

# African Research Review

---

*An International Multi-Disciplinary Journal, Ethiopia*

*Vol. 4 (3b) July, 2010*

ISSN 1994-9057 (Print)

ISSN 2070-0083 (Online)

---

## **Design for Limit Variability in Quality of Industrial Products: A Case Study of Cutix Cable Manufacturing Company, Nnewi, Nigeria** (Pp. 344-367)

**Ihuez, Chukwutoo Christopher** - Department of Industrial/Production Engineering, Nnamdi Azikiwe University, Awka, Nigeria  
E-mail: ihuezechukwutoo@yahoo.com  
GSM: +2348037065761

### **Abstract**

*This paper presents numerical approaches to assessment of process capability and product quality standards for production processes. One hundred cables produced by cable production process of CUTIX factory were sampled in order to assess their insulation quality. Twenty-five samples were used from batches in stock and each of the samples has sample size of four. The statistical method was used to establish the mean of distribution, the average standard deviation of samples and the average range of samples to establish the process control limits. The distribution of means of samples was normalized in order to ascertain the conformity of the distribution to normal distribution. The distribution was confirmed normal and transformed to standard normal distribution so that the area under the normal curve applies in the analysis of the process distribution. The area under the normal curve of the distribution was evaluated as 0.94 falling within acceptable limit for processes in control. Process control was established using classical relations and analogies to establish process capability and Process capability index. Control charts were developed for the mean and range of samples for the process monitoring period prior to the evaluation of the process population mean as 5.2, the average standard deviation of the process as 0.60, action limits 6.1 and 4.3 for the mean and for the range 2.3*

*and 0.0. The coefficient of variation was found to be 11% (0.11), indicating low variability of process. The CUTIX process for cable production therefore produces within specification. Above all, the probability of any sample observation being in the warning limit is 0.76 while the probability of any sample observation being within the control limit is 0.94, showing that the process is in control.*

**Keywords:** limit variability, quality of products, control limits, process capability

### **Introduction**

It is a common misconception that automatic machines will produce identical components. Unfortunately, real life considerations interfere with this theoretical ideal. The properties of work piece material vary along the length of the bar, the machine tool slide way must have clearances to allow them to move, and lubrication conditions will constantly be changing, and such random variable will mean that the actual size of the parts produced will vary and distributed closely around the target sizes(Black et al,1996). When the quantity of production involved is large, the pattern of variation can be studied on a statistical basis. It then becomes possible to assess the quality achieved by the process without testing every piece produced. A statistical method which reveals the pattern of variation in a product provides a more certain basis for the assessment of the quality of a large volume of work than would be provided by a detailed inspection of some parts made without reference to the pattern of variability present.

Quality failures occur due to various causes. Studies indicated that more than 50% of quality failures are due to human errors at various levels, such as understanding of customer's requirements, manufacturing, inspection, testing, packaging, and design (Sharma 2000).

To meet design and customers specifications for a product for quality assurance, Quality Engineers perform quality tests of a process before mass production. Vonderembse and White(1991) reported that the control of a process begins with the understanding of the variability of the process. Hansen and Ghare(2006) reported that the quality of a product depends upon the application of materials, men, machines and manufacturing conditions.

The process capability and control charts during the base period of manufacture are commonly used to establish the stability of process that will produce effectively. The capability of the process can be established once the

specifications and the standard deviation of process or product parameters are measured.

Classical numerical methods found in Hansen and Ghare(2006) and Stroud (1995) were used as mixed method to find the mean, range, and standard deviation of distribution of measurements from where the process control charts ,process capability and capability index were evaluated and used to specify the quality standard of production process. The statistical interpretation of area under the normal distribution curve is also employed to appraise the production process.

## **Theoretical Background**

### **Normal Distribution**

Continuous random variables and their associated density functions arise whenever our experimental data are defined over a continuous sample space. Therefore, whenever we measure time intervals, weight, height, volumes, and so fourth, our underlying population is described by a continuous distribution (Walpole, 1982). Just as there are several special discrete probability distributions, there are also numerous types of continuous distributions whose graphs may display varying amounts of skewness or in some cases may be perfectly symmetric. Among these, by far the most important is the continuous distribution whose graph is a symmetric bell-shaped curve extending indefinitely in both directions. It is this distribution that provided a basis upon which much of the theory of statistical inference has been developed.

The most important continuous probability distribution in the entire field of statistics is the normal distribution. In 1733, DeMoivre derived the mathematical equation of the normal curve. The normal distribution is often referred to as the Gaussian distribution in honor of Gauss (1777-1855), who also derived its equation from a study of errors in repeated measurements of the same quantity.

### **Basics of a control chart and Hypothesis testing control**

A control charts is designed to be a simple graphical technique to monitor and control a single variable quality characteristic. The objective is to obtain an estimate of the principal parameter that describes the variability and then use the techniques of the hypothesis testing to determine if the process is in control.

If a process is stable, then for all samples, each of consisting of  $n$  items, the average of sample items,  $x^1$  would be consistent with the implied probability distribution: normal with mean  $\mu$  and variance,  $\sigma^2/n$ . On the other hand, finding several values of  $x^1$  to be inconsistent (too large or too small) would give reason to suspect that the process is no longer stable.

For any sample  $j$ ,  $x^{1j}$  can be treated for consisting using the technique of hypothesis testing. If the desired value of the mean is  $\mu_0$ . The null hypothesis would be “process is operating as it should”. Under this the  $\mu$  would be equal to  $\mu_0$ . The alternative hypothesis would be “process is out of control” or mean  $\mu$  is not equal to  $\mu_0$ .

### **Null hypothesis**

$$H_0: \mu = \mu_0 \quad (1)$$

### **Alternative hypothesis**

$$H_1: \mu \neq \mu_0 \quad (2)$$

If the null hypothesis  $H_0$ , is true, the observed values of  $X^1$  would be distributed normal with mean  $\mu$  and standard deviation  $\sigma/\sqrt{n}$ . Consequently, the probability that any  $X^1$  would either be over  $\mu_0 + K\sigma/\sqrt{n}$  or less than  $\mu_0 - K\sigma/\sqrt{n}$  would be the the probability of defects or the capability of the defects(Hansen and Ghare 2003).

### **Hypothesis Testing with control Limits.**

The control limits are decision limits that inform the interpreter when to investigate. The hypothesis is tested accordingly by considering.

### **Null hypothesis**

$$H_0: x^1 = x^{11} \pm 3\sigma/\sqrt{n} \quad (3)$$

### **Alternative hypothesis**

$$H_1: x^1 \neq x^{11} \pm 3\sigma/\sqrt{n} \quad (4)$$

where

$x^1$	=	mean of sample sizes
$x^{11}$	=	grand mean of means(grand mean)
$\sigma$	=	population standard deviation

n	=	sample size
k	=	quality factor
$\mu$	=	population mean

If the probability of any sample is within the control limits,  $\bar{x} \pm 3\sigma/\sqrt{n}$ , the null hypothesis is accepted and the process is in control. If  $H_0$  is accepted, the process is in control.

### **Control Chart during the Base Period**

The base period stability is usually established for a manufacturing process for actual production period by setting up control limits for mean and range of samples. If any of the means,  $\bar{x}^1$  or ranges, R values during the base period (test run) were outside the control limits of a control chart, it would indicate probable lack of stability of the process. When the process is not stable in the base period, the validity of using control limits for the monitoring period would be questionable. Monitoring would require “the control limits that would have been obtained if the process were stable.” If the instability during the period is inherent in the process itself (worn-out equipment, poor quality of materials), it is best to abandon the base period results and reinitiate after the inherent causes have been corrected. If the instability is due to the newness of the product or process and/or the operators not being familiar with the product, the control limits can be derived as follows.

The steps to establish control charts in the base period for the monitoring or mass production period is as outlined below

- Step 1: Start with a base period of at least 25 samples.
- Step 2: Calculate  $\bar{X}^1$  and  $R^1$ . Calculate the control limits.
- Step 3: Check the R chart. If all observations are within control limits, go to step 4. If some observations are outside the upper control limit, remove the corresponding sample from the base period and go to step 2.
- Step 4: Check the  $\bar{x}^1$  chart. If all observations are outside either control limit, remove the corresponding sample from the base period. Otherwise, go to step 2.
- Step 5: Extend the control limits to the monitoring period. Once the control limits are established during the base period, these can be

used during the monitoring period to test the hypothesis “the process is stable .

### **Methodology and Analysis**

One hundred batteries produced by cable production process of CUTIX factory were used in order to access their quality. Twenty five samples were used and each of the samples has sample size of four as in Table1. Sampling method with theory of probability for random variable that is normally distributed is used to find the mean, range, and standard deviation of distribution of measurements from where the process capability ,capability index and control charts for mean and range of samples were evaluated (Sharma,2000, Dieter,2000).

### **Computations for Statistics Data Generation**

The methods of Walpole (1982) and Stroud (1995) were used to establish statistical parameters as follows:

### **Computations for Analysis of Observations**

**Class intervals**,  $c$  is chosen not less than 5 as recommended by Walpole (1982) so that for this study,  $c = 10$

**Class width**  $w$ , is estimated by dividing the range  $R$ , with the class interval so that by

$$R = 6.5 - 4.0 = 2.5, w = R/c = 2.5/10 = 0.25.$$

The class width will never be less than 0.25 and since the same number of significant digits are needed  $w$  is chosen as  $w = 0.3$

**Class boundaries:** The class limits are within the class boundaries. The lower class boundary to contain 4.0 is chosen as  $4.00 - 0.05 = 3.95$  and by adding the estimated class width  $w = 0.3$  to the lower class boundary 3.95 i.e.  $(3.95 + 0.3)$  the upper class boundary is evaluated as 4.25 so that the bottom class boundary is expressed as  $3.95 - 4.25$ . The subsequent class boundaries are therefore established by adding the class width,  $w = 0.3$  to the lower and upper class boundaries as follows:

**3.95– 4.25, 4.25 – 4.55, 4.55 – 4.85, 4.85 – 5.15, 5.15 – 5.45, 5.45 –5.75, 5.75 – 6.05, 6.05 – 6.35,6.35 – 6.65** and presented formally in Table 2.

The listing of the class boundary above shows that 9 class intervals is sufficient for the distribution so that we repeat the computation of the class width as  $w = 2.95/9 = 0.277$ ,

By similar assumptions made above for  $w$ ,  $w = 0.3$

**Class Intervals:** The concern here is to make sure that the class intervals found inside the class boundaries. This involves addition and subtraction of 0.05 on the lower and upper class boundaries respectively to obtain the first bottom class interval giving the lower and upper bottom class limits is hence expressed as:

$$4.0 - 4.2$$

The other class intervals are therefore obtained by adding the class width, 0.3 to the lower class limits of the first interval to obtain the class limits of the first interval. For the other classes, this procedure is continued until the class width is added to  $n - 1$  interval to obtain limits for the final interval  $n$  so that all the intervals are obtained as follows:

$$4.0 - 4.2, 4.3 - 4.5, 4.6 - 4.8, 4.9 - 5.1, 5.2 - 5.4, 5.5 - 5.7, 5.8 - 6.0, 6.1 - 6.3, 6.4 - 6.6 \text{ and presented formally in Table 2.}$$

#### **Establishment of Class mark and its Frequency**

$x$  = lower class boundary plus (+) upper class boundary divided by Two (2), expressed as

$$x = \frac{LCB + UCB}{2}$$

The results of the above computations are presented in Table 2.

#### **Estimation of Population Parameters**

Computation of mean of population,  $x^{11}$  is given as

$$x^{11} = \frac{\sum fx}{\sum f} \quad (5)$$

so that with Table 2 values in (5),  $x^{11} = 131.225/25 = 5.3$

#### **Computation of Variance and Standard deviation**

For grouped data, the computing relation for variance is expressed as

$$s^2 = \frac{(N \sum f_i x_i^2 - (\sum x_i)^2)}{N(N - 1)} \quad (6)$$

so that by substituting Table 3 values in (6) variance,  $s^2 = 0.3206$ , standard deviation,  $\sigma = s = 0.60$  and Coefficient of variation,  $v = \sigma/\bar{x} = 0.11$

### **Computation of Mean of 25 Consecutive Samples**

The mean of samples average,  $\bar{x}$  and mean range,  $R$  is computed with Table 4 as:

#### **Mean of samples average (grand average of samples)**

This is estimated with the relation

$$\bar{x} = \frac{1}{m} \sum x_i \quad (7)$$

where,

$m$  = number of samples,

By using the values of  $x_i$  from Table 4 and  $m = 25$  in (7),  $\bar{x} = 5.3$

#### **Mean range,**

This is also estimated with the relation

$$R = \frac{1}{m} \sum R_i \quad (8)$$

By using the values of  $R_i$  from Table 4 in (8),  $R = .92$

### **Process Control Model and Computations**

Many classical scholars including Basterfield(1986), Mittag and Rinne(1993) and Breyfogles(1992) presented models for estimating allowable limits of a process.

#### **(a) Evaluation of Control limits**

##### **Control Limits for Average**

The Action Limits and Warning limits for sample average are expressed respectively as

$$AL = \bar{x} \pm (3.09\sigma)/\sqrt{n} \quad (9)$$

and

$$WL = \bar{x} \pm (1.96\sigma)/\sqrt{n} \quad (10)$$

where

AL = action limit  
 WL = warning limit

By putting  $\sigma = 0.60$ ,  $\bar{x}^{11} = 5.3$ ,  $n = 4$  in (9) and (10)

The values for action limit and warning limits were obtained as:

$$UAL = \bar{x}^{11} + (3.09\sigma)/\sqrt{n} = 6.2, \quad LAL = \bar{x}^{11} - (3.09\sigma)/\sqrt{n} = 4.4$$

$$UWL = \bar{x}^{11} + (1.96\sigma)/\sqrt{n} = 5.9, \quad LWL = \bar{x}^{11} - (1.96\sigma)/\sqrt{n} = 4.7$$

where

UAL = upper action limit for mean  
 LAL = lower action limit for mean  
 UWL = upper warning limit for mean  
 LWL = lower warning limit for mean

Alternatively, the control limits for mean could be established using the relations

$$\left. \begin{aligned} CL &= \bar{x}^{11} \pm A_2 R^1 \\ UCL_X &= \bar{x}^{11} + A_2 R^1, \quad LCL_X = \bar{x}^{11} - A_2 R^1 \\ UCL_X &= \bar{x}^{11} + A_2 R^1 = 6.0, \quad LCL_X = \bar{x}^{11} - A_2 R^1 = 4.6 \end{aligned} \right\} \quad (11)$$

### **Control Limits for Range**

The upper and lower limits for the range are estimated as:

$$UCL_R = D_4 * R^1 = 2.282 * 0.92 = 2.1, \quad LCL_R = D_3 * R^1 = 0 * 0.92 = 0$$

All mean of the samples in Table 4 are compared with these control limits as follows, no sample is above the UAL<sub>x</sub> of 6.2 and no sample mean is below the LAL<sub>x</sub> of 4.4 by method 1, but by alternative method samples with mean 6.2 are above the UCL<sub>x</sub> of 6.0 and the process is not stable and hence out of control, a new control limit is established for the base period for monitoring period, by removing samples 9, 19 and 7 with mean of 4.5 below the LCL of

4.6. The three samples are dropped and new control limits established as follows:

- $x_2^{11} = \frac{\sum X^1}{\sum f} = (131.225 - (6.2+6.2+4.5))/22 = 5.2$
- $R_2^1 = (23 - (0.6+0.5+0.4))/22 = 0.97 = 1.0$

**By computing New Limits, without samples 7, 9 and 19**

$$UCL_X = x^{11} + A_2 R^1, \quad LCL_X = x^{11} - A_2 R^1$$

$$UCL_X = x^{11} + A_2 R^1 = 5.2 + .729 * 1.0 = 5.9$$

$$LCL_X = x^{11} - A_2 R^1 = 5.2 - 0.729 * 1.0 = 4.5.$$

$$UCL_R = D_4 * R_2^1 = 2.282 * 0.1 = 2.3$$

$$LCL_R = D_3 * R_2^1 = 0 * 1.0 = 0$$

where

$UCL_X$  = upper control limit for mean

$LCL_X$  = lower control limit for mean

$UCL_R$  = upper control limit for range

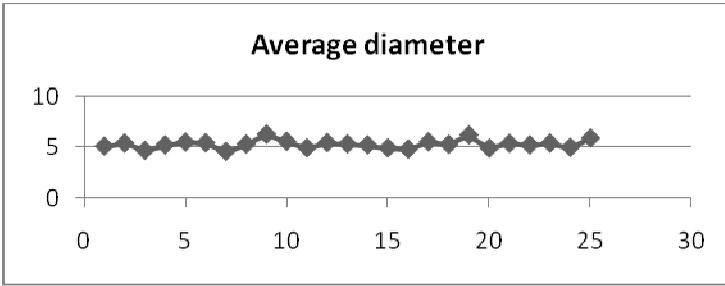
$LCL_R$  = lower control limit for range

$A_2$  = factor to determine 3 times the standard deviation of  $X^1$  from  $R^1$ .

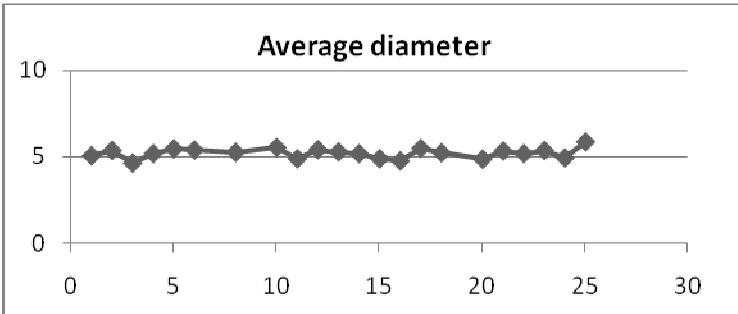
$D_3$  = factor to determine the lower control limit for R chart  
lower control limit for R chart

$D_4$  = factor to determine the upper control limit for R chart.

The process is under control because all the means and ranges are within control. The control charts for means and ranges are then developed for the monitoring period as presented below.

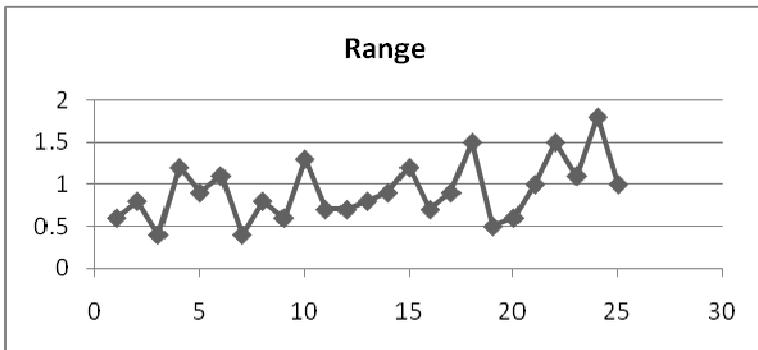


a)

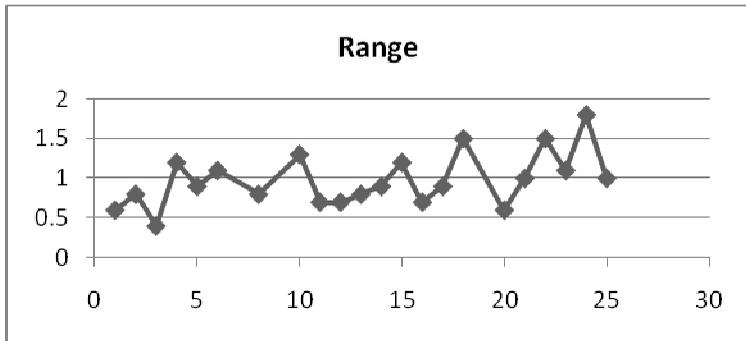


b)

Figure 1 a, b: Base Period Mean control chart for the monitoring period control limits:  $UCLx = 5.9$ ,  $LCLx = 4.5$ .



a)



b)

**Figure 2a, b: Base Period range control chart for the monitoring period for control limits:  $UCL_R = 2.3$ ,  $LCL_R = 0$**

### Process Capability Index and Tolerance Specifications

The relations for predicting the following process specification estimates are as follows:

$$USL = \bar{x}^{11} + 3 \sigma_1 \quad (12)$$

$$LSL = \bar{x}^{11} - 3 \sigma_1 \quad (13)$$

$$C_p = (USL - LSL) / 6 \sigma \quad (14)$$

where,

USL = upper specification limit,  
 LSL = lower specification limit,  
 $C_p$  = process capability index

The process capability index expressed by Dieter (2000) is the ideal or theoretical capability index, because the individual observations may not be centered on the mean, Dieter (2000) gave two relations for predicting the actual process capability index as:

$$C_{pk1} = (\bar{USL} - \bar{x}^{11}) / 3 \sigma \quad (15)$$

$$C_{pk2} = (x^{11} - LSL) / 3\sigma \tag{16}$$

By using  $x^{11} = 5.2$  and the average standard deviation of distribution 0.60,  $C_p$  is obtained as follows:

$$\sigma = 0.60, USL = 6.1, LSL = 4.3, C_p = 1$$

**The Normal Distribution and Transformation to Standard Normal Distribution**

Many physical measurements follow the symmetrical, bell-shaped curve of the normal or

**Gaussian, frequency distribution**

The equation of the normal curve is expressed as:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \tag{17}$$

Where

$f(x)$  = height of the frequency curve corresponding to an assigned value,  $x$

In order to place all normal distribution on a common basis in a standardized way, the normal curve frequently is expressed in terms of normal variable,  $z$  (Dieter 2000) as

$$z = \frac{x-\mu}{\sigma} \tag{18}$$

so that the equation of the standard normal curve becomes

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{-\left(\frac{z}{\sigma}\right)^2} \tag{19}$$

In a standardized normal curve,  $\mu = 0, \sigma = 1$ . The total area under the curve is unity .the relative frequency of a value of  $z$  falling between  $z = -\infty$  and a specified value of  $z$  is given by the total area under the curve. This is found in standard statistics texts. In this study probability method is used

with standard normal curve to appraise the capability of the process under study as follows: By employing the data of Table 2 and equations (a-c) the values of Table 4 are obtained and the graphics of the normal and the standard normal distribution are as presented in Figures 4 and 5 for the production process outputs.

**Probability of Production Within The Intervals: Computation of Areas Bounded By the Normal Curve**

This procedure is adapted from Walpole (1982) and used as presented below. For the normal curve, Cather(1993) specifies the areas for the following ranges as follows:

$$-\sigma \leq z \leq +\sigma = 0.6826(\text{Probability of outcomes in the range}) \quad (20)$$

$$-2 \sigma \leq z \leq +2\sigma = 0.9544(\text{Probability of outcomes in the range}) \quad (21)$$

$$-3 \sigma \leq z \leq +3\sigma = 0.997(\text{Probability of outcomes in the range}) \quad (22)$$

$$-\infty \leq z \leq +\infty = 1.0 (\text{Probability of outcomes in the range}) \quad (23)$$

**Assessing Production Process within the Range of Production of Samples**

The range is given by the outcomes 4.0 and 6.5 so that the area under the normal curve is evaluated as above as:

$$\begin{aligned} \Pr(4.0 \leq X \leq 6.5) &= \Pr(-2.17 \leq Z \leq 2.0) \\ &= \Pr(Z \leq 2.0) - \Pr(Z \leq -2.17) \\ &= 0.9773 - 0.0150 = 0.9623 \equiv 96\% \end{aligned}$$

**Assessing Production Process within  $\pm\sigma(0.60)$**

$$\begin{aligned} \Pr(0.06 \leq Z \leq 0.06) &= \Pr(Z \leq 0.06) - \Pr(Z \leq -0.06) \\ &= 0.7257 - 0.2743 \\ &= 0.4514 \equiv 45\% \end{aligned}$$

**Assessing Production Process within  $\pm 1.96\sigma(1.18)$ : Warning Limit Specification**

$$\Pr(-1.18 \leq Z \leq 1.18) = \Pr(Z \leq 1.18) - \Pr(Z \leq -1.18)$$

$$0.7620 \equiv \begin{aligned} &= 0.8810 - 0.1190 = \\ &76\% \end{aligned}$$

The warning limit is normally set at  $\pm 1.96\sigma$  so that this 76% is the probability of any sample observation being within the warning limit.

**Assessing Production Process within  $\pm 3.09\sigma(1.9)$ : Control Limit Specification**

$$\begin{aligned} \Pr(-1.9 < Z < 1.9) &= \Pr(Z < 1.9) - \Pr(Z < -1.9) \\ &= 0.9713 - 0.0287 = \\ 0.9426 &\equiv 94\%. \end{aligned}$$

**Design Specification and Process Capability**

The three major steps in the production of any item are, design, production and inspection so that the essence of process control is to ensure that the process produces within design specifications.

**Control limits and Specifications**

The control limits are normally set at  $\mu \pm 3\sigma/\sqrt{n}$  of the normal curve so that the upper and lower control limits for any population sample becomes

$$x^1 = \mu \pm 3\sigma/\sqrt{n} \tag{24}$$

so that for sample size  $n = 4$ ,  $\mu = 5.2$  the upper and lower control(specification) limits becomes respectively

$$\begin{aligned} x^1 &= \mu + \frac{3\sigma}{\sqrt{n}} = 5.2 + \frac{3 * 0.60}{\sqrt{4}} = 6.1 = \text{USL} = \text{UCL}_x \\ x^1 &= \mu - \frac{3\sigma}{\sqrt{n}} = 5.2 - \frac{3 * 0.60}{\sqrt{4}} = 4.3 = \text{LSL} \\ &= \text{LCL}_x \end{aligned}$$

**Determination of Process Capability and Defective Process Output**

It may be defined as the range of variation that will include almost all the product coming out of the process. When the assumption of normal probability distribution is valid, 99.73% of product measurements would lie in the range of mean  $\pm 3$  standard deviation (Hansen and Ghare, 2006).

Process Capability is also defined as the best quality attainable or the smallest fraction defective that can be achieved by manipulating the process and is evaluated by the methods of (Hansen and Ghare, 2006)

$$PC = 6\sigma \quad (25)$$

So that by using  $\sigma = 0.60$  in (22)  $PC = 3.6$ . The probability of defective production is evaluated as:

0.23%

where

$$\mu = 5.2, \sigma = 0.60$$

Therefore the best quality or smallest fraction defective is 3.6 or 0.23%, this means that area beyond  $\mu \pm 3\sigma/\sqrt{n}$  is 0.0023, remember that for the standard normal curve  $\mu = 0, \sigma = 1$

### **Validation of Process**

Figure 4 describing process variability and product specification limits and graphics of Figure 5 describing the coverage of a population by multiples of the standard deviation  $\sigma$  from the mean,  $\mu$  show process in control. Table B-1 of Dieter (2000) presents the area under the normal curve as unity as probability of the normally distributed events. The standard deviation and mean of population were used to evaluate the ranges of the normal curve  $\pm \sigma$ ,  $\pm 2\sigma$  and  $\pm 3\sigma$  representing the dispersion of outcome around the mean, warning limits and control limits respectively. The probabilities of the ranges were 0.45, 0.76 and 0.94 respectively and were found to be within the ranges represented by the following probabilities 0.682, 0.9544 and 0.9973 recommended for process in control (Cather, 1993). Figure 4 representing the normal distribution curve of the study, falls within the range  $\pm 3\sigma$  of standard normal curve showing that the process is in control.

The upper and lower production limits were evaluated as 5.9 and 4.5 or 6.1 and 4.3 by different methods as upper and lower specification limits respectively. When Table 5 average values for the mean of samples and graphics of Figure 1b were compared with the control limits for the mean the null hypothesis was accepted since all the average values are within the control limits. Similarly, for the sample ranges all the ranges of samples were within the control limits.

### **Hypothesis Confirmatory Deductions**

The control limits are decision limits that informs the interpreter when to investigate. The hypothesis is tested accordingly by considering,

$$\text{Null Hypothesis } H_0: x^1 = \mu \pm 3\sigma/\sqrt{n} \quad (26)$$

$$\text{Alternative Hypothesis } H_1: x^1 \neq \mu \pm 3\sigma/\sqrt{n} \quad (27)$$

If the probability of any sample is within the control limits,  $x^1 = \mu \pm 3\sigma/\sqrt{n}$ , the null hypothesis is accepted and the process is in

control. If  $H_0$  is accepted, the implication is that the process is in control. When the probability of any class mark of Table 3 is compared with the probability of lying within the upper and lower control limits, evaluated as 6.1 and 4.3 the process was found to be in control, this is estimated with the area under the normal curve as follows:

$$\Pr(-1.5 \leq Z \leq 1.5) = \Pr(Z \leq 1.5) - \Pr(Z \leq -1.5) = 0.9331 - 0.0668 = 0.8662$$

This is the probability of producing within specifications. The 0.8662 is in agreement with 0.94, the area evaluated within the standard normal curve at  $\pm 3\sigma$ , indicating that the process is in control.

### **Discussions of Results**

The variability of the process estimated by measuring the standard deviation of distribution as 0.60 shows that variability of the process is low with the coefficient of variation estimated as 11%. The process capability index and tolerance specification evaluation show that the process variability is low and the  $C_p = 1$ , showing that the process is producing within the upper and lower specification limit evaluated as 6.1 and 4.3.

Table1 shows the 100 measurements of 25 samples of cables taken from a manufacturing stock of CUTIX cable manufacturing Industry of Nnewi, Nigeria in order to asses the process for cable manufacturing. The grouped frequency distribution of table 2 describes the relative sizes of cables within the interval while table-shows the distribution of cables manufactured as normal and standard normal distribution.

Both the graphics of figure-and figure-shows also the normal distribution of cables manufactured. Normal distribution analysis shows that the process produces the range of 4.0 - 6.5. The areas within  $\pm 2 \sigma$  and  $\pm 3 \sigma$  of normal standard normal curve are usually used to specify process limits while this study used the same limits to establish the probabilities of the process working within the limits. Black (1996) specified the areas within  $\pm 1.96 \sigma$  and  $\pm 3.09\sigma$  as 95% and 99.8% of area enclosed, while the mean size is evaluated at  $\pm \sigma$  (68.27% of total area enclosed).

This study found the percentage of the areas enclosed within  $\pm\sigma$ ,  $\pm 1.96 \sigma$  and  $\pm 3.09 \sigma$  of the area under the standard normal curve as 45%, 76% and 95% respectively. This means that of 100 measurements 45, 76 and 94 fell within the limits  $\pm \sigma$ ,  $\pm 1.96 \sigma$  and  $\pm 3.09 \sigma$  respectively.

The upper and lower control limits of the process were evaluated as 5.9 and 4.5 while the upper and lower warning limits were evaluated by classical relations as 5.9 and 4.7. These limit values and percentages of production within the lower and upper control limits as well as the probabilities of producing within the range 4.0-6.5, showing that the process is under control.

Above all, the probability of any sample observation being in the warning limit is 0.76 while the probability of any sample observation being within the control limit is 0.94 while the probability of consistent production is evaluated as 0.8662, showing that the process is in control.

### **Conclusion**

With the capability of process known (or estimated through a preliminary study), jobs can be quality scheduled more efficiently. Of course, knowing the capability of a process will not always eliminate the necessity for using a process that will produce defective products because of stringen specification requirements.

Above all, the process capability index of one(1) of this study and tolerance specification evaluation of this study showing process variability as being low showed that the process is producing within the upper and lower control limits evaluated as 6.1 and 4.3. Since the probability of any sample observation is within the control limit of 0.94 and probability of producing defects is 0.23 indicating that the probability of consistent production to specifications of all ranges is 0.9967, the process of production is hence appropriate and is in control.

**Table 1: CUTIX Cable Diameters**

4.9 4.8 5.1 5.4	5.3 5.8 5.4 5.1	5.2 4.7 5.7 5.8
5.0 5.8 5.3 5.3	4.9 5.3 5.2 5.7	4.0 4.8 5.1 5.8
4.4 4.7 4.8 4.6	5.2 5.4 4.6 5.5	5.3 5.8 6.0 6.3
4.6 5.8 5.4 4.9	5.4 4.8 4.2 5.1	
5.2 5.3 6.1 5.2	4.6 4.4 4.9 5.1	
5.0 5.9 5.8 4.8	5.7 5.4 6.0 4.8	
4.3 4.6 4.7 4.5	5.1 4.3 5.7 5.8	
4.9 4.9 5.5 5.7	5.9 6.4 6.2 6.1	
5.9 6.4 6.1 6.5	5.0 5.1 4.5 4.8	
5.3 5.9 6.1 4.8	4.9 5.9 5.3 5.2	
4.6 4.6 5.3 5.0	5.4 5.9 4.4 5.0	

**Table 2 Class frequency Distribution Data**

Class interval	Class boundaries	Class mark, x	Class frequency, f
4.0-4.2	3.95-4.25	4.1	2
4.3-4.5	4.25-4.55	4.4	7
4.6-4.8	4.55-4.85	4.7	18
4.9-5.1	4.85-5.15	5	20
5.2-5.4	5.15-5.45	5.3	22
5.5-5.7	5.45-5.75	5.6	7
5.8-6.0	5.75-6.05	5.9	15
6.1-6.3	6.05-6.35	6.2	6
6.4-6.6	6.35-6.65	6.5	3

**Table 3 Computation of Variance Data**

Class interval	Class mark, x	Class frequency, f	$x^2$	Fx	$fx^2$
4.0-4.2	4.1	2	16.81	8.2	33.62
4.3-4.5	4.4	7	19.36	30.8	135.52
4.6-4.8	4.7	18	22.09	84.6	397.62
4.9-5.1	5	20	25	100	500
5.2-5.4	5.3	22	28.09	116.6	617.98
5.5-5.7	5.6	7	31.36	39.2	219.52
5.8-6.0	5.9	15	34.81	88.5	522.15
6.1-6.3	6.2	6	38.44	37.2	230.64
6.4-6.6	6.5	3	42.25	19.5	126.75
Sum		100	258.21	524.6	2783.8

**Table 4 Samples Data for Evaluation of Control Limits in the Base Period**

Sample number	Individual diameters				Total diameters	Average diameter	Range
1	4.9	4.8	5.1	5.4	20.2	5.05	0.6
2	5	5.8	5.3	5.3	21.4	5.35	0.8
3	4.4	4.7	4.8	4.6	18.5	4.625	0.4
4	4.6	5.8	5.4	4.9	20.7	5.175	1.2
5	5.2	5.3	6.1	5.2	21.8	5.45	0.9
6	5	5.9	5.8	4.8	21.5	5.375	1.1
7	4.3	4.6	4.7	4.5	18.1	4.525	0.4
8	4.9	4.9	5.5	5.7	21	5.25	0.8
9	5.9	6.4	6.1	6.5	24.9	6.225	0.6
10	5.3	5.9	6.1	4.8	22.1	5.525	1.3
11	4.6	4.6	5.3	5	19.5	4.875	0.7
12	5.3	5.8	5.4	5.1	21.6	5.4	0.7
13	4.9	5.3	5.2	5.7	21.1	5.275	0.8
14	5.2	5.4	4.6	5.5	20.7	5.175	0.9
15	5.4	4.8	4.2	5.1	19.5	4.875	1.2
16	4.6	4.4	4.9	5.1	19	4.75	0.7
17	5.7	5.4	6	4.8	21.9	5.475	0.9
18	5.1	4.3	5.7	5.8	20.9	5.225	1.5
19	5.9	6.4	6.2	6.1	24.6	6.15	0.5
20	5	5.1	4.5	4.8	19.4	4.85	0.6
21	4.9	5.9	5.3	5.2	21.3	5.325	1
22	5.4	5.9	4.4	5	20.7	5.175	1.5
23	5.2	4.7	5.7	5.8	21.4	5.35	1.1
24	4	4.8	5.1	5.8	19.7	4.925	1.8
25	5.3	5.8	6	6.3	23.4	5.85	1
Subtotal						131.225	23

**Table 5 Process in Control Samples for the Monitoring Period**

Sample number	Individual diameters				Total diameters	Average diameter	Range
1	4.9	4.8	5.1	5.4	20.2	5.05	0.6
2	5	5.8	5.3	5.3	21.4	5.35	0.8
3	4.4	4.7	4.8	4.6	18.5	4.625	0.4
4	4.6	5.8	5.4	4.9	20.7	5.175	1.2
5	5.2	5.3	6.1	5.2	21.8	5.45	0.9
6	5	5.9	5.8	4.8	21.5	5.375	1.1
8	4.9	4.9	5.5	5.7	21	5.25	0.8
10	5.3	5.9	6.1	4.8	22.1	5.525	1.3
11	4.6	4.6	5.3	5	19.5	4.875	0.7
12	5.3	5.8	5.4	5.1	21.6	5.4	0.7
13	4.9	5.3	5.2	5.7	21.1	5.275	0.8
14	5.2	5.4	4.6	5.5	20.7	5.175	0.9
15	5.4	4.8	4.2	5.1	19.5	4.875	1.2
16	4.6	4.4	4.9	5.1	19	4.75	0.7
17	5.7	5.4	6	4.8	21.9	5.475	0.9
18	5.1	4.3	5.7	5.8	20.9	5.225	1.5
20	5	5.1	4.5	4.8	19.4	4.85	0.6
21	4.9	5.9	5.3	5.2	21.3	5.325	1
22	5.4	5.9	4.4	5	20.7	5.175	1.5
23	5.2	4.7	5.7	5.8	21.4	5.35	1.1
24	4	4.8	5.1	5.8	19.7	4.925	1.8
25	5.3	5.8	6	6.3	23.4	5.85	1
Subtotal						114.325	21.5

**Table 5 . Normal and standard normal distribution data**

Interval	Class mark	z	f(x)	f(z)
1	4.10	-2.00	0.05	0.03
2	4.40	-1.50	0.12	0.07
3	4.70	-1.00	0.23	0.14
4	5.00	-0.50	0.33	0.20
5	5.30	0.00	0.34	0.23
6	5.60	0.50	0.33	0.20
7	5.90	1.00	0.23	0.14
8	6.20	1.50	0.12	0.07
9	6.50	2.00	0.05	0.03

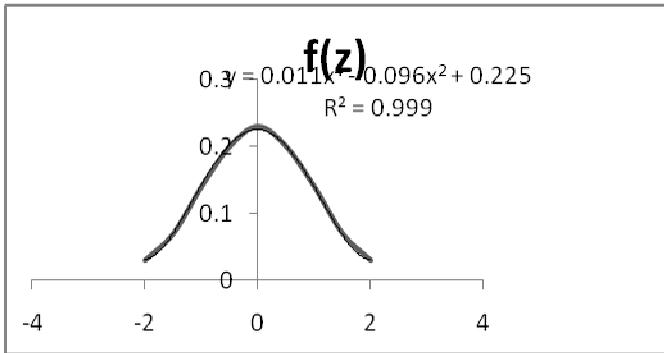
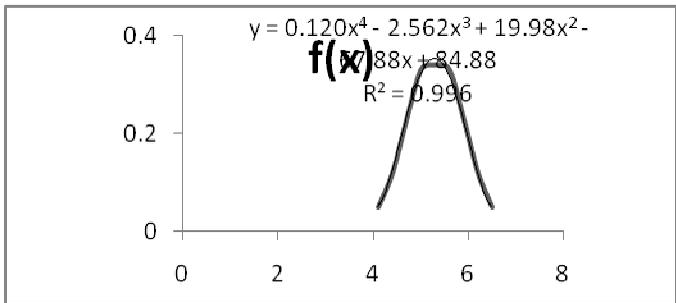


Figure 4a,b Normal and standard normal distribution of cable diameters

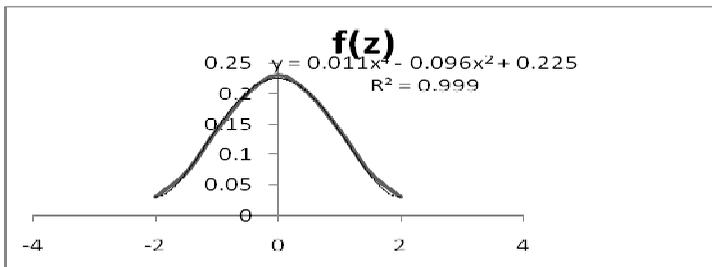


Figure 5 Area properties of a normal curve:

### References

- Black; S.C., Chiles, V, Lissaman, A. J and Martin, S. J. (1996). *Principles of Engineering Manufacture*, (3<sup>rd</sup> edn.). London: ARNOLO Publishers.
- Sharma, S.C. (2000). *Materials Management and Materials Handling* 3<sup>rd</sup> edn. Delhi: Khanna Publishers,
- Vonderembse, M. A. and White, G.P (1991). *Operations Management Concepts: Methods and Strategies*. 2<sup>nd</sup> ed. New York: West Publishing Company.
- Hansen, B.L. and Ghare, P. M (2006). *Quality control and Application*. New Delhi: Prentice-Hall of India Private Limited.
- Walpole, E. (1982). *Introduction to Statistics*. 3<sup>rd</sup> edn. Macmillan Publishing New York: Co. Inc.
- Dieter, G.E. (2000) *Engineering Design: A Materials and Processing Approach*, 3ed, McGraw-Hill International Edition Mechanical Engineering series,
- Stroud, K. A. (1995). *Engineering Mathematics*, 4<sup>th</sup> edn. London: McMillan Press limited.
- Basterfield, D. H. (1986). *Quality Control*, Prentice Hall, Inc. London.
- Mittag, H.J and Rinne, H, (1993). *Statistical Methods of Quality Control*. London: Chapman and Hall Publishers,
- Breyfogles, F.W. (1992). *Statistical Methods for Testing, Development and Manufacture*. John Wiley and sons Inc.
- Cather,H.L (1993) *Contol of Quality*. In Dal. Koshal (ed).*Manufacturing Engineers Reference book* . Butterworth-Heinemann Publisher.