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Multivariate Analysis of Rwanda Economic Indicators using Vector Autoregressive (VAR) Model

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Abstract. .

Rwanda's economy has been growing fast due to important economic and structural reforms over the last decade. Consumer Price Index (CPI), Exchange Rate and Nominal Growth Domestic Product (NGDP) constitute some of the major economic indicators in emerging market economies that require monetary authorities to elaborate tools and policies to prevent high volatility in prices. Thus, understanding CPI, exchange rate and NGDP dynamics is a key to the design of fund programs to help stabilize the economy of a developing country such as Rwanda. In this study, secondary data from the National Bank of Rwanda, depicting quarterly time series of the 3 indicators from 1997$Q1$ to 2014$Q4$ has been used. Appropriate Vector Autoregressive (VAR) model with maximum 3 lagged values for the underlying variables was selected, based on the smallest value of Bayesian Schwartz information criterion and diagnostic tests for disturbances performed. The Granger causality and Impulse response function analysis confirms that in the fitted VAR model, CPI, exchange rate and NGDP are endogeneous variables, each one related to its lagged values and/or of the lag values of other variables. In addition, the results suggest that the CPI is not directly related to the exchange but to the NGDP.

Key words: VAR model, Exchange rate; Nominal Growth Domestic Product; Consumer Price Index.
AMS 2010 Mathematics Subject Classification : 62M10; 62P20; 97K80; 62-07.

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Résumé. (French) L'économie du Rwanda a connu une croissance rapide en raison d'importantes réformes économiques et structurelles au cours de la dernire décennie. L'indice des prix à la consommation (IPC), le taux de change et la croissance nominale du produit intérieur (PNGD) constituent quelques-uns des principaux indicateurs économiques des pays émergents exigeant des autorités monétaires qu'elles élaborent des outils et des poli- tiques pour éviter une volatilité élevée des prix. Ainsi, la compréhension de l'IPC, du taux de change et de la dynamique du PNDP est un élément clé de la conception de programmes de fonds pour aider à stabiliser l'économie d'un pays en développement comme le Rwanda. Dans cette étude, des données secondaires de la Banque Nationale du Rwanda, représentant des séries temporelles trimestrielles des 3 indicateurs de 1997 à 2014 ont été utilisées. Un modle VAR (Vector Autoregressive) approprié avec un maximum de 3 valeurs décalées pour les variables sous-jacentes a été sélectionné, basé sur la plus petite valeur du critre d'information bayésien Schwartz et des tests diagnostiques pour les perturbations effectuées. L'analyse de la causalité de Granger et de la fonction de réponse impulsionnelle confirme que dans le modle VAR ajusté, CPI, taux de change et NGDP sont des variables endognes, chacune liée à ses valeurs retardées et / ou aux valeurs de retard des autres variables. En outre, les résultats suggrent que l'IPC n'est pas directement lié à la bourse mais au PNDG. .

1. Introduction

Consumer Price Index (CPI), exchange rate and Nominal Growth Domestic Product (NGDP) are important economic indicators in an economically developing country such as Rwanda. Understanding these indicators helps designing structural reforms and strategies to make the Rwanda's economy stable.

A Vector Autoregressive model can be used to identify the relationship between each of these economic indicators, means CPI, exchange rate and NGDP and its lagged values as well as the lagged values of others.

In a VAR model, all variables have equations linking the change in that variable to its own past values and the past values of all the variables in the model, as it describes the dynamic evolution of a number of variables from their common history Verbeek (2004). The VAR model allows treating each variable as endogenous thus avoiding restrictions. VAR models have been ever and then used in determining and analyzing the relationship between economic indicators. First important feature of the VAR model is its flexibility and the simplicity in generalization.

Ever since Sims (1972) originated the VAR approach, it has been the standard tool for examining the monetary transmission mechanism. The VAR has proven to be especially useful for describing the dynamic behavior of economic and financial time series and for forecasting Rusek (1994).

The forecasts generated by VARs are often better than traditional structural models. It has been argued in a number of articles (see, for example, Sims (1980)) that large-scale structural models performed badly in terms of their out-of-sample forecast accuracy. The VAR is just a multiple time-series generalization of the AR model and is easy to estimate because we can use the Ordinary Least Square method.

For instance, dynamics of exchange and inflationary rate and money supply in Nigeria was examined by Akinbobola (2012) using quarterly data from 1986$Q1$ to 2008$Q4$. The model was estimated using VAR model through its VEC form. The models shows that in the long run, there is a negative and significant association between inflationary rate and foreign exchange rate as well money supply.

On the other hand, Tucker (2003) had used cointegrating VAR to find a long-run stability of money demand in Sierra Leone as part of the West African Monetary Zone. Using the data for $1981Q1 - 2000Q4$, he examined the relationship between real broad money balance, real GDP, three-month deposit rates, three-month Treasury bill discount rates and inflation. He found that the relationship between money balance, real GDP and T-bill rates was stable in the long-run and concluded that the proposed West African Central Bank could conduct a single monetary policy to achieve price stability within the zone.

Ndung'u (1993) estimated a six-variables VAR money supply, domestic price level, exchange rate index, foreign price index, real output, and the rate of interest in an attempt to explain the inflation movement in Kenya. He observed that the rate of inflation and exchange rate explained each other.

The rest of the paper is as follows: Section 2, is the identification of the model, followed by Section 3, which is the estimation of parameters using OLS method, ...

2. Model identification

The vector autoregressive (VAR) model is an economic model used to capture the linear interdependencies among multiple time series, VARs were popularised in econometrics by Sims (1980). Vector autoregressive (VAR) models generalize the univariate autoregressive (AR) model by allowing for more than one evolving variable. All variables in a VAR are treated symmetrically in a structural sense, although the estimated quantitative responses will not in general be the same, the model have been used to analyze Rwanda economic indicators in which each variable has an equation explaining its evolution based on its own lags and the lags of the other variables. Vector autoregressive (VAR) modeling does not require as much knowledge about the forces influencing a variable as do structural models with simultaneous equations: the only prior knowledge required is a list of variables which can be hypothesized to affect each other. In VAR model the simplest model is bivariate where there are only two variables, for Instance; y_{1t} and y_{2t}, each of whose current values depend on different combinations of the previous k values of both variables, and error terms.

$$y_{1t} = \beta_{10} + \beta_{11} y_{1t-1} + \ldots + \beta_{1k} y_{1t-k} + \alpha_{11} y_{2t-1} + \ldots + \alpha_{1k} y_{2t-k} + u_{1t} \tag{1}$$

$$y_{2t} = \beta_{20} + \beta_{21} y_{2t-1} + \ldots + \beta_{2k} y_{2t-k} + \alpha_{21} y_{1t-1} + \ldots + \alpha_{2k} y_{1t-k} + u_{2t}, \tag{2}$$

where u_{it} is error term with mean zero and zero correlation between error term.

VAR system could be extended to include g variables $y_{1t}, y_{2t}, \ldots, y_{gt}$, instead of only two variables. The main goal of this study was to fit appropriate VAR model to Rwanda economic indicators and to get appropriate model. The stationarity of variable series have been conducted through Dickey Fuller (DF) and Augmented Dickey Fuller (ADF) tests. Furthermore, tests to select order of VAR model information criteria selection have been adopted, for instance: Akaike information criterion, Hannan-Quinn criterion, Schwarz criteria, to determine which sets of variables have a significant effects on each dependent variable, and causality tests have been conducted by restricting the coefficient of the lags of particular variable to zero and finally the impulse response function analysis have been carried out to measure the effect of a unit shock of the variable i at time t on the variable j in the later periods.

3. Model Results

3.1. Data Description

The data used in this study are about Rwanda Consumer Price Index (CPI), Exchange rate (EXRATE) and Nominal Growth Domestic Price (NGDP), collected from National Bank of Rwanda. They are quarterly data from $1997Q1$ to $2012Q2$ and consist of 64 observations.
Generally, the graphs of these three variables show an increasing trend. It is clear that the variance and the mean are not constant. CPI graph shows trend from 1997 up to 2012, while EXRATE shows trend from 1998 up to 2012, and finally NGDP graph shows trend from 1999 up to 2012. To remove trend in the series, first difference transformation of the series has been made.

3.2. Model fitting

To select suitable vector Autoregressive on chosen Rwanda economic indicators with appropriate order of lag we first fits VAR model of order 5 (see Table 1) as the VAR model with maximum number of lags.
Table 1 shows a general model with 5 lags. It provides the values of the parameters together with the standard errors in parentheses () and the test statistics in brackets []. Some parameters are significant while others are not. In the estimation of a VAR model, we have first to determine its appropriate order, and this is done using the information criteria technique. With different information criteria, we estimate different VAR (p) models and the one with smallest Bayesian Schwartz information criterion is chosen to be the best. Based on the information criteria, it is clear that the best VAR model is VAR (3) see Table 2.
Based on the values of R squared in Table 3, we see that the chosen significant independent variables explain slightly the dependent variable in the first and second equations i.e. where DCPI and DEXRATE are endogenous variables respectively, since R^2 values are small, while in the third equation, the chosen significant variables explain well the dependent variable NGDP since the value of R squared is high with more than 99%. We may say that NGDP depends on its previous values as well as the previous values of DEXRATE and DCPI. The Table 3 also provides the values of the parameters of the model together with the standard errors in parentheses () and the test statistics in brackets []. First equation: with DCPI being the dependent variable, only DCPI (-2), DCPI (-3), NGDP (-2) and NGDP (-3) are significant, this means, they are related to DCPI. Second equation: with DEXRATE being

the dependent variable, only DEXRATE (-1). This means that DEXRATE is only related to itself.

3.2.1. Granger Causality Tests

Granger Causality/ Block Exogeneity Wald tests are performed (see Table 4) to determine which sets of variables have significant effects on each dependent variable based on the p-values. Those variables which have significant effects may be used to understand or predict the future values of the dependent variables.

When DCPI is dependent, it is related to NGDP and not related to DEXRATE. This means that DCPI is only related to the previous values of NGDP, when DEXRATE is dependent, it is not related to any of DCPI and NGDP. This means that DEXRATE is not related to any of the previous values of NGDP and DCPI and when NGDP is dependent, it is related to DCPI, but not related to DEXRATE. This means that NGDP is only related to the previous values of DCPI.

3.2.2. The Impulse Response Function Analysis

To measure the effect of a unit shock of the variable, impulse response function has been used, Thus, for each variable from each equation separately, a unit shock is applied to the error term and the effects upon the VAR system over time are noted. It traces the effect of one standard deviation shock to one of the innovations on the current and future values of the endogenous variables in the system. We can examine how long and to what degree a shock to a given equation is on all of the variables in the system. The Figure 2 shows how long and to what degree the shock of every variable when considered to be dependent, has been affected by others considered to be independent. Here, every dependent variable is the response to independent variables.

Considering the first subfigure at the top of Figure 2 on the left, when the response variable is CPI, we see that the shock of CPI innovation has a significant effect on itself since the graph shows a positive effect up to the third period and decreases up to minus 0.5 percent where there is a negative shock and this effect of the shock will finish after six periods. The second and the third subfigure show that the shock of exchange rate and NGDP innovations has no effects on CPI since the graphs show that the values fluctuate around the line zero. Similarly, on the fourth and sixth subfigures, when the response variable is EXRATE, the shocks of CPI and NGPD have no effects on exchange rate since the graph starts from zero even if CPI seems to have a small negative effect, and shows that the values fluctuate around the line zero. On the other hand, when the response is exchange rate, we see that the shock of EXRATE innovation has a significant effect only on itself since the graph shows a positive effect up to the seventh period. When the response is NGDP, every response of NGDP on CPI and on itself is all positive at each time responsive period. EXRATE has obvious fluctuation.

Therefore, in general the impulse response plots in Figure 2 show that the shocks of the variables under our study have a great effect on themselves than on one another. This means that they are more related to their previous values than on the previous values of another variable.

4. Conclusion

This study was undertaken with the main objective of finding the relationship between each underlying economic indicator and its previous values as well as the previous values of other indicators, the indicators being CPI (DCPI), exchange rate (DEXRATE) and NGDP. The objective has been achieved using a Vector Autoregressive (VAR) Model described in this study. Granger Causality/Block Exogeneity Wald Tests and Impulse response function analysis presented in Table 4 and Figure 2 respectively, confirmed that in the VAR model estimated, from each equation, that is, when DCPI, DEXRATE and NGDP are endogenous variables, each one is at least related to its previous value(s) and some of the previous values of other variables. This led us to the confirmation that the relationship exists. Furthermore, the results suggest that CPI is not directly related to the exchange but to the nominal GDP and vice versa.

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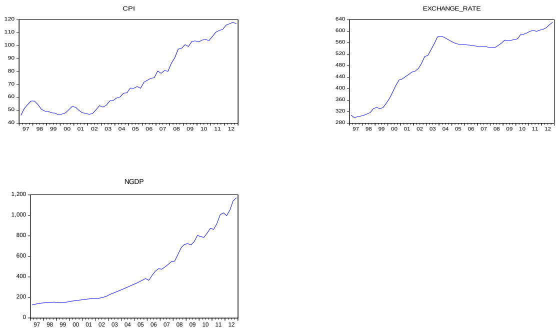

Fig. 1. Graphical description of the data variables used in the study.

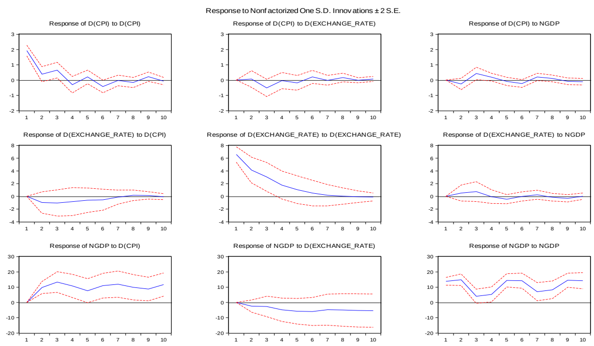

Fig. 2. Impulse response plots.

Table 1. General model estimation with maximum lags of 5.

Vector Autoregression Estimates
Sample (adjusted): 1998Q3 to 2012Q4
Included observations: 58 after adjustments
Standard errors in ( ) and t-statistics in [ ]

	DCPI	DEXRATE	NGDP
DCPI(-1)	0.358488	-0.785717	4.837201
	(0.15257)	(0.56119)	(1.20924)
	[2.34965]	[-1.40009]	[4.00020]
DCPI(-2)	0.222925	0.013252	1.350047
	(0.17511)	(0.64410)	(1.38790)
	[1.27305]	[0.02057]	[0.97273]
DCPI(-3)	-0.420148	0.357254	1.039018
	(0.16575)	(0.60967)	(1.31371)
	[-2.53480]	[0.58598]	[0.79090]
DCPI(-4)	0.340155	-0.664701	-0.762111
	(0.17771)	(0.65367)	(1.40851)
	[1.91407]	[-1.01688]	[-0.54107]
DCPI(-5)	-0.211552	-0.480331	0.318341
	(0.15371)	(0.56538)	(1.21828)
	[-1.37630]	[-0.84957]	[0.26130]
DEXRATE(-1)	-0.023624	0.679839	-0.295189
	(0.04196)	(0.15435)	(0.33258)
	[-0.56298]	[4.40465]	[-0.88757]
DEXRATE(-2)	-0.060197	0.017653	0.124282
	(0.05056)	(0.18597)	(0.40072)
	[-1.19062]	[0.09493]	[0.31014]
DEXRATE(-3)	0.122588	-0.239998	-0.349551
	(0.05021)	(0.18469)	(0.39797)
	[2.44137]	[-1.29944]	[-0.87833]
DEXRATE(-4)	-0.102774	0.296647	0.202254
	(0.05528)	(0.20332)	(0.43811)
	[-1.85925]	[1.45901]	[0.46165]
DEXRATE(-5)	0.011779	-0.177591	-0.089464
	(0.04279)	(0.15740)	(0.33915)
	[0.27526]	[-1.12831]	[-0.26379]
NGDP(-1)	-0.015436	0.007025	0.996772
	(0.02049)	(0.07535)	(0.16236)
	[-0.75351]	[0.09324]	[6.13925]
NGDP(-2)	0.037911	-0.011736	-0.756219
	(0.02811)	(0.10339)	(0.22279)
	[1.34872]	[-0.11351]	[-3.39436]
NGDP(-3)	-0.025733	-0.001457	0.763425
	(0.03041)	(0.11186)	(0.24104)
	[-0.84614]	[-0.01302]	[3.16717]
NGDP(-4)	-0.012732	0.025141	-0.017765
	(0.03190)	(0.11733)	(0.25281)
	[-0.39914]	[0.21428]	[-0.07027]

NGDP(-5)	0.017202	-0.018893	0.065639
	(0.02223)	(0.08177)	(0.17619)
	[0.77382]	[-0.23105]	[0.37255]
C	0.810991	3.707220	4.276693
	(0.59504)	(2.18869)	(4.71616)
	[1.36292]	[1.69381]	[0.90682]
R squared	0.472565	0.511251	0.998352
Adj. R squared	0.284196	0.336698	0.997763
Sum sq. resids	142.3877	1926.417	8944.537
S.E. equation	1.841245	6.772523	14.59333
F statistic	2.508716	2.928914	1695.848
Log likelihood	-108.3437	-183.8847	-228.4107
Akaike AIC	4.287712	6.892575	8.427956
Schwarz SC	4.856110	7.460973	8.996354
Mean dependent	1.078045	5.500095	474.9386
S.D. dependent	2.176279	8.315624	308.5419
Determinant resid covariance (dof adj.)	27729.99		
Determinant resid covariance	10529.64		
Log likelihood	-515.4918		
Akaike information criterion	19.43075		
Schwarz criterion	21.13595		

Table 2. Information criteria.

VAR Lag Order Selection Criteria
Endogenous variables: DCPI DEXRATE NGDP
Exogenous variables: C
Sample: 1997Q1 2012Q4
Included observations: 58

Lag	LogL	LR	FPE
0	-742.7731	NA	29582494
1	-566.5645	328.1126	92737.37
2	-560.9113	9.941785	104383.2
3	-527.9723	54.51972*	46038.41*
4	-520.1818	12.08873	48616.83
5	-515.4918	6.792384	57591.82
Lag	**AIC**	**SC**	**HQ**
0	25.71631	25.82289	25.75783
1	19.95050	20.37680	20.11655
2	20.06591	20.81193	20.35650
3	19.24043*	20.30617*	19.65556*
4	19.28213	20.66760	19.82180
5	19.43075	21.13595	20.09496

*indicates lag order selected by the criterion
LR: sequential modified LR test statistic (each test at 5% level)
FPE: Final prediction error
AIC: Akaike information criterion
SC: Schwarz information criterion
HQ: Hannan-Quinn information criterion

Table 3. Appropriate fitted model

Vector Autoregression Estimates Sample (adjusted): 1998Q1 to 2012Q4 Included observations: 60 after adjustments Standard errors in ( ) and t-statistics in [ ]			
	DCPI	DEXRATE	NGDP
DCPI(-1)	0.199082	-0.505847	5.071817
	(0.12701)	(0.43340)	(0.91207)
	[1.56742]	[-1.16715]	[5.56079]
DCPI(-2)	0.395772	-0.311640	0.273543
	(0.13725)	(0.46832)	(0.98555)
	[2.88369]	[-0.66544]	[0.27755]
DCPI(-3)	-0.490936	0.026210	0.115803
	(0.13706)	(0.46769)	(0.98421)
	[-3.58195]	[0.05604]	[0.11766]
DEXRATE(-1)	0.010361	0.628108	-0.362401
	(0.04160)	(0.14194)	(0.29870)
	[0.24909]	[4.42521]	[-1.21326]
DEXRATE(-2)	-0.093152	0.087333	0.158907
	(0.04950)	(0.16890)	(0.35545)
	[-1.88192]	[0.51706]	[0.44707]
DEXRATE(-3)	0.072046	-0.104995	-0.104065
	(0.04099)	(0.13988)	(0.29437)
	[1.75751]	[-0.75060]	[-0.35352]
NGDP(-1)	-0.018363	0.037352	1.070988
	(0.01327)	(0.04528)	(0.09528)
	[-1.38391]	[0.82496]	[11.2402]
NGDP(-2)	0.054108	-0.019315	-0.746547
	(0.02011)	(0.06861)	(0.14438)
	[2.69114]	[-0.28152]	[-5.17069]
NGDP(-3)	-0.034785	-0.020736	0.723073
	(0.01457)	(0.04971)	(0.10460)
	[-2.38792]	[-0.41716]	[6.91245]
C	0.355938	3.226837	2.359501
	(0.55628)	(1.89821)	(3.99464)
	[0.63985]	[1.69994]	[0.59067]
R squared	0.347080	0.450235	0.998293
Adj. R squared	0.229555	0.351278	0.997986
Sum sq. resids	186.4231	2170.678	9613.091
S.E. equation	1.930922	6.588896	13.86585
F-statistic	2.953234	4.549780	3248.727
Log likelihood	-119.1465	-192.7898	-237.4324
Akaike AIC	4.304884	6.759661	8.247747
Schwarz SC	4.653942	7.108718	8.596804
Mean dependent	0.995888	5.445574	464.0904
S.D. dependent	2.199855	8.180565	308.9366
Determinant resid covariance (dof adj.)	28399.66		
Determinant resid covariance	16434.99		
Log likelihood	-546.6240		
Akaike information criterion	19.22080		
Schwarz criterion	20.26797		

Table 4. Granger Causality tests

VAR Granger Causality/Block Exogeneity Wald Tests			
Sample: 1997Q1 to 2012Q4			
Included observations: 60			
Dependent variable: DCPI			
Excluded	Chi sq.	Df	Prob.
DEXRATE	4.653305	3	0.1990
NGDP	9.229128	3	0.0264
All	13.91212	6	0.0306
Dependent variable: DEXRATE			
Excluded	Chi sq.	Df	Prob.
DCPI	2.008114	3	0.5707
NGDP	0.992273	3	0.8031
All	2.934651	6	0.8170
Dependent variable: NGDP			
Excluded	Chi sq.	Df	Prob.
DCPI	34.55643	3	0.0000
DEXRATE	1.999050	3	0.5726
All	37.02261	6	0.0000