OPTIMAL SCHEDULING OF PETROLEUM PRODUCTS DISTRIBUTION IN NIGERIA

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ABSTRACT

One major problem in Nigeria is the persistent scarcity and inequitable distribution of petroleum products even with the establishment of three refineries, many storage depots and pipelines interconnecting them. This paper is applied to the situation in the downstream petroleum sector in order to determine an efficient and equitable distribution of three blends of petroleum products, namely; Premium motor spirit (PMS) Dual purpose kerosene (DPK) and Automotive Gasoline (AGO). For effective distribution, three refineries and seventeen storage depots were considered because of their interconnections with pipelines. The model was formulated as a linear programming problem with 20 constraints and 51 variables and was solved using LINDO Optimization software which uses simplex approach. Post-optimality analysis was performed investigating the effect of varying supply from refineries and demand at storage depots. The optimal solution was obtained for each of the product in the target year "2010" and the result was compared with the result of optimal solution obtained during uncertainty period say "2015". The study reveals that any variation in supply, demand and transportation cost changes the optimal solution.

Keyword: Distribution, petroleum products, refineries, storage depots, linear programming.

NOMENCLATURE

AGO	–	Automotive Gasoline
BPD(B/D)	–	Barrel per day
C_{ij}	–	Cost of transporting a unit metric tonne of the product from the supply point i to the demand point j.
D_j	–	Demand point requirement j.
DPK	–	Dual purpose kerosene
MT	–	Metric tonne
NNPC	–	Nigeria National Petroleum Corporation
PMS	–	Premium motor Spirit
PPMC	–	Pipeline and Product Marketing Company
Si	–	Supply point availability i.

INTRODUCTION

The production and distribution of petroleum products in Nigeria is an important factor in her domestic economy. From 1970 to date, the nation has invested substantially in refineries, storage depots, pipelines etc. The total pipeline network is about 4500Km [1]. Unfortunately, within the past few years, the supply of these petroleum product blends to storage depots and then to consumers have not been enough to meet the increasing demand. The areas of demand include: Domestic sector, Industrial sector, Transport and Agricultural sectors [2].

Proper schedule of the distribution through pipeline networks can facilitate the economical integration of refinery locations and storage depots for easy shipment of the products from refineries to depot locations and then to consumers at minimum delivery cost.

The refineries are situated in Port Harcourt, Warri and Kaduna; and storage depots are located in Aba, Enugu, Makurdi, Yola, Benin, Ore, Mosimi, Satellite town in Lagos, Ibadan, Ilorin, Suleja, Minna, Jos, Gombe, Maiduguri, Kano and Gusau.

The objective of the study is to develop a transportation model based on the linear programming technique that will schedule the distribution which minimizes the cost of delivery of these blends of products from refineries to storage depots. The system should be robust yet simple to support routine scheduling of monthly and annual distribution of petroleum products.

The supply model, the demand model, primary distribution (moving products from refineries or other supply sources to depots) at least cost, perhaps continuously by pipeline and secondary distribution (moving products from depots to consumers such as petrol stations) by lorry tankers are represented as a linear programming problem by Mehring and Gutherman [3] in which total cost of delivery are minimized (or profit contribution is maximized). Supply and distribution models are apparently of the few model in the oil industry that integrates several functional areas [4, 5].

Dantizig [6] formulated the transportation problem as linear programming problem and then developed the simplex algorithm for solving such problems. The simplex algorithm is an iterative procedure, that is, one that repeats the same steps over and over again, producing a sequence of "basic feasible" solutions where each solution improves on the preceding one until no further improvement is possible. The solution finally arrived at is the desired optimal solution.

Chen et al [7] developed a primal-dual simplex algorithm for the general linear programming problem and applied it to cost reduction problems.

HU [8] and HU and Johnson [9] developed a primal-dual simplex algorithm that is designed to take advantage of both primal and dual feasible solutions. Klabjan [10] developed an algorithm known as parallel primal-dual simplex which is capable of solving linear programs with thousands of rows and millions of columns.

2.0 MODEL AND PROBLEM FORMULATION

In formulating the model, the objective is to minimize the cost of delivery of these blends of petroleum products from refineries to storage depots through pipeline network.

The objective function of the model is

$$\text{Minimize } Z = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} X_{ij} \qquad (1)$$

Subject to the constraints

$$\sum_{i=1}^{n} X_{ij} \leq S_i \quad \text{all source nodes i} \qquad (2)$$

$i = 1,2,3, \ldots, n, n = 17$

$$\sum_{i=1}^{n} X_{ij} \geq D_j \quad \text{all demand nodes j} \qquad (3)$$

$i = 1,2, \ldots, m, m = 3$

$$X_{ij} \geq 0 \qquad (4)$$

Where X_{ij} = quantity of product transported from supply point i to demand point j

C_{ij} = Cost of transporting each unit product from supply point i to demand point j.

S_i = Supply point availability

D_j = Demand point requirement

There are several assumptions that accompany the above model

1) It is assumed that multiple sourcing is allowed. This occurs when a particular demand point requirements is met by more than one supply point.

2) The model assumes that the transportation cost on a given route is directly proportional to direct distance between supply points and demand points.

The equation (1) of the model is the objective function which expresses the goal of the problem.

Equation (2) in the model is the supply availability. This indicates that the quantities of final blended products leaving the refineries are less than or equal to the quantities being produced by the refineries.

Equation (3) of the model is the demand requirement. The demand model indicates that the quantities of blended products leaving the refineries to storage depots are greater than or equal to the demand of each depot.

Equation (4) of the model is the non-negativity constraint which explains that the quantity of each final blended product transported from refineries to the depots must be greater than or equal to zero.

2.1 Problem formulation

Formulation of cost minimization problem for distribution of petroleum products must begin with identification of decision variables. There are 51 decision variables which are defined as follows X_{ij} = quantity of PMS, DPK or AGO transported from supply point i (refineries) at unit cost (c_{ij}) to demand point j (storage depots). Table 1 show the decision variables used for the three refineries and seventeen storage depots.

Table 1: Decision variables used for the three refineries and seventeen storage depots.

Storage Depots/Refineries	Port Harcourt	Warri	Kaduna
Aba	$X_{1,1}$	$X_{2,1}$	$X_{3,1}$
Enugu	$X_{1,2}$	$X_{2,2}$	$X_{3,2}$
Makurdi	$X_{1,3}$	$X_{2,3}$	$X_{3,3}$
Yola	$X_{1,4}$	$X_{2,4}$	$X_{3,4}$
Benin	$X_{1,5}$	$X_{2,5}$	$X_{3,5}$
Ore	$X_{1,6}$	$X_{2,6}$	$X_{3,6}$
Mosimi	$X_{1,7}$	$X_{2,7}$	$X_{3,7}$
Satellite	$X_{1,8}$	$X_{2,8}$	$X_{3,8}$
Ibadan	$X_{1,9}$	$X_{2,9}$	$X_{3,9}$
Ilorin	$X_{1,10}$	$X_{2,10}$	$X_{3,10}$
Suleja	$X_{1,11}$	$X_{2,11}$	$X_{3,11}$
Minna	$X_{1,12}$	$X_{2,12}$	$X_{3,12}$
Jos	$X_{1,13}$	$X_{2,13}$	$X_{3,13}$
Gombe	$X_{1,14}$	$X_{2,14}$	$X_{3,14}$
Maiduguri	$X_{1,15}$	$X_{2,15}$	$X_{3,15}$
Kano	$X_{1,16}$	$X_{2,16}$	$X_{3,16}$
Gusau	$X_{1,17}$	$X_{2,17}$	$X_{3,17}$

3.0 METHODOLOGY

3.1 Data Collection

Six years' data on products demand, supply and consumptions were obtained from the NNPC's data bank in Port Harcourt and PPMC.

3.2 Statistical Analysis

Projections of the products demand, supply and consumption were made based on the above data.

For the period (1994-1999), the actual percentage depot demand and refinery supply for each product is given by the equation

$$b\% = \frac{X_{di}}{X_{dt}} \; x \; 100\% \qquad (5)$$

Where b% is the actual percentage demand or supply of each product at each depot or refinery in a particular year

X_{di} is the actual demand or supply of each product in a particular depot or refinery.

X_{dt} is the total demand or supply of each product in all the depots or refinery throughout the country.

The average demand or supply over the period is then given by

$$K = \frac{\sum b}{n} \qquad (6)$$

Where K is the average demand over the period

$\sum b$ is the summation of actual percentage demand or supply of each product at each depot or refinery for n years.

n is the number of years (n = 6)

Using the projected quantity consumed (C_p) from data available from NNPC and population size(P_t) from the National Population Commission (NPC), the projected per capita consumption of the blends of petroleum products(Y) will be

$$Y = \frac{C_p}{p_t} \qquad (9)$$

Where, C_p is the projected consumption of each product; Y is the per capita consumption of each product; and P_t is the population in a particular year. The projected per capita consumption against year for the blends of petroleum products is shown in figure 1.

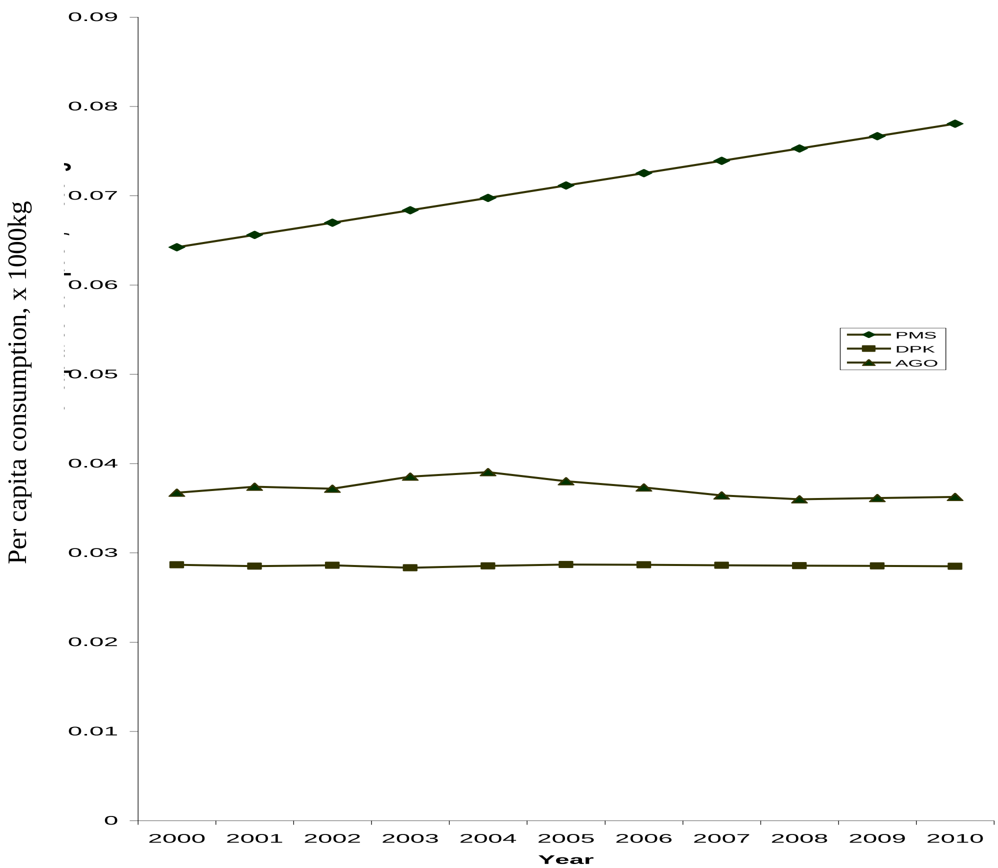

Figure 1: Projected per capita consumption against year for the blends of petroleum product

The demand of each product in a particular depot can be estimated by the relationship:

$$S_d = K.C_p \qquad (10)$$

where S_d = projected product demand at each depot in a particular year.

Similarly, the procedure can also be used for products supply projection from a particular refinery up to the target year.

Thus, $Z = m.C_p$ (11)

where Z is the projected supply of each product from each refinery up to the target year.

The unit transportation cost (C_{ij}) for each route is obtained.

Thus,

$$C_{ij} = f \times d_{ij}$$

Where f = cost factor for transporting one metric tonne of each product per km from refineries to storage depots as determined by the corporation. It is the same for all locations (f > 0). The parameter, d_{ij} = pipeline distance from refineries to storage depots.

3.3 Data Presentation

The projected petroleum products blend supplied by refineries to storage depots, projected petroleum products demand at storage depots for the target year "2010" (Tables 2 and 3) and the unit cost of transporting one metric tonne of each product from refineries to storage depots (Table 4) are input data to the linear programming model as given by equations (1) to (4). The model was solved using LINDO optimization software [11].

Table 2: Projected supply of blends of petroleum products for the year ''2010'' in metric tonnes (mt)

Refineries	PMS	DPK	AGO
Port Harcourt	6,590,712	2,126,824	3,162,249
Warri	2,921,276	1,291,379	1,572,285
Kaduna	2,382,457	924,027	790,010
Total	11,894,445	4,342,230	5,524,544

Table 3: Projected demand of blends of petroleum products in the year "2010" in metric tonnes (mt)

Storage depots	PMS	DPK	AGO
Aba	721,921	339,128	276,227
Enugu	869,397	330,009	449,145
Makurdi	398,424	179,768	188,939
Yola	141,530	33,869	79,553
Benin	545,900	267,481	307,717
Ore	223,593	85,542	98,337
Mosimi	933,622	277,469	395,864
Satellite	1,690,032	270,521	340,864
Ibadan	1,204,787	246,204	403,330
Ilorin	356,798	94,661	271,255
Suleja	312,793	86,410	124,302
Minna	58,277	67,739	57,455
Jos	210,511	115,503	123,750
Gombe	132,015	59,489	106,071
Maidguri	156,991	62,962	133,142
Kano	259,273	209,730	240,318
Gusau	90,389	62,094	63,532
Total	8,306,249	2,778,580	3,658,943

Table 4: Unit transportation cost (C_{ij}) (Naira) from refineries to storage depots

Refineries / Storage depots	Port Harcourt	Warri	Kaduna
Aba	54	522	725
Enugu	210	366	569
Makurdi	390	546	749
Yola	895	1051	1254
Benin	486	90	507
Ore	600	204	621
Mosimi	751	355	772
Satellite	794	398	815
Ibadan	830	434	851
Ilorin	1000	604	1021
Suleja	629	447	150
Minna	709	527	230
Jos	944	762	165
Gombe	1209	1027	430
Maidguri	1506	1324	727
Kano	1002	820	223
Gusau	1039	857	260

For post optimality, an uncertainty period say ‘2015” was used to compare the result of optimal solution of “2010”. The projected refineries supply and depots demand (Tables 5 and 6) and unit transportation cost (Table 4) are used as input data to equation (1) through equation (4). The model equation was solved using the same software.

Table 5: Projected supply of blends of petroleum products in the year ‘’2015’’ in metric tonnes(mt)

Refineries	PMS	DPK	AGO
Port Harcourt	9,265,576	2,422,694	3,687,075
Warri	3,663,644	1,471,027	1,833,230
Kaduna	2,987,899	1,052,571	921,125
Total	14,917,119	4,946,292	6,441,430

Table 6: Projected demand of blends of petroleum products in the year "2015" in metric tonnes (mt).

Storage depots	PMS	DPK	AGO
Aba	905,379	386,305	322,072
Enugu	1,090,322	375,918	523,688
Makurdi	499,674	204,776	220,297
Yola	77,496	38,581	92,757
Benin	684,627	304,692	358,788
Ore	280,414	97,442	114,657
Mosimi	1,170,877	316,068	460,562
Satellite	2,119,511	308,154	397,436
Ibadan	1,510,953	280,455	470,224
Ilorin	447,469	107,829	316,274
Suleja	392,281	98,431	144,932
Minna	73,087	77,162	66,991
Jos	264,007	131,571	144,288
Gombe	165,563	67,764	123,675
Maidguri	196,886	71,721	155,238
Kano	325,161	238,906	280,202
Gusau	113,359	70,732	74,076
Total	10,417,075	3,176,508	4,266,160

4.0 RESULTS AND DISCUSSION

The result of the study considered two important factors in the schedule of the distribution. The factors are

(1) The pipeline distance of storage depots from refineries.

(2) The product availability in refineries.

In the Premium Motor Spirit (PMS) schedule, the best schedule was obtained after performing 19 iterations by this algorithm.

For example, it is more economical to supply the 721,921 metric tonnes that will be needed in Aba depot in the year "2010" from the refinery in Port Harcourt ($X_{1,1}$) instead of pumping from refineries in Warri ($X_{2,1}$) or Kaduna ($X_{3,1}$).

Similarly, the 1,690,032metric tonnes that will be demanded at the depot in Satellite town in Lagos for the target year should be pumped from Warri refinery ($X_{2,8}$) only. Again, it was found that out of the 1,204,787 metric tonnes needed in the depot at Ibadan, it will be optimal to supply 1,098,970metric tonnes from Port Harcourt refinery($X_{1,9}$) and the remaining 104,816 metric tonnes from Warri ($X_{2,9}$). It is not economical to pump the product from Kaduna refinery to this depot ($X_{3,9}$).

Also, the minimum cost can be achieved when the 259,273metric tonnes needed in Kano depot is supplied from only

Kaduna refinery ($X_{3,16}$). The optimal cost of delivery for this schedule will be ₦3.493 billion.

Under uncertainty (2015), the pattern of schedule will be the same for minimum delivery cost and this will be ₦4.381 billion (Table 7).

In Dual Purpose Kerosene (DPK) schedule, the best schedule was achieved after performing 17 iterations. From the optimization results, it is economical to pump the 330,009 metric tonnes of this product that will be needed at Enugu depot from Port Harcourt refinery only ($X_{1,2}$) .In the same vein, cost can again be minimized if 267,48 metric tones expected at the depot in Benin is supplied alone from Warri refinery ($X_{2,5}$). It will be expensive if supply is obtained from either Port Harcourt refinery ($X_{1,5}$) or Kaduna refinery ($X_{3,5}$). Also, the 86,410 metric tonnes of this product that will be required at Suleja depot in the target year will be best met by Kaduna refinery ($X_{3,11}$) only because of its minimum cost effectiveness. The minimum cost of delivery for optimal solution will be ₦0.781 billion.

For uncertainty, the pattern of schedule will be the same for cost minimization to be achieved and the optimal cost of delivery will be ₦0.890 billion (Table 8).

In the Automotive Gasoline (AGO) schedule, the best distribution schedule was achieved after performing 20 iterations. From the result, the 188,939 metric tonnes meant for Makurdi depot in the year "2010" should be supplied by Port Harcourt refinery ($X_{1,3}$) only for minimum delivery cost to be achieved. Similarly, out of the 271,255 metric tonnes needed in Ilorin depot, 244,223 metric tonnes will be supplied by Port Harcourt refinery ($X_{1,10}$) while the remaining 27,032 metric tonnes will be supplied by the Warri refinery ($X_{2,10}$).

Again, the total of 124,302 metric tonnes that will be needed in Suleja depot in the target year will be distributed in such a way that 58,560 metric tonnes will be pumped by the Port Harcourt refinery ($X_{1,11}$). The remaining 65,747 metric tonnes that will be needed in Suleja depot will be supplied by Kaduna refinery ($X_{3,11}$) for minimum cost delivery. The minimum cost of delivery for optimality will be ₦ 1.306 billion.

Under uncertainty, the same pattern of schedule will be the same for minimum cost to be achieved and optimal cost will be ₦ 1.522 billion (Table 9).

Table 7: Premium Motor Spirit (PMS) result schedule for year '2010' and '2015'

	2010			**2015**		
Refineries / Storage depots	**Port Harcourt**	**Warri**	**Kaduna**	**Port Harcourt**	**Warri**	**Kaduna**
Aba	721,920	0.00	0.00	905,379	0.00	0.00
Enugu	869,397	0.00	0.00	1,090,323	0.00	0.00
Makurdi	398,424	0.00	0.00	499,674	0.00	0.00
Yola	141,530	0.00	0.00	177,496	0.00	0.00
Benin	0.00	545,900	0.00	0.00	684,627	0.00
Ore	0.00	223,593	0.00	0.00	280,413	0.00
Mosimi	933,621	0.00	0.00	1,170,877	0.00	0.00
Satellite	0.00	1,690,031	0.00	0.00	2,119,510	0.00
Ibadan	1,099,970	104,817	0.00	1,379,500	131,453	0.00
Ilorin	0.00	356,798	0.00	0.00	447,469	0.00
Suleja	0.00	0.00	312,793	0.00	0.00	392,281
Minna	0.00	0.00	58,276	0.00	0.00	73,087
Jos	0.00	0.00	210,511	0.00	0.00	264,007
Gombe	0.00	0.00	132,015	0.00	0.00	165,564
Maidguri	0.00	0.00	156,990	0.00	0.00	196,866
Kano	0.00	0.00	259,272	0.00	0.00	325,161
Gusau	0.00	0.00	90,389	0.00	0.00	113,359
Optimal Cost(₦)	**3,493,383,337.00**			**4,381,137,181.00**		

Table 8: Dual Purpose Kerosene (DPK) result schedule for year '2010' and '2015'

	2010			**2015**		
Refineries / Storage depots	**Port Harcourt**	**Warri**	**Kaduna**	**Port Harcourt**	**Warri**	**Kaduna**
Aba	339,128	0.00	0.00	386,305	0.00	0.00
Enugu	330,010	0.00	0.00	375,918	0.00	0.00
Makurdi	170,768	0.00	0.00	204,776	0.00	0.00
Yola	33,869	0.00	0.00	38,581	0.00	0.00
Benin	0.00	267,481	0.00	0.00	304,692	0.00
Ore	0.00	85,542	0.00	0.00	97,442	0.00
Mosimi	0.00	277,469	0.00	0.00	316,068	0.00
Satellite	0.00	270,521	0.00	0.00	308,154	0.00
Ibadan	0.00	246,204	0.00	0.00	280455	0.00
Ilorin	0.00	94,661	0.00	0.00	107,829	0.00
Suleja	0.00	0.00	86,410	0.00	0.00	98,431
Minna	0.00	0.00	67,739	0.00	0.00	77,162
Jos	0.00	0.00	115,503	0.00	0.00	131,571
Gombe	0.00	0.00	59,489	0.00	0.00	67,764
Maidguri	0.00	0.00	62,962	0.00	0.00	71,721
Kano	0.00	0.00	209,730	0.00	0.00	238,906
Gusau	0.00	0.00	62,094	0.00	0.00	70,732
Optimal Cost(₦)	**781,625,278.00**			**890,359,607.00**		

Table 9: Automotive Gasoline (AGO) result schedule for year '2010' and '2015'

	2010			2015		
Refineries / Storage depots	**Port Harcourt**	**Warri**	**Kaduna**	**Port Harcourt**	**Warri**	**Kaduna**
Aba	276,227	0.00	0.00	322,072	0.00	0.00
Enugu	449,145	0.00	0.00	532,688	0.00	0.00
Makurdi	188,939	0.00	0.00	220,297	0.00	0.00
Yola	79,553	0.00	0.00	92,757	0.00	0.00
Benin	0.00	307,717	0.00	0.00	358,788	0.00
Ore	0.00	98,337	0.00	0.00	114,657	0.00
Mosimi	0.00	395,005	0.00	0.00	460,562	0.00
Satellite	0.00	340,864	0.00	0.00	397,436	0.00
Ibadan	0.00	403,292	0.00	0.00	470,224	0.00
Ilorin	244,223	27,032	0.00	284,712	31,562	0.00
Suleja	58,560	0.00	65,742	68,279	0.00	76,653
Minna	0.00	0.00	57,445	0.00	0.00	66,991
Jos	0.00	0.00	123,750	0.00	0.00	144,288
Gombe	0.00	0.00	106,071	0.00	0.00	123,675
Maidguri	0.00	0.00	133,142	0.00	0.00	155,238
Kano	0.00	0.00	240,318	0.00	0.00	280,202
Gusau	0.00	0.00	63,532	0.00	0.00	74,076
Optimal Cost(₦)	**1,306,207,808.00**			**1,522,957,530.00**		

In this section also, the model is examined by varying one model parameter or all models parameter at a time and plotting the optimal cost function against year for the three blends of petroleum products. The supply and demand on the right-hand side of the constraints were increased to 2015 and the unit transportation cost used for the target year 2010 was applied to it. The optimal cost variation of these blends over these years are shown in figure 2.

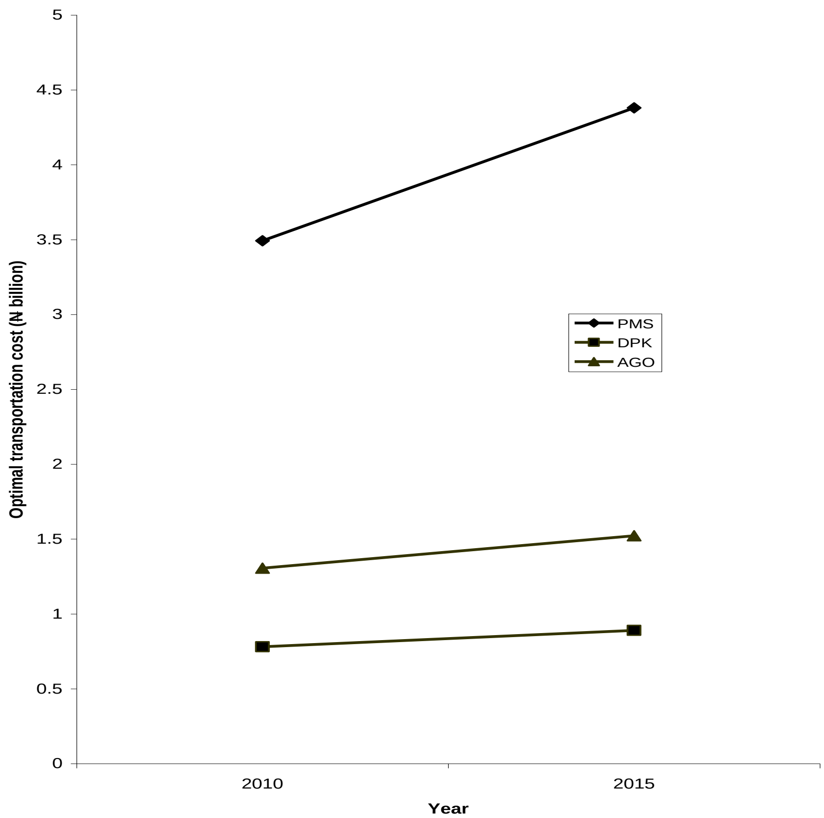

Fig. 2.: Optimal cost against year for the blends of petroleum products

CONCLUSIONS AND RECOMMENDATION

The study proposes a cost minimization model for description of the transportation of three blends of petroleum products along pipelines from three refineries to seventeen storage depots. From the optimal distribution schedule, some routes are feasible for distribution of petroleum products at minimum delivery cost while others either unfeasible or expensive.

Increasing supply and demand to accommodate uncertainty reveals the same pattern of distribution schedule but a change in the optimal solution.

Besides, additional crude should be allocated and refined in Warri refinery in order to meet up all PMS demand at Mosimi depot, some PMS demand at Ibadan depot and all AGO demand at Ilorin depot at minimum cost.

Similarly, more crude should also be allocated to Kaduna refinery so that all AGO requirements in Suleja depot will be met at minimum cost.

Furthermore, proper data keeping and documentation by NNPC and PPMC is strongly recommended for further studies.

One of the limitations of this model is that sensitivity analysis could not be investigated on observable constraints such as pipeline vandalization, plant failures, environmental degradation etc because they cannot be quantified. Again, the model did not provide for suppressed demand to depots or supply from refineries.

Finally, plant capacities of the nation’s refineries should be adjusted to meet with the projected productions to satisfy the demand.

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