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Modelling and Forecasting Periodic Electric Load for a Metropolitan City in Nigeria

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Eneje, I. S. - Department of Mechanical Engineering, University of Ibadan, Ibadan, Nigeria

Fadare D. A. - Department of Mechanical Engineering, University of Ibadan, Ibadan, Nigeria

E-mail: esimmar@yahoo.com

Simolowo O.E. Department of Mechanical Engineering, University of Ibadan, Ibadan, Nigeria

Falana, A. - Department of Mechanical Engineering, University of Ibadan, Ibadan, Nigeria

Abstract

In this work, three models are used to analyze the electric load capacity of a fast growing urban city and to estimate its future consumption. Ikorodu, the case-study location is a highly populated city whose energy demand is continuously increasing. The ultimate focus of this study is to establish a basis for the comparison of different electric load consumption for the

existing populace and to provide estimates for the future planning of the city. In this work, three different models have been used to present more accurate load predictions and to enhance proper comparison of results. Among numerous mathematical and scientific models that are applicable to this kind of task, the compound-growth method, the linear model approach and the cubic model have been chosen to enhance diversity in load analysis. The futuristic scheme to be harnessed will fall within the ranges of values obtained from the three different models used in forecasting. This paper concludes with issues pertaining to economics of load utilization as it affects substantive planning.

Key words: Electric-load, Linear trend, Compound-growth, Cubic model, forecasting

Introduction

Energy is considered as one of the most important resources of any community or country. The rate of industrial growth of any country is a function of the amount of energy available in that country and the extent to which this energy is utilized (Saab S. et al, 2000). Investigations are being conducted continually in the aspect of energy forecasting to present new methods of load predictions (Fadare, 2010; Antonio *et al.*, 2004; Breipohl and Douglas, 1998,). These works are based on the knowledge of the sources and essentials of energy generation (Musa, 2004; Ogbonnaya, *et al.*, 2006). Electricity load consumption in Nigeria is of great concern and its government is putting in all efforts towards solving the energy problems. Poor planning is one of the basis for the under-supply of electric energy in developing nations (Ogbonnaya *et al.*, 2006). The entailment of planning is nothing other than load planning on generation, transmission, distribution and utilization (Badran et al, 2008). This means that it involves the load forecasting and estimation for a given period and for a given people. Load forecasting is classified into; short range (some number of days), medium range (several weeks to one month) , and long range (several years).

Each class of load forecasting uses different models to meet the specific objectives of the application (Zaid et al, 2003). Among the models and forecasting parameters which had been in existence are regression methods (linear and quadratic) and Artificial Neural Network (ANN), Static state estimation method, the Gaussian Process models, time series, expert systems, fuzzy logic, the reference forecast, forecast by

the use of national economic and demographic variables and many others (Douglas *et al.*, 2004).

Load forecasting is the operation of predicting, with the help of previous data, what the future consumption will be. Forecasting of the electric load at a future time involves enormous tasks and challenging problems as a result of diversities of uncertainties that surround the study (Volkan, *et al*, 2001). The models used in this work are Linear models, the Compound-growth model, and the Cubic model approach and for the test of which suits the forecast two tools were used. They are Pearson's rank of correlation coefficient and mean-absolute-percentage-error.

Research method

All information and data were collected from the Power Holding Company of Nigeria (PHCN). A procedure which is uncommon with energy forecast of the cold temperate regions of the world was applied in this study. In the cold regions, winter is a time of high demand of energy for heating purposes and the summer a time of less consumption. In the case study environment for this work, most of the energy required is used for cooling or for domestic chores. However, there are instances of heating as in the cooking of meals with electricity warm livestock farms, and other slight applications. In Ikorudu, there are months during which load consumption is always high and there are months when it is normalized. It is therefore imperative to carry out this study with respect to the manner of changes in load according to month. In all the years, December and January have the highest load values while the months of the mid-year do not show much increase in load consumption. The reason could be firstly, due to the hot earth surface temperature. In this situation, the weather is very warm and cooling systems are used at this time. Secondly, there are several activities going on at such a time and much migration of people living outside the country or town returning home for festivities. These contribute to escalated load consumption. Based on this, the load varies according to month and not every month has a uniform load figure. However, the forecast made in this work is on the basis of total load consumption. This is because the energy needed for any system is based on the total load required to run that system. In this analysis the load consumption is summed up for the entire 12 months and then forecast is done on a yearly plan for 2011, 2012, 2013, and so on. Also, the total load figures for both residential and non-residential of the last 5 years

presented in Tables 1 and 2 will be used to forecast what the figures will be for the next 5 (years).

Research theories

The Linear Model Approach: This comprises linear trend and Excel algorithm, *The linear trend* has the form of representation of two specific variables, the independent and dependent variables. Y is load at a given year X, it states that, $Y = a + bX$ where *a* and *b* can be obtained from equation (1)

$$\sum Y = na + b \sum X \text{ and } \sum XY = a \sum X + b \sum X^2 \dots\dots\dots (1)$$

The Excel algorithm is a linear regression model and the expression for this is common to normal straight line equation which is $y = mx + c$, *n* is the number of years and the constants *m* and *c* can be resolved from equation (2) and (3) ;

$$m = \frac{n \sum xiyi - (\sum xi)(\sum yi)}{n \sum xi^2 - (\sum xi)^2} \dots\dots\dots (2)$$

$$C = \frac{(\sum yi)(\sum xi^2) - (\sum xi)(\sum xiyi)}{n \sum xi^2 - (\sum xi)^2} \dots\dots\dots (3)$$

The Compound-Growth Model can be expressed as equation (4);

$$Y = \text{antilog}(c + dx) \dots\dots\dots (4)$$

Where the constants *c* and *d* can be found when solved simultaneously from

$$\sum \log Y = nc + d \sum X \text{ and } \sum X(\log)Y = c \sum X + d \sum X^2$$

The Cubic regression model relates peak load (y) and the years (X) in the form given in equation (5)

$$y = a_0 + a_1X + a_2 X^2 \dots\dots\dots (5)$$

While the values of *a*₀ , *a*₁ and *a*₂ can be solved from the equations (6);

$$\begin{aligned}
 a_0 n + a_1 \sum x_i + a_2 \sum x_i^2 &= \sum y_i \\
 a_0 \sum x_i + a_1 \sum x_i^2 + a_2 \sum x_i^3 &= \sum x_i y_i \quad \dots\dots\dots (6) \\
 a_0 \sum x_i^2 + a_1 \sum x_i^3 + a_2 \sum x_i^4 &= \sum x_i^2 y_i
 \end{aligned}$$

Tools used to test the reliability of Models

The Rank of correlation coefficient : The rank of correlation coefficient is a tool for verifying the validity and reliability of a chosen model. It tells how truthful the model can be in its prediction. Its optimal value is unity. It is stated as equation (7)

$$r_{xy} = \frac{\sum x_i y_i - n \bar{x} \bar{y}}{n s_x s_y} = \frac{n \sum x_i y_i - \sum x_i \sum y_i}{\sqrt{n \sum x_i^2 - (\sum x_i)^2} \sqrt{n \sum y_i^2 - (\sum y_i)^2}} \quad \dots\dots\dots (7)$$

Mean absolute percentage error (MAPE) is a measure of accuracy in a fitted time series value in statistical trending. It usually expresses accuracy as a percentage and is expressed as equation (8)

$$M = \frac{1}{n} \sum_{t=1}^n \left| \frac{A_t - F_t}{A_t} \right| \quad \dots\dots\dots (8)$$

where A_t is the actual value and F_t is the forecast value. The least value of M is optimal unlike highest value of rank r (Fung and Tummala, 1993).

Data analysis and results

Tables 1 and 2 show the total load consumption/utility values for Ikorodu and is computed ideally for the entire feeder for residential and non-residential groups. The residential comprises of 14 feeders and the non-residential are spread on 3 feeders. (Note, the mean annual load for each feeder was calculated and then the total for all the feeder is obtained for each year and that gave rise to Table 1 an 2).

The Gross Loads for all years were calculated as the sum of both Residential and non-residential. A gross load presentation (Table 3) that reveals the nature of load growth in Ikorodu with respect to the two groups was made. Knowledge of the growth pattern will assist in making managerial decision during forecasting.

The Prediction of residential load consumption

Presented in Table 4 is the generation of all the values that were used for residential load computation. Appropriate values in table 4 are to be extracted when applying all the models described in section 2.0 in forecasting.

Linear trend method

Recalling that $Y = a + bX$ with constraints as $\sum Y = na + b \sum X$ and $\sum XY = a \sum X + b \sum X^2$

From Table 4, $1814.774 = 5a + 15b$ and $5765.547 = 15a + 55b$. Solving simultaneously, $b = 32.123$, and $a = 266.59$. This builds the main linear expression for the linear model as:

$$Y = 266.59 + 32.123X \dots\dots\dots (9)$$

To check the reliability of this model, we substitute all 1st year to 5th year, and we have for residential;

- Y(2006) = $266.59 + 32.123(1) = 298.713$ (forecast value for 2006)
- Y(2007) = $266.59 + 32.123(2) = 330.836$ (forecast value for 2007)
- Y(2008) = $266.59 + 32.123(3) = 362.959$ (forecast value for 2008)
- Y(2009) = $266.59 + 32.123(4) = 395.082$ (forecast value for 2009)
- Y(2010) = $266.59 + 32.123(5) = 427.205$ (forecast value for 2010)

MAPE Test : To Test this model(i.e. if it is best suitable for this prediction of load for 6th to 10th year which is 2011 to 2015.)

Using MAPE,

$$M = \frac{1}{n} \sum_{t=1}^n \left| \frac{A_t - F_t}{A_t} \right|$$

| Year | Actual Value, At | Forecast Value, Ft. |
|------|------------------|---------------------|
| 1 | 284.79 | 298.713 |
| 2 | 345.035 | 330.836 |
| 3 | 368.374 | 362.959 |
| 4 | 397.31 | 395.082 |
| 5 | 419.265 | 427.205 |

Hence, $M = 0.02585$

The rank of correlation coefficient test: This is stated as

$$r_{xy} = \frac{\sum x_i y_i - n\bar{x}\bar{y}}{n s_x s_y} = \frac{n \sum x_i y_i - \sum x_i \sum y_i}{\sqrt{n \sum x_i^2 - (\sum x_i)^2} \sqrt{n \sum y_i^2 - (\sum y_i)^2}}$$

For this , its r value is (see Table 4 for values) for linear trend;

$$\text{Rank, } r = 0.976941$$

$$\text{Hence, } r^2 = 0.95441$$

The compound-growth model

Having stated that $Y = \text{antilog}(c + dx)$ and the constants c and d can be found by solving the equations below;

$$\Sigma \log Y = nc + d\Sigma X ; \quad \Sigma X(\log)Y = c\Sigma X + d\Sigma X \dots\dots\dots (10)$$

Lifting values from Table 4;

$$12.7803 = 5c + 15d \dots\dots\dots (11)$$

$$38.7381 = 15c + 55d \dots\dots\dots (12)$$

Solving together, $d = 0.03972$ and $c = 2.4363$, inserting into the main expression gives $Y(\text{nth}) = \text{antilog}(2.4363 + 0.03972X) \dots\dots\dots (13)$

Putting the years into the model to get the forecast values

$$\text{1st year} = Y(2006) = \text{antilog}[2.4363 + 0.03972(1)] = \text{antilog}(2.47602) = 299.24$$

$$\text{2nd year} = Y(2007) = \text{antilog}[2.4363 + 0.03972(2)] = 327.899 \approx 327.9$$

$$\text{3rd year} = Y(2008) = \text{antilog}[2.4363 + 0.03972(3)] = 359.302 \approx 359.3$$

$$\text{4th year} = Y(2009) = \text{antilog}[2.4363 + 0.03972(4)] = 393.713 \approx 393.7$$

5th year=Y(2010) = antilog [2.4363 + 0.03972 (5)] = 431.4197 ≈ 431.42

To Test this model(i.e. if it is best suitable for this prediction of load for 6th to 10th year which is 2011 to 2015.) using MAPE. MAPE = 0.0326212. Similarly for the rank of compound-growth $r = 0.998540221$; Hence, $r^2 = 0.99708$.

Cubic model

The model relates the peak load (Y) and the years(X). Using this method: $y = a_0 + a_1 X + a_2 X^2$. While the values of a_0 , a_1 and a_2 can be solved from the equations;

$$a_0 n + a_1 \sum x_i + a_2 \sum x_i^2 = \sum y_i$$

$$a_0 \sum x_i + a_1 \sum x_i^2 + a_2 \sum x_i^3 = \sum x_i y_i$$

$$a_0 \sum x_i^2 + a_1 \sum x_i^3 + a_2 \sum x_i^4 = \sum x_i^2 y_i$$

From Table 4, the values for the variables in the equations above are obtained. The 3 x 3 matrix becomes;

$$5 a_0 + 15 a_1 + 55 a_2 = 1814.774 \dots\dots\dots (14)$$

$$15 a_0 + 55 a_1 + 225 a_2 = 5765.547 \dots\dots\dots (15)$$

$$55 a_0 + 225 a_1 + 979 a_2 = 21818.881 \dots\dots\dots (16)$$

Solving simultaneously, we have that, $a_0 = 231.0958$; $a_1 = 62.5438$; and $a_2 = - 5.0702$. The cubic model is given as

$$Y = 231.0958 + 62.5438 X - 5.0702 X^2 \dots\dots\dots(17)$$

From the 1st year to the 5th year, the predictions are thus;

$$1\text{st year} = Y (2006) = 231.0958 + 62.5438(1) - 5.0702(1)^2 = 288.5694$$

$$2\text{nd year} = Y (2007) = 231.0958 + 62.5438(2) - 5.0702(2)^2 = 335.9026$$

$$3\text{rd year} = Y (2008) = 231.0958 + 62.5438(3) - 5.0702(3)^2 = 373.0954$$

$$4\text{th year} = Y (2009) = 231.0958 + 62.5438(4) - 5.0702(4)^2 = 400.1478$$

$$5\text{th year} = Y(2010) = 231.0958 + 62.5438(5) - 5.0702(5)^2 = 417.0598$$

To test this model for its suitability in predicting residential load for 6th to 10th year which is 2011 to 2015. MAPE = 0.012992; for rank, r^2 of cubic = 0.9877. Having obtained the errors in the three models used, the model whose MAPE value is smallest is the best model to be used to forecast the residential load consumption for the year 2011, 2012, 2013, 2014, and 2015 which is the purpose of this work. In summary, comparison of values of MAPE and Rank, r (residential) are displayed in table 5

Looking at Table 5, the model with least error and high rank is the most important. The Compound-growth model is chosen for predicting the residential load consumption for year 2011 to 2015. This reason is based on the fact that it has the best rank closest to unity (one). Shown in figure 1 is a plot showing the comparison of the results from the three models used in predicting the load consumption for the residential area. Predicting the residential Load consumption using the compound growth model for year 2011 to 2015 symbolized by 6th year to 10th year is given as equation (18)

$$Y(\text{nth}) = \text{antilog}(2.4363 + 0.03972X) \dots \dots \dots (18)$$

$$6\text{th Year} = \text{antilog}[2.4363 + 0.03972(6)] = 472.74 \text{ MW}$$

$$7\text{th Year} = \text{antilog}[2.4363 + 0.03972(7)] = 518.0 \text{ MW}$$

$$8\text{th Year} = \text{antilog}[2.4363 + 0.03972(8)] = 567.62 \text{ MW}$$

$$9\text{th Year} = \text{antilog}[2.4363 + 0.03972(9)] = 621.99 \text{ MW}$$

$$10\text{th Year} = \text{antilog}[2.4363 + 0.03972(10)] = 681.55 \text{ MW}$$

Figure 2 is a plot of the actual and forecast values of residential load consumption using the compound growth.

The prediction of non-residential load consumption

The same procedures of forecasting the residential load consumption were adhered to in forecasting the non-residential load consumption for the case study town Ikorodu. Table 6 presents all the values for the three models used in calculating the non-residential load consumption. Values in Table 6 were calculated based on load consumption in Table 2

Shown in Figure 3 is a plot showing the comparison of values obtained

finally using the three models in predicting the non-residential load consumption. Having compared the MAPE and Rank 'r' values for non-residential load consumption, the linear model was considered the best choice to forecast the non-residential load for year 2011(6thyear), 2012(7th), 2013(8th), 2014(9th) and 2015(10th). The linear model for non-residential load forecast is given in equation (19). This is obtained following similar steps described in section 3.1

$$Y = 144.62 + 11.958 (X)..... (19)$$

Hence,

$$6\text{th year} = Y (2011) = 144.62 + 11.958 (6) = 216.368 \text{ MW}$$

$$7\text{th year} = Y (2012) = 144.62 + 11.958 (7) = 228.326 \text{ MW}$$

$$8\text{th year} = Y (2013) = 144.62 + 11.958 (8) = 240.284 \text{ MW}$$

$$9\text{th year} = Y (2014) = 144.62 + 11.958 (9) = 252.242 \text{ MW}$$

$$10\text{th year} = Y (2015) = 144.62 + 11.958 (10) = 264.2 \text{ MW}$$

Figure 4 is a plot of the actual and forecast values of load consumption for the non-residential area using the linear model. The non-residential consumption is purely industrial load and cannot be vouched for in the sense that there could be increase in production capacity or industrial wind-up due to economic recession. Some industries may or may not be functioning at certain period or might relocate to other parts.

Forecast utilization and planning

Records show that there is no steady increase in number of factories in the case study town Ikorodu per decade, but rather an increase in production capacity. Forecast for the residential and non-residential load are done separately since their load consumption pattern are different. It is observed that the annual growth for the year 2011 to 2015 is steadily decreasing considering the forecast using the linear models. For the compound-growth, the annual growth is staggering, increasing up and decreasing but that does not affect the periodic load growth for each additional year. For example, the residential load is considered to steadily increased from 472.74 MW to 681.55 MW (Fig. 2) without dropping while its complementary growth annually does not follow a specific size of increase. This is very likely for population or city that

has a vague data of actual size of the people, business and industries due to poor machinery and lack of acquisition of statistical data.

There is a high need to consolidate the energy supply to the residential because of the large gap in the load values. This showed that the residential energy demand is almost twice that of the non-residential energy demand . It is not disputable due to the presence of few industries in Ikorodu. Priority should be given to the residents when strategies on energy is deliberated upon or done. There is either none or insignificant commercial consumption in the city.

Conclusion

In this work, the ultimate focus of establishing a basis for the comparison of different load estimates for the existing populace of Ikorodu town in Nigeria and to further provide estimates for future energy requirement of the city has been achieved. Three different models namely, the compound-growth, the linear and cubic were tested and the most suitable were chosen for the two scenarios of the non-residential and residential load consumptions. For a good and reliable forecast to be achieved for a given area or system, it is important to get enough past load trends or have a prior knowledge of what the input and output of the location has been before the forecast. These load trend requirements were obtained from a reliable source, the Power Holding Company of Nigeria (PHCN). Also, it is vital to weigh the authenticity of every chosen model to know that which is best suited for a particular forecast. The validity of the trends used were tested using two methods, namely, Pearson`s Rank of Correlation Coefficient and Mean-Absolute-Percentage-Error (MAPE).

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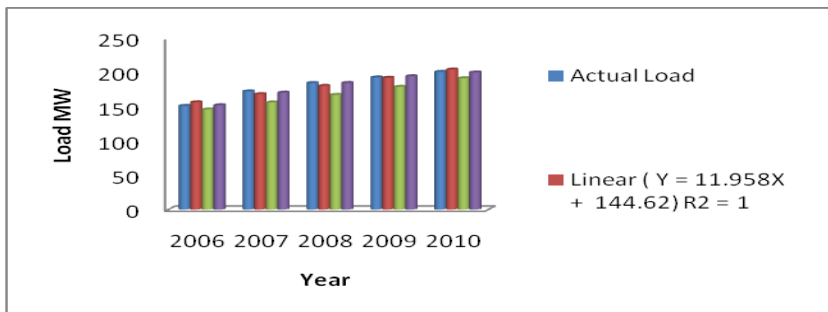


Figure 1: Comparison of results of model predictions for residential area

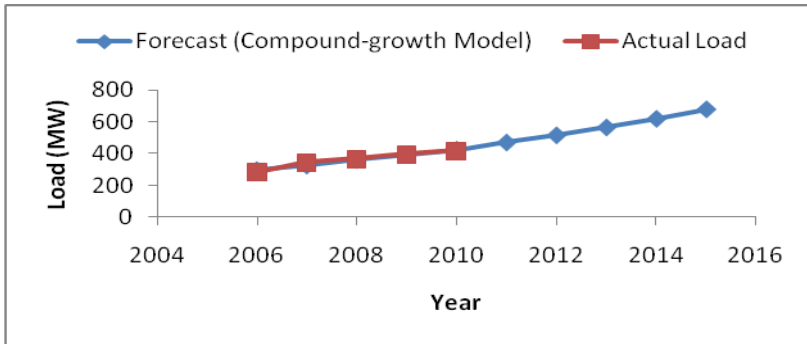


Figure 2: Actual and forecast values of residential load by compound growth model for the next five years.

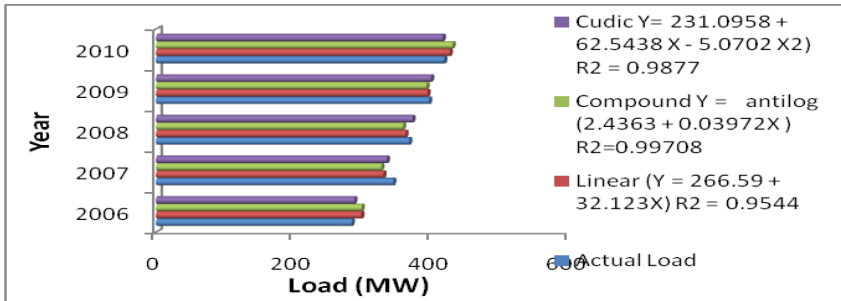


Figure 3: Comparison of results of model predictions for non-residential area

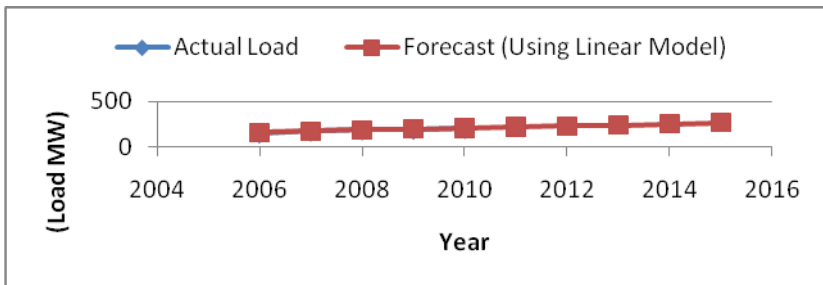


Figure 4: Actual and forecast load values for non-residential area using the linear model for the next five years.

Table 1 Residential load from PHCN load chart

| X (Year) | Y (load)in MW |
|-----------|---------------|
| 1 (2006) | 284.79 |
| 2 (2007) | 345.035 |
| 3 (2008) | 368.374 |
| 4 (2009) | 397.31 |
| 5 (2010) | 419.265 |

Table 2: Non – Residential load from PHCN load chart .

| X (Year) | Y (load) in MW |
|-----------|----------------|
| 1 (2006) | 151.38 |
| 2 (2007) | 172.56 |
| 3 (2008) | 184.673 |
| 4 (2009) | 192.861 |
| 5 (2010) | 201.02 |

Table 3:- Gross Load presentation

| N (X) | Year | Residential Peak Load(Y)in MW | % Annual growth(residential) | Non-residential Peak Load(Y)in MW | % Annual growth(Non-residential) |
|-------|------|-------------------------------|------------------------------|-----------------------------------|----------------------------------|
| 1 | 2006 | 284.79 | -- | 151.38 | -- |
| 2 | 2007 | 345.035 | 21.15 | 172.56 | 13.99 |
| 3 | 2008 | 368.374 | 6.76 | 184.673 | 7.02 |
| 4 | 2009 | 397.31 | 7.86 | 192.861 | 4.43 |
| 5 | 2010 | 417.265 | 5.02 | 201.02 | 4.23 |

Table 4: Table of values for computation of residential prediction for all models

| X(nth year) | Y(load) | X ₂ | Log Y | Xlog Y | XY | X ³ | X ⁴ | X ² Y | n | Y ² |
|--------------------|----------|----------------|---------|---------|---------|----------------|----------------|------------------|-----|----------------|
| 1 | 284.79 | 1 | 2.4545 | 2.4545 | 284.79 | 1 | 1 | 284.79 | | 81105.34 |
| 2 | 345.035 | 4 | 2.5379 | 5.0758 | 690.07 | 8 | 16 | 1380.14 | | 119049.15 |
| 3 | 368.374 | 9 | 2.5663 | 7.6989 | 1105.12 | 27 | 81 | 3315.366 | | 135699.4 |
| 4 | 397.31 | 16 | 2.5991 | 10.3964 | 1589.24 | 64 | 256 | 6356.96 | | 157855.24 |
| 5 | 419.265 | 25 | 2.6225 | 13.1125 | 2096.32 | 125 | 625 | 10481.625 | | 175783.14 |
| Total, $\Sigma=15$ | 1814.774 | 55 | 12.7803 | 38.7381 | 5765.54 | 225 | 979 | 21818.881 | n=5 | 669492.3 |

Table 5: Comparison of values of MAPE and Rank, **r** (Residential) :

| Tool | Linear | Compound-growth | Cubic |
|-----------------------------|----------|-----------------|----------|
| MAPE | 0.025856 | 0.0326212 | 0.012992 |
| Rank , r² | 0.95441 | 0.99708 | 0.9877 |

Table 6: Table of values for computation of non-residential prediction for all models

| X(nth year) | Y(load) | X ² | Log Y | Xlog Y | XY | X ³ | X ⁴ | X ² Y | N |
|--------------------|---------|----------------|---------|--------|---------|----------------|----------------|------------------|-----|
| 1 | 151.38 | 1 | 2.1800 | 2.1800 | 151.38 | 1 | 1 | 151.38 | |
| 2 | 172.56 | 4 | 2.2369 | 4.4738 | 345.12 | 8 | 16 | 690.24 | |
| 3 | 184.673 | 9 | 2.2664 | 6.7992 | 554.019 | 27 | 81 | 1662.057 | |
| 4 | 192.861 | 16 | 2.2852 | 9.1408 | 771.444 | 64 | 256 | 3085.776 | |
| 5 | 201.02 | 25 | 2.3032 | 11.516 | 1005.1 | 125 | 625 | 5025.5 | |
| Total, $\Sigma=15$ | 902.494 | 55 | 11.2717 | 34.109 | 2827.06 | 225 | 979 | 10614.953 | n=5 |