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Improved Randomized Response Technique for Two Sensitive Attributes

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Abstract. We proposed new and more efficient estimators for estimating population proportion of respondents belonging to two related sensitive attributes in survey sampling by extending the work of Mangat(1994). Our proposed estimators are more efficient than Lee *et al* (2013) simple and crossed model estimators as the population proportion of possessing the sensitive attribute increases.

Résumé. Ce travail concerne l'estimation des proportions de répondants classés par rapport à deux attributs de sensibilité dans une enquête. Nous proposons des nouveaux estimateurs qui se trouvent être plus performants que ceux de Mangat(1994) dont nos résultats sont un extension. Nos résultats sont aussi plus performants que ceux de Lee et al. (2013) aussi bien dans le cas simple que dans le cas croisé.

Key words: Efficiency, Proportion Estimation, Randomized Response Techniques, Reliable Information, Unbiased Estimation.
AMS 2010 Mathematics Subject Classification : 62D05, 62G05

1. Introduction

Reliability of data is compromised when sensitive topics on embarrassing or illegal acts such as drunk driving, abortion, alcoholism, illicit drugs usage, tax evasion, illegal possession of arms are required in direct method of data collection in sample survey. Of a major concern is the impact of the response distortion on the survey or test results. Surveys on human population have established the fact that the direct question about sensitive characters often result in either refusal to respond or falsification of the answer [Sidhu *et al.* (2009)]. However, obtaining valid and reliable information is a prerequisite for obtaining meaningful

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results. Hence, there is need to ensure confidentiality of respondents which will in-turn lead to more reliable information. Warner (1965) developed an interviewing procedure designed to reduce or eliminate this bias and called it Randomized Response Technique (RRT). This allows researchers to obtain sensitive information while guaranteeing respondents' privacy. Many works have been done to improve the strategy introduced by Warner (1965), see Bouza *et al.* (2010) for review.

Mangat (1994) proposed a strategy in which the respondent is instructed to say *yes* if he/she belongs to an attribute A. If not, he/she is required to use the Warner (1965) randomized device consisting of two statements:

– I belong to attribute A with probability P
– I do not have attribute A with probability 1 - P

The probability of a *yes* answer for the procedure is given by:

$$\alpha = \pi + (1 - \pi)(1 - P)$$

where π is the population proportion of respondents possessing the sensitive attribute. The proposed unbiased Mangat (1994) estimator for π is:

$$\hat{\pi}_M = \frac{\hat{\alpha} - 1 + P}{P}$$

where $\hat{\alpha}$ is the observed proportion of *yes* answers obtained from sampled individuals. The variance of the estimator is given as:

$$V(\hat{\pi}_M) = \frac{\hat{\alpha}(1 - \hat{\alpha})}{(n-1)P^2}$$

where n is the sample size.

In order to improve the works of Warner (1965) and Odumade and Singh (2009) suggested a new randomized model using two decks of cards. Each of these decks of cards is the same as in Warner (1965) model with varied probabilities. All the respondents were made to go through the device twice for a single attribute. Recently, Batool *et al.* (2015) modified Abdelfatah *et al.* (2013) procedure by using six decks of cards and went further to show that Warner (1965), Mangat and Singh (1990), Odumade and Singh (2009) and Abdelfatah *et al.* (2013) are special cases of their proposed procedure. Other works in this regards include Bouza *et al.* (2010), Singh and Sedory (2012), Song and Kim (2012).

It is important to note that Warner (1965)'s and others' designs can only capture one attribute at a time and many potential information were lost. This is a limitation, as no researcher sets out to collect only one item in a survey but multiple items. Therefore, the research work focuses on providing models for measuring hidden respondent characteristics on two sensitive items and their interaction and also investigate the performance of the developed strategies.

Christofides (2005) developed a new strategy of estimating the proportion of individuals having two sensitive characteristics at the same time. Lee *et al.* (2013) proposed a simple model which is a special case of Christofides (2005)with probabilities of (yes,yes), (yes,no), (no,yes) and (no,no) as denoted by $\theta_{11}, \theta_{10}, \theta_{01}$ and θ_{00} respectively where

$$\theta_{11} + \theta_{10} + \theta_{01} + \theta_{00} = 1$$

$$\begin{aligned}
\theta_{11} &= (2P-1)(2T-1)\pi_{AB} + (2P-1)(1-T)\pi_A + (1-P)(2T-1)\pi_B + (1-P)(1-T), \\
\theta_{10} &= -(2P-1)(2T-1)\pi_{AB} + (2P-1)T\pi_A + (1-P)(2T-1)\pi_B + (1-P)T, \\
\theta_{01} &= -(2P-1)(2T-1)\pi_{AB} + (2P-1)(1-T)\pi_A + P(2T-1)\pi_B + P(1-T) \\
\theta_{00} &= (2P-1)(2T-1)\pi_{AB} + (2P-1)T\pi_A + P(2T-1)\pi_B + PT.
\end{aligned}$$

The unbiased estimators of the population proportion are given by:

$$\hat{\pi}_{A(SM)} = \frac{\hat{\theta}_{11} + \hat{\theta}_{10} - \hat{\theta}_{01} - \hat{\theta}_{00} + (2P-1)}{2(2P-1)},$$

$$\hat{\pi}_{B(SM)} = \frac{\hat{\theta}_{11} - \hat{\theta}_{10} + \hat{\theta}_{01} - \hat{\theta}_{00} + (2T-1)}{2(2T-1)}$$

$$\hat{\pi}_{AB(SM)} = \frac{\hat{\theta}_{11}(P+T) + \hat{\theta}_{10}(T-P) + \hat{\theta}_{01}(P-T) + \hat{\theta}_{00}(2-P-T) - T(1-P) - P(1-T)}{2(2P-1)(2T-1)},$$

for $P \neq 0.5$ and $T \neq 0.5$ where $\hat{\theta}_{11} = \frac{n_{11}}{n}, \hat{\theta}_{10} = \frac{n_{10}}{n}, \hat{\theta}_{01} = \frac{n_{01}}{n}$, and $\hat{\theta}_{00} = \frac{n_{00}}{n}$.

The variances of the estimators are given as:

$$v(\hat{\pi}_{A(SM)}) = \frac{\pi_A(1-\pi_A)}{n} + \frac{P(1-P)}{n(2P-1)^2}$$

$$v(\hat{\pi}_{B(SM)}) = \frac{\pi_B(1-\pi_B)}{n} + \frac{T(1-T)}{n(2T-1)^2}$$

and

$$v(\hat{\pi}_{AB(SM)}) = \frac{\pi_{AB}(1-\pi_{AB})}{n} + \frac{(2P-1)^2T(1-T)\pi_A + P(1-P)(2T-1)^2\pi_B + PT(1-P)(1-T)}{n(2P-1)^2(2T-1)^2},$$

for $P \neq 0.5$ and $T \neq 0.5$.

Lee *et al.* (2013) also proposed new model called *Crossed Model*. This they established to be more efficient than the simple model. Perri *et al.* (2015) applied the crossed model in estimating induced abortion and foreign irregular presence in Calabria, Italy.

The unbiased estimators of the population proportions π_A, π_B, and π_{AB} for the crossed model are given as:

$$\hat{\pi}_{A(CM)} = \frac{1}{2} + \frac{(T-P+1)(\hat{\theta}^*_{11} - \hat{\theta}^*_{00}) + (P+T-1)(\hat{\theta}^*_{10} - \hat{\theta}^*_{01})}{2(P+T-1)}$$

$$\hat{\pi}_{B(CM)} = \frac{1}{2} + \frac{(P-T+1)(\hat{\theta}^*_{11} - \hat{\theta}^*_{00}) + (P+T-1)(\hat{\theta}^*_{01} - \hat{\theta}^*_{10})}{2(P+T-1)}$$

and

$$\hat{\pi}_{AB(CM)} = \frac{PT\hat{\theta}^*_{11} - (1-P)(1-T)\hat{\theta}^*_{00}}{[PT + (1-P)(1-T)](P+T-1)}$$

for $P + T \neq 1$

The variances of the estimators for the crossed model are given as:

$$v(\hat{\pi}_{A(CM)}) = \frac{\pi_A(1-\pi_A)}{n} + \frac{(1-P)[T[PT + (1-P)(1-T)](1 - \pi_A - \pi_B + 2\pi_{AB})]}{n(P+T-1)^2}$$

and

$$v(\hat{\pi}_{B(CM)}) = \frac{\pi_B(1-\pi_B)}{n} + \frac{(1-T)[P[PT + (1-P)(1-T)](1 - \pi_A - \pi_B + 2\pi_{AB})]}{n(P+T-1)^2},$$

$$\begin{aligned} V(\hat{\pi}_{AB(CM)}) \;=\; & \frac{\pi_{AB}(1-\pi_{AB})}{n} + \frac{1}{n\{PT + (1-P)(1-T)\}(P+T-1)^2} \\ & \times \left[\pi_{AB}\{P^2T^2 + (1-P)^2(1-T)^2 - \{PT + (1-P)(1-T)(P+T-1)^2\} \right. \\ & \left. +\pi_{AB}(1-\pi_{AB})\right] \end{aligned}$$

for $P + T \neq 1$.

2. Proposed Design

We consider selecting a sample from a finite population using simple random sample with replacement. Two related sensitivity questions "A" and "B" are posed at each respondent in order to estimate proportion of respondents belonging to character "A" or "B" or "AB". Let population proportion of respondents belonging to character A, B or AB be π_A, π_B, and π_{AB} respectively. A procedure similar to that of Mangat (1994) strategy is being presented in which each respondent is instructed to answer "yes" if he or she belongs to attribute/character "A". If not, he or she is required to draw a card from deck I of cards containing two statements:

- I belong to character A with probability P
- I do not belong to character A with probability 1-P

and answer "yes" or "no" accordingly without reporting the statement on the card to the interviewer. Also, the respondent proceeds to the next stage by answering "yes" if he or she belongs to character "B". If not, he or she is required to draw another card from deck II of cards containing either of the two statements:

- I belong to character B with probability λ
- I do not belong to character B with probability 1-λ

Respondent answer *yes* or *no* accordingly without reporting the statement on the card to the interviewer.

The observed responses is being categorized into four different places: n_{11}, n_{10}, n_{01} and n_{00} where $n_{11} + n_{10} + n_{01} + n_{00} = n$ i.e $\Sigma_{i=0}^{1}\Sigma_{i=0}^{1} n_{ij} = n$.

Therefore we represent the true probabilities of (yes, yes), (yes, no), (no, yes), and (no, no) with θ_{11}, θ_{10}, θ_{01} and θ_{00} respectively where $\theta_{11} + \theta_{10} + \theta_{01} + \theta_{00} = 1$ i.e $\sum_{i=0}^{I}\sum_{j=0}^{J}\theta_{ij} = 1$.

Note that, the observed probabilities are $\hat{\theta}_{11} = \frac{n_{11}}{n}, \hat{\theta}_{10} = \frac{n_{10}}{n}, \hat{\theta}_{01} = \frac{n_{01}}{n}$, and $\hat{\theta}_{00} = \frac{n_{00}}{n}$.

Using the proposed procedure, we have:

$$\theta_{11} = \alpha_1\pi_{AB} + \alpha_2\pi_A + \alpha_3\pi_B + \alpha_4 \tag{1}$$

$$\theta_{10} = -\alpha_1\pi_{AB} + \alpha_1\pi_A - \alpha_3\pi_B + \alpha_3 \tag{2}$$

$$\theta_{01} = -\alpha_1\pi_{AB} - \alpha_2\pi_A + \alpha_1\pi_B + \alpha_2 \tag{3}$$

$$\theta_{00} = \alpha_1\pi_{AB} - \alpha_1\pi_A - \alpha_1\pi_B + \alpha_1 \tag{4}$$

where $\alpha_1 = P\lambda, \alpha_2 = P(1-\lambda), \alpha_3 = (1-P)\lambda, \alpha_4 = (1-P)(1-\lambda)$

The distance between the observed and the true probability is minimized using the following expression:

$$\phi = \frac{1}{2}\Sigma_{i=0}^{1}\Sigma_{j=0}^{1}(\theta_{ij} - \hat{\theta}_{ij})^2 \tag{5}$$

We further differentiate ϕ with respect to π_A, π_B, and π_{AB} and equate to zero, then solve simultaneously in order to obtain unbiased estimators of π_A, π_B, and π_{AB} .

Theorem 1. *The proposed unbiased estimators of π_A, π_B, and π_{AB} are given by:*

$$\hat{\pi}_{A(EA)} = \frac{\hat{\theta}_{11} + \hat{\theta}_{10} - \hat{\theta}_{01} - \hat{\theta}_{00} + (2p-1)}{2p} \tag{6}$$

$$\hat{\pi}_{B(EA)} = \frac{\hat{\theta}_{11} - \hat{\theta}_{10} + \hat{\theta}_{01} - \hat{\theta}_{00} + (2\lambda-1)}{2\lambda} \tag{7}$$

$$\begin{aligned} \hat{\pi}_{AB(EA)} = & \Big[\hat{\theta}_{11}(2p+2\lambda-1) - \hat{\theta}_{10}(2p-2\lambda+1) + \hat{\theta}_{01}(2p-2\lambda-1) \\ & -\hat{\theta}_{00}(2p+2\lambda-3) + (2p-1)(2\lambda-1)\Big]\frac{1}{4p\lambda} \end{aligned} \tag{8}$$

for p, $\lambda > 0$

Lemma 1. *The variance of $\hat{\pi}_A$, $\hat{\pi}_B$, and $\hat{\pi}_{AB}$ can be obtained using the following:*

$$\begin{gathered} V(x_{11}) = \theta_{11}(1-\theta_{11}), V(x_{10}) = \theta_{10}(1-\theta_{10}) \\ V(x_{01}) = \theta_{01}(1-\theta_{01}), V(x_{00}) = \theta_{00}(1-\theta_{00}) \\ c(x_{11}, x_{10}) = \theta_{11}\theta_{10}, c(x_{11}, x_{01}) = \theta_{11}\theta_{01}, c(x_{11}, x_{00}) = \theta_{11}\theta_{00} \\ c(x_{10}, x_{01}) = \theta_{10}\theta_{01}, c(x_{10}, x_{00}) = \theta_{10}\theta_{00}, c(x_{01}, x_{00}) = \theta_{01}\theta_{00}, \end{gathered}$$

where V and C are the operators of variance and covariance over the randomized response device respectively. And also;

x_{11} *is obtained when we have yes response for character A and yes response for character B*
x_{10} *is obtained when we have yes response for character A and no response for character B*
x_{01} *is obtained when we have no response for character A and yes response for character B*
x_{00} *is obtained when we have no response for character A and no response for character B*

Theorem 2. *The variances of $\hat{\pi}_A$, $\hat{\pi}_B$, and $\hat{\pi}_{AB}$ are given respectively as:*

$$v(\hat{\pi}_{A(EA)}) = \frac{\pi_A[(2p-1) - p\pi_A] + (1-p)}{np} \tag{9}$$

$$v(\hat{\pi}_{B(EA)}) = \frac{\pi_B[(2\lambda-1) - \lambda\pi_B] + (1-\lambda)}{n\lambda} \tag{10}$$

$$\begin{aligned} v(\hat{\pi}_{AB(EA)}) \;=\; & [\pi_{AB}[(2p-1)(2\lambda-1) - p\lambda\pi_{AB}] + (2p-1)(1-\lambda)\pi_A \\ & +(1-p)(2\lambda-1)\pi_B + (1-p)(1-\lambda)]\,\frac{1}{np\lambda}, \end{aligned} \tag{11}$$

for $p \neq 0$ *and* $\lambda \neq 0$

3. Efficiency Comparison

The proposed estimators $\hat{\pi}_{A(EA)}, \hat{\pi}_{B(EA)}$, and $\hat{\pi}_{AB(EA)}$ will be more efficient than estimators (simple model) $\hat{\pi}_{A(SM)}, \hat{\pi}_{B(SM)}$, and $\hat{\pi}_{AB(SM)}$ which are due to Lee *et al.* (2013);

$$\pi_A > \frac{(1-3p)(1-p)}{(2p-1)^2}, \tag{12}$$

for $p \neq 1/2$,

$$\pi_B > \frac{(1-3\lambda)(1-\lambda)}{(2\lambda-1)^2} \tag{13}$$

for $\lambda \neq 1/2$, and

$$\begin{aligned} \pi_{AB} > & \\ & \frac{(2p-1)^2(1-\lambda)\pi_A[(2p-1)(2\lambda-1)^2 - p\lambda^2] + (1-p)(2\lambda-1)^2\pi_B[(2p-1)^2(2\lambda-1) - p^2\lambda]}{(2p-1)^2(2\lambda-1)^2[p\lambda-(2p-1)(2\lambda-1)]} \\ & + \frac{(1-p)(1-\lambda)[(2p-1)^2(2\lambda-1)^2 - p^2\lambda^2]}{(2p-1)^2(2\lambda-1)^2[p\lambda-(2p-1)(2\lambda-1)]}, \end{aligned} \tag{14}$$

$p = \lambda \neq 1/2$.

It is important to note that $\pi_{A(EA)}$ is the estimator of π_A we propose here while $\pi_{A(SM)}$ and $\pi_{A(CM)}$ are the simple and crossed model estimators of π_A respectively as proposed by Lee *et al.* (2013). These are applicable to other estimators.

Hence, we obtain the relative efficiency of the proposed estimators $\hat{\pi}_{A(EA)}$, $\hat{\pi}_{B(EA)}$ and $\hat{\pi}_{AB(EA)}$ with respect to the estimators $\hat{\pi}_{A(SM)}$, $\hat{\pi}_{B(SM)}$ and $\hat{\pi}_{AB(SM)}$ respectively:

$$RE(\hat{\pi}_{A(EA)}, \hat{\pi}_{A(SM)}) = \frac{MSE(\hat{\pi}_{A(SM)})}{MSE(\hat{\pi}_{A(EA)})}, \tag{15}$$

$$RE(\hat{\pi}_{B(EA)}, \hat{\pi}_{B(SM)}) = \frac{MSE(\hat{\pi}_{B(SM)})}{MSE(\hat{\pi}_{B(EA)})}, \tag{16}$$

and

$$RE(\hat{\pi}_{AB(EA)}, \hat{\pi}_{AB(SM)}) = \frac{MSE(\hat{\pi}_{AB(SM)})}{MSE(\hat{\pi}_{AB(EA)})} \tag{17}$$

4. Application

We set p = 0.6 and $\lambda = 0.7$ in order to ensure moderate confidentiality and obtain reliable information from respondents. We fixed $\pi_{AB} < \pi_A$, $\pi_{AB} < \pi_B$ and $\pi_A + \pi_B < 0.99$; the value of π_{AB} were also fixed at 0.05, 0.1 and 0.2 as proposed by Lee *et al.* (2013) while π_A and π_B were changed from 0.1 to 0.9 with a step of 0.1. It is important to note that the sample size does not influence the relative efficiency.

The result of the analysis shows that the proposed estimators $\hat{\pi}_{A(EA)}$, $\hat{\pi}_{B(EA)}$ and $\hat{\pi}_{AB(EA)}$ performed better than Lee *et al.* (2013) simple and crossed model estimators$\hat{\pi}_{A(SM)}$, $\hat{\pi}_{B(SM)}$ and $\hat{\pi}_{AB(SM)}$; and $\hat{\pi}_{A(CM)}$, $\hat{\pi}_{B(CM)}$ and $\hat{\pi}_{AB(CM)}$ respectively under the conditions stated above. The summary is presented in the figure and tables in the Appendix

5. Discussion

In computing relative efficiency of the proposed estimators over Lee *et al.* (2013), equations above were used. It was observed that the relative efficiency of $RE(\hat{\pi}_{A(EA)}, \hat{\pi}_{A(SM)})$ and $RE(\hat{\pi}_{B(EA)}, \hat{\pi}_{B(SM)})$ increases with increase in the values of π_A and π_B respectively. Also, we observed that there is an insignificant drop in $RE(\hat{\pi}_{A(EA)}, \hat{\pi}_{A(SM)})$ as π_{AB} value increases from 0.05 to 0.1 while a noticeable increase occurs in the relative efficiency, $RE(\hat{\pi}_{A(EA)}, \hat{\pi}_{A(SM)})$ as the value of π_{AB} increases from 0.1 to 0.2. For the crossed model, it was observed that as the value of π_{AB} increases from 0.05 to 0.2, the proposed estimators perform better. This implies that as the proposed method captures more people bearing the joint sensitive characters, the estimators become more efficient.

The crossed model by Lee *et al.* (2013) was adjudged to be better than the simple model which is a special case of Christofides (2005). Also Perri *et al.* (2015) application of crossed model appears encouraging as reported by them but the authors saw need to re-define it.

6. Conclusion

We have presented new and more efficient estimators of estimating proportion of people having two related sensitive attributes by extending the work of Mangat (1994). The proposed estimators perform better than previous ones and they become more efficient as the design captures more people possessing the sensitive attributes.

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Appendix : Tables and figures

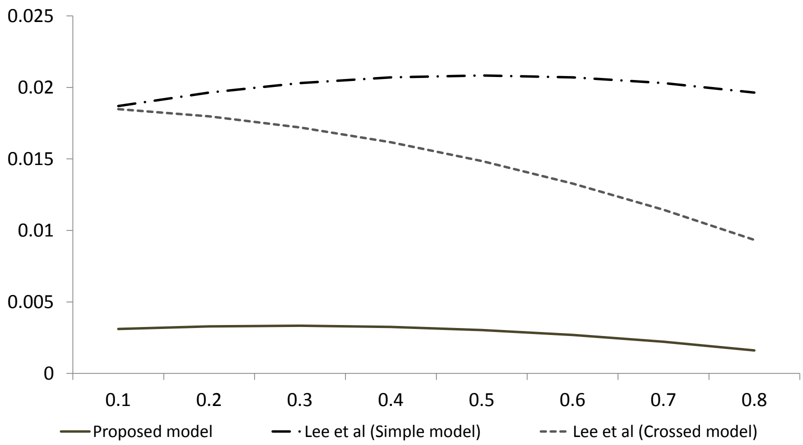

Fig. 1. Comparison of the variances of the estimators

Table 1. Relative efficiency of the proposed estimators with respect to Lee *et al.* (2013) simple model estimators

π_A	π_B	$RE(\hat{\pi}_{A(EA)}, \hat{\pi}_{A(SM)})$	$RE(\hat{\pi}_{B(EA)}, \hat{\pi}_{B(SM)})$	π_{AB}	$RE(\hat{\pi}_{AB(EA)}, \hat{\pi}_{AB(SM)})$
0.1	0.1	24.52	6.02	0.05	25.07
0.1	0.2	24.52	5.98	0.05	24.15
0.1	0.3	24.52	6.09	0.05	23.39
0.1	0.4	24.52	6.37	0.05	22.75
0.1	0.5	24.52	6.87	0.05	22.22
0.1	0.6	24.52	7.70	0.05	21.76
0.1	0.7	24.52	9.18	0.05	21.36
0.1	0.8	24.52	12.23	0.05	21.01
0.2	0.1	24.68	6.02	0.05	24.44
0.2	0.2	24.68	5.98	0.05	23.61
0.2	0.3	24.68	6.09	0.05	22.92
0.2	0.4	24.68	6.37	0.05	22.35
0.2	0.5	24.68	6.87	0.05	21.85
0.2	0.6	24.68	7.70	0.05	21.43
0.2	0.7	24.68	9.18	0.05	21.06
0.3	0.1	25.49	6.02	0.05	23.86
0.3	0.2	25.49	5.98	0.05	23.11
0.3	0.3	25.49	6.09	0.05	22.49
0.3	0.4	25.49	6.37	0.05	21.96
0.3	0.5	25.49	6.87	0.05	21.51
0.3	0.6	25.49	7.70	0.05	21.12
0.4	0.1	27.08	6.02	0.05	23.32
0.4	0.2	27.08	5.98	0.05	22.64
0.4	0.3	27.08	6.09	0.05	22.08
0.4	0.4	27.08	6.37	0.05	21.60
0.4	0.5	27.08	6.87	0.05	21.19
0.5	0.1	29.76	6.02	0.05	22.82
0.5	0.2	29.76	5.98	0.05	22.21
0.5	0.3	29.76	6.09	0.05	21.69
0.5	0.4	29.76	6.37	0.05	21.25
0.6	0.1	34.21	6.02	0.05	22.35
0.6	0.2	34.21	5.98	0.05	21.80
0.6	0.3	34.21	6.09	0.05	21.33
0.7	0.1	42.07	6.02	0.05	21.91
0.7	0.2	42.07	5.98	0.05	21.41
0.8	0.1	58.33	6.02	0.05	21.51

Table 2. Relative efficiency of the proposed estimators with crossed model estimators when π_{AB} is fixed at 0.05

π_A	π_B	π_{AB}	$RE(\hat{\pi}_{A(EA)}, \hat{\pi}_{A(CM)})$	$RE(\hat{\pi}_{B(EA)}, \hat{\pi}_{B(CM)})$	$RE(\hat{\pi}_{AB(EA)}, \hat{\pi}_{AB(CM)})$
0.1	0.1	0.05	6.4	4.6	3.0
0.1	0.2	0.05	5.8	4.2	2.4
0.1	0.3	0.05	5.1	3.9	1.9
0.1	0.4	0.05	4.4	3.6	1.6
0.1	0.5	0.05	3.7	3.5	1.2
0.1	0.6	0.05	3.1	3.3	0.9
0.1	0.7	0.05	2.4	3.2	0.7
0.1	0.8	0.05	1.7	3.1	0.5
0.2	0.1	0.05	6.0	4.1	2.6
0.2	0.2	0.05	5.4	3.7	2.1
0.2	0.3	0.05	4.7	3.4	1.6
0.2	0.4	0.05	4.0	3.2	1.3
0.2	0.5	0.05	3.3	3.0	1.0
0.2	0.6	0.05	2.7	2.8	0.7
0.2	0.7	0.05	2.0	2.6	0.5
0.3	0.1	0.05	5.7	3.6	2.2
0.3	0.2	0.05	5.0	3.3	1.7
0.3	0.3	0.05	4.3	3.0	1.4
0.3	0.4	0.05	3.6	2.8	1.0
0.3	0.5	0.05	2.9	2.5	0.8
0.3	0.6	0.05	2.2	2.3	0.5
0.4	0.1	0.05	5.4	3.2	1.8
0.4	0.2	0.05	4.7	2.8	1.4
0.4	0.3	0.05	4.0	2.6	1.1
0.4	0.4	0.05	3.2	2.3	0.8
0.4	0.5	0.05	2.5	2.0	0.6
0.5	0.1	0.05	5.2	2.7	1.5
0.5	0.2	0.05	4.4	2.4	1.1
0.5	0.3	0.05	3.6	2.1	0.8
0.5	0.4	0.05	2.8	1.9	0.6
0.6	0.1	0.05	5.0	2.2	1.2
0.6	0.2	0.05	4.1	2.0	0.9
0.6	0.3	0.05	3.2	1.7	0.6
0.7	0.1	0.05	4.8	1.8	0.9
0.7	0.2	0.05	3.7	1.5	0.6
0.8	0.1	0.05	4.7	1.3	0.7

Table 3. Relative efficiency of the proposed estimators with respect to Lee *et al.* (2013) crossed model estimators when π_{AB} is fixed at 0.1

π_A	π_B	π_{AB}	$RE(\hat{\pi}_{A(EA)}, \hat{\pi}_{A(CM)})$	$RE(\hat{\pi}_{B(EA)}, \hat{\pi}_{B(CM)})$	$RE(\hat{\pi}_{AB(EA)}, \hat{\pi}_{AB(CM)})$
0.1	0.1	0.1	7.1	5.0	3.5
0.1	0.2	0.1	6.4	4.6	2.9
0.1	0.3	0.1	5.8	4.3	2.4
0.1	0.4	0.1	5.1	4.1	2.0
0.1	0.5	0.1	4.4	3.9	1.6
0.1	0.6	0.1	3.7	3.9	1.3
0.1	0.7	0.1	3.1	3.9	1.0
0.1	0.8	0.1	2.4	4.0	0.8
0.2	0.1	0.1	6.7	4.6	3.1
0.2	0.2	0.1	6.0	4.2	2.5
0.2	0.3	0.1	5.4	3.9	2.1
0.2	0.4	0.1	4.7	3.6	1.7
0.2	0.5	0.1	4.0	3.5	1.4
0.2	0.6	0.1	3.3	3.3	1.1
0.2	0.7	0.1	2.7	3.2	0.8
0.3	0.1	0.1	6.4	4.1	2.7
0.3	0.2	0.1	5.7	3.7	2.2
0.3	0.3	0.1	5.0	3.4	1.8
0.3	0.4	0.1	4.3	3.2	1.4
0.3	0.5	0.1	3.6	3.0	1.1
0.3	0.6	0.1	2.9	2.8	0.9
0.4	0.1	0.1	6.1	3.6	2.3
0.4	0.2	0.1	5.4	3.3	1.9
0.4	0.3	0.1	4.7	3.0	1.5
0.4	0.4	0.1	4.0	2.8	1.2
0.4	0.5	0.1	3.2	2.5	0.9
0.5	0.1	0.1	6.0	3.2	2.0
0.5	0.2	0.1	5.2	2.8	1.6
0.5	0.3	0.1	4.4	2.6	1.2
0.5	0.4	0.1	3.6	2.3	0.9
0.6	0.1	0.1	5.9	2.7	1.7
0.6	0.2	0.1	5.0	2.4	1.3
0.6	0.3	0.1	4.1	2.1	1.0
0.7	0.1	0.1	6.0	2.2	1.4
0.7	0.2	0.1	4.8	2.0	1.0
0.8	0.1	0.1	6.3	1.8	1.1

Table 4. Relative efficiency of the proposed estimators with respect to Lee *et al.* (2013) crossed model estimators when π_{AB} is fixed at 0.2

π_A	π_B	π_{AB}	$RE(\hat{\pi}_{A(EA)}, \hat{\pi}_{A(CM)})$	$RE(\hat{\pi}_{B(EA)}, \hat{\pi}_{B(CM)})$	$RE(\hat{\pi}_{AB(EA)}, \hat{\pi}_{AB(CM)})$
0.2	0.2	0.2	7.4	5.0	3.5
0.2	0.3	0.2	6.7	4.7	3.0
0.2	0.4	0.2	6.0	4.5	2.5
0.2	0.5	0.2	5.4	4.4	2.1
0.2	0.6	0.2	4.7	4.4	1.8
0.3	0.2	0.2	7.1	4.6	3.1
0.3	0.3	0.2	6.4	4.3	2.6
0.3	0.4	0.2	5.7	4.1	2.2
0.3	0.5	0.2	5.0	3.9	1.8
0.4	0.2	0.2	6.9	4.2	2.8
0.4	0.3	0.2	6.1	3.9	2.3
0.4	0.4	0.2	5.4	3.6	1.9
0.5	0.2	0.2	6.8	3.7	2.4
0.5	0.3	0.2	6.0	3.4	2.0
0.6	0.2	0.2	6.8	3.3	2.1