^{\alpha_0+n-2} e^{-\theta(n\bar{x}+\beta_0)} \right\} I_{(0,+\infty)}(\theta) \quad (12)$$

with

$$\frac{1}{C(\alpha_0, \beta_0)} = \Gamma(\alpha_0 + n - 1) \left\{ \frac{1}{(n\bar{x}r + \beta_0)^{\alpha_0+n-1}} - \frac{1}{(n\bar{x} + \beta_0)^{\alpha_0+n-1}} \right\}. \quad (13)$$

The Bayesian estimator of θ is then

$$\hat{\theta}(\alpha_0, \beta_0) = C(\alpha_0, \beta_0)\Gamma(\alpha_0 + n) \left\{ \frac{1}{(n\bar{x}r + \beta_0)^{\alpha_0+n}} - \frac{1}{(n\bar{x} + \beta_0)^{\alpha_0+n}} \right\}. \quad (14)$$

As for ρ, following the methodology of Męczarski and Zieliński (1991), we keep the value β_0 fixed and consider the parameter $\alpha_1 \leq \alpha \leq \alpha_2$ instead of α_0 with α_1 and α_2 fixed. Then, we compute the oscillation of the posterior risk of the Bayesian estimator of θ denoted $PR2\theta(\alpha)$ when α varies from α_1 until α_2. We have

$$PR2(\alpha) = E(\theta^2|x) \; - \; 2\, \hat{\theta}(\alpha_0, \beta_0)\, E(\theta|x) \; + \; \hat{\theta}(\alpha_0, \beta_0)^2 \quad (15)$$

where

$$\begin{aligned} E(\theta|x) &= C(\alpha, \beta_0)\Gamma(\alpha + n) \left\{ \frac{1}{(n\bar{x}r + \beta_0)^{\alpha+n}} - \frac{1}{(n\bar{x} + \beta_0)^{\alpha+n}} \right\} \\ E(\theta^2|x) &= C(\alpha, \beta_0)\Gamma(\alpha + n + 1) \left\{ \frac{1}{(n\bar{x}r + \beta_0)^{\alpha+n+1}} - \frac{1}{(n\bar{x} + \beta_0)^{\alpha+n+1}} \right\} \end{aligned}$$

and $\hat{\theta}(\alpha_0, \beta_0)$ is the Bayesian estimator of θ given above. The oscillation of the posterior risk of the Bayesian estimator of θ is then,

$$R2(0) = |\max_{\alpha_1 \leq \alpha \leq \alpha_2} PR2(\alpha) - \min_{\alpha_1 \leq \alpha \leq \alpha_2} PR2(\alpha)|. \quad (16)$$

3.2. Generalized quadratic loss function

Now, consider generalized quadratic loss function of the form $L(d, t) = t^k(d - t)^2$ where d is the Bayesian estimator, t is to be estimated and $k \in N$ is the parameter of the generalized loss function. Notice that, when $k = 0$, we obtain the basic quadratic loss function. Now, the oscillation of the PR's depends on k for both ρ and θ. Our aim is to check if there exist values of $k \neq 0$ such that the oscillation is smaller than using basic quadratic loss function ($k = 0$). In other words, we check if there are values of k which improve robustness properties of the Bayesian estimators of ρ and θ. In the following, we give the formulas of the Bayesian estimators and the corresponding PR's the Bayesian estimators of ρ and θ respectively.

3.2.1. **Bayesian stability for ρ**

Using the Beta prior $Beta(a_0, b_0)$ under generalized quadratic loss function, the Bayesian estimator of ρ is given by the formula

$$\hat{\rho}(a_0, b_0, k) = \frac{E(\rho^{k+1})}{E(\rho^k)} = \frac{(a_0 + k)\, \rho_0\, F_1[a_0 + k + 1, n, -b_0, a_0 + k + 2, 1 - r, \rho_0]}{(a_0 + k + 1)\, F_1[a_0 + k, n, -b_0, a_0 + k + 1, 1 - r, \rho_0]}. \quad (17)$$

Following the methodology given above (***see 3.1.1), the PR is as follows

$$PR1(a,k) = E(\rho^{k+2}|x) \; - \; 2\;\hat{\rho}(a_0,b_0,k)\;E(\rho^{k+1}|x) \; + \; E(\rho^{k}|x)\;\hat{\rho}(a_0,b_0)^2 \tag{18}$$

with

$$E(\rho^j|x) = \frac{a\;\rho_0^j\;F_1[a+j,n,-b_0,a+j+1,1-r,\rho_0]}{(a+j)\;F_1[a,n,-b_0,a+1,1-r,\rho_0]} \quad \text{for} \quad j=k,\; k+1,\; k+2$$

The oscillation of the posterior risk of the Bayesian estimator of ρ can be written as follows,

$$R1(k) = |\max_{\alpha_1\le\alpha\le\alpha_2} PR1(\alpha,k) - \min_{\alpha_1\le\alpha\le\alpha_2} PR1(\alpha,k)| \tag{19}$$

where $R1(0)$ corresponds to the oscillation obtained using basic quadratic loss function.

3.2.2. **Bayesian stability for θ**

Using the gamma prior $G(\alpha_0,\beta_0)$ under generalized quadratic loss function, the Bayesian estimator of θ is given by the formula

$$\hat{\theta}(\alpha_0,\beta_0,k) = \frac{\alpha_0+n+k-1}{(n\bar{x}r+\beta_0)(n\bar{x}+\beta_0)}\;\frac{(n\bar{x}+\beta_0)^{\alpha_0+n+k}-(n\bar{x}r+\beta_0)^{\alpha_0+n+k}}{(n\bar{x}+\beta_0)^{\alpha_0+n+k-1}-(n\bar{x}r+\beta_0)^{\alpha_0+n+k-1}}. \tag{20}$$

Following the methodology given above, the PR of the Bayesian estimator of θ is

$$PR2(\alpha,k) = E(\theta^{k+2}|x) \; - \; 2\;\hat{\theta}(\alpha_0,\beta_0,k)\;E(\theta^{k+1}|x) \; + \; E(\theta^{k}|x)\;\hat{\theta}(\alpha_0,\beta_0)^2 \tag{21}$$

with

$$E(\theta^j|x) = C(\alpha_0,\beta_0)\;\Gamma(\alpha{+}n{+}j{-}1)\left\{\frac{(n\bar{x}+\beta_0)^{\alpha+n+j-1}-(n\bar{x}r+\beta_0)^{\alpha+n+j-1}}{(n\bar{x}+\beta_0)^{\alpha+n+j-1}(n\bar{x}r+\beta_0)^{\alpha+n+j-1}}\right\}\; j=k,\; k{+}1,\; k{+}2$$

and $C(\alpha_0,\beta_0)$ given by the formula (13). The oscillation of the posterior risk of the Bayesian estimator of θ can be written as follows,

$$R2(k) = |\max_{\alpha_1\le\alpha\le\alpha_2} PR2(\alpha,k) - \min_{\alpha_1\le\alpha\le\alpha_2} PR2(\alpha,k)| \tag{22}$$

where $R2(0)$ corresponds to the oscillation obtained using basic quadratic loss function.

4. Monte Carlo experiments

In this section, we consider three samples for $n = 10$, 30 and 50 for $\rho = 0.6$ and $\theta = 1.0$. We obtain the following values of ρ_0, S, $n\bar{x}$ and r for the three situations.

Now, let us study the stability of the Bayesian estimators of ρ and θ under the generalized quadratic loss given above, when k varies. Our aim is to check if the oscillation can be smaller when the loss is not the basic one ($k \neq 0$).

n	ρ_0	S	$n\bar{x}$	r
10	0.6450434	26.77185	24.22747	0.2872139
30	0.6414586	64.23298	65.65518	0.3724364
50	0.6206054	126.7354	127.9633	0.3853501

Table 1. Values of ρ_0, S, $n\bar{x}$ and r for $n = 10, 0, 50$

	n		
k	10	30	50
0	1.49578	1.49981	1.49994
1	1.49907	1.49988	1.49996
2	1.49976	1.49993	1.49998
3	1.49993	1.49996	1.49999
4	1.49998	1.49998	1.49999
5	1.49999	1.49999	1.50000
8	1.50000	1.50000	1.50000
10	1.50000	1.50000	1.50000

Table 2. Variation of the oscillation of the PR's for ρ and $n = 10, 30, 50$ when k varies

4.1. Bayesian stability for ρ

Suppose that $a_0 = 1.0$ and $b_0 = 1.0$. Notice that this corresponds to uniform prior on $(0, 1)$ for ρ. The Bayesian estimates of ρ for $n = 10$, 30 and 50 are 0.64504, 0.6414 and 0.6206 respectively. The Table 2 presents the oscillation of the PR's when $0.5 \leq a \leq 1.5$ and $b_0 = 1.0$ fixed, for different values of k.
Notice that the oscillation does not change significatively with k. then, one can say that generalized quadratic don't improve the stability of the Bayesian estimation of ρ. Now, let us check if is the same for the parameter θ.

4.2. Bayesian stability for θ

In this case, assume that $\alpha_0 = 1.0$ and $\beta_0 = 1.0$ corresponding to the standard prior for the parameter θ. The Bayesian estimates of θ for $n = 10$, 30 and 50 are 0.87946, 1.17867 and 0.993825 respectively. The Table 3 presents the oscillation of the PR's when $0.5 \leq \alpha \leq 1.5$ and $\beta_0 = 1.0$ fixed, for different values of k.

Here, the situation is different since, the oscillation decreases until a value of k and then grows (see Figure1). In our example, the value of k where the oscillation is minimal is equal to 9, 10 for $n = 10, 30$ respectively. However, for $n = 50$, the value of the minimal oscillation is achieved for $k = 21$ as showed in the following table.

Thus, one can say that, using generalized quadratic loss, we can improve the robustness of the Bayesian estimators of ρ and θ. Also, using this methodology one can provides a suitable value of k such that the stability of the Bayesian estimation becomes optimal.

	n		
k	10	30	50
0	1.42653	1.45446	1.48044
1	1.43215	1.44543	1.48037
2	1.43138	1.43247	1.47991
3	1.42456	1.41377	1.47903
4	1.41043	1.3865	1.4777
5	1.38578	1.34616	1.47585
6	1.34430	1.28542	1.47335
7	1.27407	1.19229	1.47008
8	1.15228	1.04664	1.4658
9	1.00000	1.0000	1.46023
10	1.59357	1.0000	1.45297

Table 3. Variation of the oscillation of the PR's for θ and $n = 10, 30, 50$ when k varies

k	$R2(k)$
11	1.4434
12	1.4308
13	1.4141
14	1.391
15	1.3610
16	1.3190
17	1.2608
18	1.1791
19	1.0631
20	1.0
21	1.0
22	1.2602

Table 4. Variation of the oscillations of the PR's when $n = 50$ and $k = 11, 12, \ldots, 22$

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5. Conclusion

In this work, robustness of the Bayesian estimation of the autoregressive models with exponential innovations is studied in terms of stability of the oscillation of the posterior risks. Using the well known robustness methodology of Męczarski and Zieliński (1991), we proved that, under generalized quadratic loss function, we can construct an optimal and robust Bayesian estimator of the parameters corresponding to the smallest oscillation of the posterior risks. This makes the derived Bayesian estimators robust in terms of sensitivity to

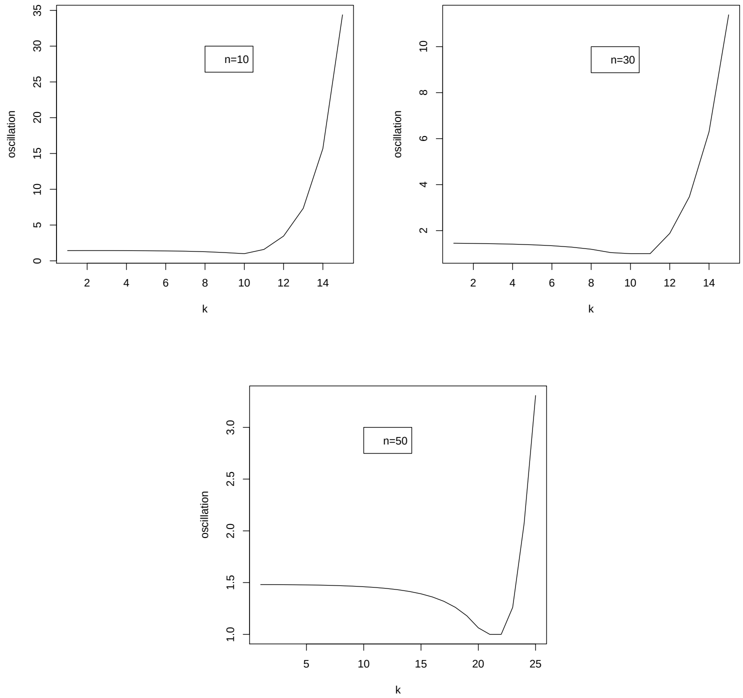

Fig. 1. Variation of the oscillation of the PR's with k for θ when $n = 10, 30, 50$

priors. To improve this work, one can study the performance of theses estimators under, for example, mixture classes of priors.

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