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Concomitants of record values arising from the Morgenstern type bivariate Lindley distribution

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Abstract. . In this work, we developed the distribution theory of concomitants of record values arising from the Morgenstern type bivariate Lindley distribution and derived best linear unbiased estimator (BLUE) of parameter associated with the study variate involved in it are generated. The efficiencies of the BLUE with respect to the unbiased estimator are also evaluated in this work.

Résumé. Dans ce travail, nous avons développé la théorie de la distribution des observations records concomitants de données issues de la distribution de Lindley à deux variables de type Morgenstern. Ensuite, nous en déduisons le meilleur estimateur linéaire sans biais (BLUE) du paramètre associé. L'efficacité de ce BLUE, comparé aux estimateurs biaisés, est évaluée.

Key words: Concomitants of record values; Best linear unbiased estimator; Morgenstern type bivariate Lindley distribution
AMS 2010 Mathematics Subject Classification Objects : 62B10; 62G30.

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1. Introduction

The idea of introducing mixture distribution is one of the important way to obtain new probability distributions in applied probability and several research areas. Lindley (1958) proposed a new distribution to establish the difference between fiducial distribution and posterior distribution and its probability density function (pdf) is given by,

$$h(x;\sigma) = \frac{\sigma^2}{1+\sigma}(1+x)e^{-\sigma x};\ x>0,\ \sigma>0. \tag{1}$$

Clearly $h(x;\sigma)$ is a mixture of exponential (θ) and gamma (2,θ) distributions. Ghitany *et al.* (2008) studied various properties of Lindley distribution and proved that the advantages of Lindley distribution compared to the exponential distribution using real life data sets. Sankaran (1970) developed a distribution known as the Poisson-Lindley distribution using Lindley distribution as the mixing distribution of a Poisson parameter. Zakerzadeh and Dolati (2009) have obtained a generalized Lindley distribution and elucidated its various properties and applications. Ghitany *et al.* (2013) and Nadarajah *et al.* (2013) have recently proposed two parameter extensions of the Lindley distribution named as the generalized Lindley and power Lindley distributions respectively. A discrete form of Lindley distribution was introduced by Gomez and Ojeda (2011) by discretizing the continuous Lindley distribution. Nedjar and Zehdoudi (2016) introduced gamma Lindley distribution and studied its important properties. Location parameter extension of Lindley distribution is extensively discussed by Monsef (2015). Shibu and Irshad (2016) proposed an extended version of generalized Lindley distribution and showed that it includes all the existing Lindley models. Another extension of generalized Lindley distribution is elaborately elucidated by Irshad and Maya (2017). Also Maya and Irshad (2017) introduced generalized Stacy-Lindley mixture distribution and thoroughly studied its reliability properties. But only few bivariate Lindley models are available in the existing literature. However the articles of Zakerzadeh and Dolati (2009) and Vaidhyanathan and Varghese (2016) threw some light on this area. Hence in this paper our prime objective is to introduce a bivariate Lindely distribution using Morgenstern approach.

2. Morgenstern Family of Distributions

When we consider a model corresponding to a bivariate data, the previous knowledge is in the form of marginal distributions, it is one merit to select families of bivariate distributions. Morgenstern Family of Distributions (MFD) are constructed by specified univariate distribution functions $F_W(w)$ and $F_Z(z)$ of random variables W and Z respectively and a dependence parameter ϕ. A bivariate random vector (X,Y) is said to possess a Morgenstern type bivariate distribution if its cumulative distribution function *cdf* $F(w,z)$ can be represented in the form,

$$F_{W,Z}(w,z) = F_W(w)F_Z(z)\{1+\phi[1-F_W(w)][1-F_Z(z)]\}, -1\leq\phi\leq 1, \tag{2}$$

where $F_W(w)$ and $F_Z(z)$ are the univariate distribution functions of the random variables W and Z respectively. The class of all bivariate distributions with cdf

$F(w, z)$ holding a representation of the form (2) is called in the literature as Morgenstern Family of Distributions (MFD). The joint pdf corresponding to the cdf defined in (2) is given by,

$$f(w, z) = f_W(w) f_Z(z)\{1 + \phi[1 - 2F_W(w)][1 - 2F_Z(z)]\}, -1 \le \phi \le 1, \tag{3}$$

where $f_W(w)$ and $f_Z(z)$ are the pdf's corresponding to the marginal random variables W and Z respectively.

Let $X_1, X_2, ...$ be independent sequence of samples coming from a population. An observation X_j will be called an upper record value if its value exceeds that of all preceding observations. Thus X_j is a record if $X_j > X_i$, $\forall\ i < j$. The first observation X_1 is taken as the initial record R_1. The next record R_2 is the observation following R_1 which is larger than R_1 and so on. The records $R_1, R_2, \cdots$ as defined above are sometimes referred to as the sequence of upper records. In a very similar manner, an observation X_j will be called a lower record value if its value is less than that of all preceding sample values.

Statistical dealing based on Record values are of very much importance in several real world situations involving weather, economic and sports data. The statistical study of record values started by Chandler (1952) and has now spread in different areas. For a detailed discussion on the developments in the theory and applications of record values, one can refer Arnold *et al.* (1998), Nevzorov and Balakrishnan (1998) and Ahsanullah (1995).

Suppose $(W_1, Z_1), (W_2, Z_2), \cdots$ be a sequence of bivariate observations from a population. Let R_n denote the n^{th} upper record corresponding to the W variable. Then the random variable which occur together as a Z component in the ordered pair with W component equal to R_n is termed as the concomitant of the n^{th} upper record and it may be denoted by $R^*_{[n]}$. Let L_n denote the n^{th} lower record with respect to the W variable. Then the random variable which occur together as a Z component in the ordered pair with W component equal to L_n is termed as the concomitant of the n^{th} lower record and may be denoted by $L^*_{[n]}$. The study of concomitants of records was started by Houchens (1984).

The role of concomitants of record values arise in many ways for studying bivariate situations. Suppose things are to be selected based on the measurement of an one variable Z whose high value is desirable, but it is not easy to measure. Suppose a variable W which is associated to Z is easily measurable. So the things are measured on the basis of the variable W and only those W variable having higher (or smaller) than all preceding values on W qualify to be measured for their Z values. Then the resulting samples made on Z are called record concomitants. In similar way one can find many examples where concomitants of record values are useful. In this paper, our study based on concomitants of upper record values only.

Let $R^*_{[n]}$ be the concomitant of the n^{th} upper record value. Houchens (1984) derived the pdf $f^*_{[n]}(z)$ of $R^*_{[n]}$ arising from the MFD with *cdf* defined in (2) is given by

$$f^*_{[n]}(z) = f_Z(z) + \phi(1 - 2^{1-n})[f_{2:2}(z) - f_Z(z)], n \geq 1 \tag{4}$$

where $f_{2:2}(z)$ is the probability density function *pdf* of the largest order statistic $Z_{2:2}$ of a random sample of size two drawn from the marginal distribution of random variable Z. Beg and Ahsanullah (2004) derived the joint *pdf* of $R^*_{[m]}$ and $R^*_{[n]}$ for $m < n$ and is given by ,

$$\begin{aligned} f^*_{[m,n]}(z_1, z_2) &= f_Z(z_1)f_Z(z_2) + \phi\left\{\frac{1}{2^{m-1}} - 1\right\}\{f_Z(z_1) - f_{2:2}(z_1)\}f_Z(z_2) \\ &+\phi\left\{\frac{1}{2^{n-1}} - 1\right\}\{f_Z(z_2) - f_{2:2}(z_2)\}f_Z(z_1) \\ &+\phi^2\left\{\frac{1}{3^m}\frac{1}{2^{n-m-2}} - \frac{1}{2^{m-1}} - \frac{1}{2^{n-1}} + 1\right\} \\ &\times\{f_Z(z_1) - f_{2:2}(z_1)\}\{f_Z(z_2) - f_{2:2}(z_2)\}. \end{aligned} \tag{5}$$

Using (4), we obtain the equation for the k^{th} moment of the concomitant of n^{th} upper record value and is given by,

$$E[{R^*_{[n]}}^k] = \mu^{(k)} + \phi(1 - 2^{1-n})(\mu^{(k)}_{2:2} - \mu^{(k)}), \tag{6}$$

where $\mu^{(k)} = E[Z^k]$ and $\mu^{(k)}_{2:2} = E[Z^k_{2:2}]$.

Now the product moment of concomitants of m^{th} and n^{th} upper record values can be obtained explicitly by using (5) and is given by

$$\begin{aligned} E[{R^*_{[m]}}^k, {R^*_{[n]}}^k] = \mu^2 + \phi\mu(\mu - \mu_{2:2})\left(\frac{1}{2^{m-1}} + \frac{1}{2^{n-1}} - 2\right) \\ + \phi^2\left(\frac{1}{3^m}\frac{1}{2^{n-m-2}} - \frac{1}{2^{m-1}} - \frac{1}{2^{n-1}} + 1\right)(\mu - \mu_{2:2})^2. \end{aligned} \tag{7}$$

Development of the theory and applications of concomitants of record values makes a very rich source of mathematical results which are useful for the study of bivariate distributions.

Hence in this study our aim is to develop a bivariate version of one parameter Lindley distribution which is a member of well known Morgenstern family introduced by Morgenstern (1956), called it as Morgenstern type bivariate Lindley distribution (MTBLDD). The distribution theory of concomitants of record values arising from the MTBLDD have been derived and obtain BLUE of σ_2 involved in MTBLDD. The efficiency comparison of the proposed BLUE with usual unbiased estimator of σ_2 based on the concomitants of largest upper record values are also made in this paper.

3. Morgenstern Type Bivariate Lindley Distribution

A bivariate random variable (W, Z) is said to possess a Morgenstern type bivariate Lindley distribution (MTBLDD), if its *pdf* is given by,

$$f(w,z) = \begin{cases} \frac{1}{2\sigma_1}\left(1+\frac{w}{\sigma_1}\right)e^{-\frac{w}{\sigma_1}}\frac{1}{2\sigma_2}\left(1+\frac{z}{\sigma_2}\right)e^{-\frac{z}{\sigma_2}}\left\{1+\phi\left[2e^{-\frac{w}{\sigma_1}}\left(1+\frac{w}{2\sigma_1}\right)-1\right]\right. \\ \left.\left[2e^{-\frac{z}{\sigma_2}}\left(1+\frac{z}{2\sigma_2}\right)-1\right]\right\}, \quad if \quad x>0, y>0; -1\le\phi\le 1 \\ 0, \qquad otherwise. \end{cases} \tag{8}$$

The corresponding *cdf* is given by

$$\begin{aligned} F(w,z) = &\left\{1-e^{-\frac{w}{\sigma_1}}\left(1+\frac{w}{2\sigma_1}\right)\right\}\left\{1-e^{-\frac{z}{\sigma_2}}\left(1+\frac{z}{2\sigma_2}\right)\right\} \\ &\times\left\{1+\phi e^{-\frac{w}{\sigma_1}-\frac{z}{\sigma_2}}\left(1+\frac{w}{2\sigma_1}\right)\left(1+\frac{z}{2\sigma_2}\right)\right\}. \end{aligned} \tag{9}$$

Clearly the marginal distributions of W and Z variables are univariate Lindley distributions with probability density functions (pdfs) are respectively given by

$$f_W(w) = \begin{cases} \frac{1}{2\sigma_1}\left(1+\frac{w}{\sigma_1}\right)e^{-\frac{w}{\sigma_1}}, & if \quad w>0, \sigma_1>0; \\ 0, & otherwise. \end{cases}$$

and

$$f_Z(z) = \begin{cases} \frac{1}{2\sigma_2}\left(1+\frac{z}{\sigma_2}\right)e^{-\frac{z}{\sigma_2}}, & if \quad z>0, \sigma_2>0; \\ 0, & otherwise. \end{cases}$$

3.1. Estimation of the parameter σ_2 using concomitants of upper record values

Suppose $(W_i, Z_i), i = 1, 2, \cdots$ be a bivariate sequence of observations arising from (8). The sequence of upper record values corresponding to the marginal sequence $\{W_i\}$ of observations be $\{R_n^*, n \ge 1\}$. Let $R_{[n]}^*$ be the concomitant of the n^{th} upper record value corresponding to the upper record value R_n^*. Then the pdf $h_{[n]}^*(z)$ of $R_{[n]}^*$ and the joint *pdf* $h_{[m,n]}^*(z_1, z_2)$ of $R_{[m]}^*$ and $R_{[n]}^*$ for $m < n$ are obtained from (4) and (5) respectively and are given by

$$h_{[n]}^*(z) = \frac{1}{2\sigma_2}\left(1+\frac{z}{\sigma_2}\right)e^{-\frac{z}{\sigma_2}}\left\{1+\phi(1-2^{1-n})\left(1-2e^{-\frac{z}{\sigma_2}}\left\{1+\frac{z}{2\sigma_2}\right\}\right)\right\}, n\ge 1 \tag{10}$$

and for $m < n$,

$$\begin{aligned}
h^*_{[m,n]}(z_1,z_2) = & \frac{1}{4\sigma_1\sigma_2}\left(1+\frac{z_1}{\sigma_2}\right)\left(1+\frac{z_2}{\sigma_2}\right)e^{-\frac{z_1}{\sigma_2}-\frac{z_2}{\sigma_2}}\left\{1-\phi\left(\frac{1}{2^{m-1}}-1\right)\right. \\
& \times\left(1-2e^{-\frac{z_1}{\sigma_2}}\left\{1+\frac{z_1}{2\sigma_2}\right\}\right)-\phi\left(\frac{1}{2^{m-1}}-1\right) \\
& \left(1-2e^{-\frac{z_2}{\sigma_2}}\left\{1+\frac{z_2}{2\sigma_2}\right\}\right)+\phi^2\left(\frac{1}{3^m}\frac{1}{2^{n-m-2}}-\frac{1}{2^{m-1}}-\frac{1}{2^{n-1}}+1\right) \\
& \left.\times\left(1-2e^{-\frac{z_1}{\sigma_2}}\left\{1+\frac{z_1}{2\sigma_2}\right\}\right)\left(1-2e^{-\frac{z_2}{\sigma_2}}\left\{1+\frac{z_2}{2\sigma_2}\right\}\right)\right\}, z_1,z_2>0.
\end{aligned} \tag{11}$$

Using (10) and (11), we get the following.

For $n \geq 1$, we have,

$$\begin{aligned}
E[{R^*_{[n]}}^k] = \sigma_2^k\Gamma(k+1)\left\{1+\frac{k}{2}-\phi\left(1-2^{1-n}\right)\left(\frac{1}{2^{k+1}}+\frac{(k+1)}{2^{k+2}}+\frac{(k+1)}{2^{k+3}}\right.\right. \\
\left.\left.+\frac{(k+1)(k+2)}{2^{k+4}}-\frac{k}{2}-1\right)\right\}
\end{aligned} \tag{12}$$

and for $m < n$,

$$\begin{aligned}
& E[R^*_{[m]}R^*_{[n]}] = \\
& \sigma_2^2\left\{\frac{9}{4}-\frac{33}{32}\phi\left(\frac{1}{2^{m-1}}+\frac{1}{2^{n-1}}-2\right)+\frac{121}{256}\phi^2\left(\frac{1}{3^m}\frac{1}{2^{n-m-2}}-\frac{1}{2^{m-1}}-\frac{1}{2^{n-1}}+1\right)\right\}
\end{aligned} \tag{13}$$

From (12), we get the mean and variance of $R^*_{[n]}$ and are given below.
For $n \geq 1$,

$$E[R^*_{[n]}] = \sigma_2\left\{\frac{3}{2}+\frac{11}{16}\phi\left(1-2^{1-n}\right)\right\} \tag{14}$$

and

$$Var[R^*_{[n]}] = \sigma_2^2\left\{\frac{7}{4}+\frac{3}{4}\phi\left(1-2^{1-n}\right)-\frac{121}{256}\phi^2\left(1-2^{1-n}\right)^2\right\}. \tag{15}$$

From (12) and (13), we have obtained the covariance between the $R^*_{[m]}$ and $R^*_{[n]}$ and is given for $m < n$, by

$$\begin{aligned}
Cov[R^*_{[m]}R^*_{[n]}] = \frac{33}{32}\phi\left(2^{1-n}+2^{1-m}-\frac{1}{2^{m-1}}-\frac{1}{2^{n-1}}\right)\sigma_2^2 \\
+\frac{121}{256}\phi^2\left(\frac{1}{3^m}\frac{1}{2^{n-m-2}}-\frac{1}{2^{m-1}}-\frac{1}{2^{n-1}}-(1-2^{1-m})(1-2^{1-n})+1\right)\sigma_2^2
\end{aligned} \tag{16}$$

If we put,

$$\zeta_n = \frac{3}{2} + \frac{11}{16}\phi\left(1 - 2^{1-n}\right), n \geq 1, \tag{17}$$

$$\eta_{n,n} = \frac{7}{4} + \frac{3}{4}\phi\left(1 - 2^{1-n}\right) - \frac{121}{256}\phi^2\left(1 - 2^{1-n}\right)^2, n \geq 1 \tag{18}$$

and for $m < n$

$$\eta_{m,n} = \frac{33}{32}\phi\left(2^{1-n} + 2^{1-m} - \frac{1}{2^{m-1}} - \frac{1}{2^{n-1}}\right)$$
$$+ \frac{121}{256}\phi^2\left(\frac{1}{3^m}\frac{1}{2^{n-m-2}} - \frac{1}{2^{m-1}} - \frac{1}{2^{n-1}} - (1-2^{1-m})(1-2^{1-n}) + 1\right). \tag{19}$$

Using (17), (18) and (19), we have the following. For $n \geq 1$,

$$E[R^*_{[n]}] = \sigma_2\zeta_n, \tag{20}$$

$$Var[R^*_{[n]}] = \sigma_2^2\eta_{n,n} \tag{21}$$

and for $m < n$,

$$Cov[R^*_{[m]}, R^*_{[n]}] = \sigma_2^2\eta_{m,n}. \tag{22}$$

Let $\mathbf{R}^*_{[n]} = (R^*_{[1]}, R^*_{[2]}, \cdots, R^*_{[n]})'$ denote the vector of concomitants of first n upper record values arising from (8). Then from (20), (21) and (22), we can write,

$$E[\mathbf{R}^*_{[n]}] = \sigma_2\zeta \tag{23}$$

and the dispersion matrix of $\mathbf{R}^*_{[n]}$,

$$D[\mathbf{R}^*_{[n]}] = \sigma_2^2\mathbf{H}, \tag{24}$$

where $\zeta = (\zeta_1, \zeta_2, \cdots, \zeta_n)'$ and $\mathbf{H} = ((\eta_{m,n}))$. If ϕ contained in ζ and $\mathbf{H}$ are known then (23) and (24) together defines a generalized Gauss-Markov setup and then the BLUE of σ_2 is given by

$$\tilde{\sigma_2} = (\zeta'\mathbf{H}^{-1}\zeta)^{-1}\zeta'\mathbf{H}^{-1}\mathbf{R}^*_{[n]} \tag{25}$$

and the variance of σ_2 is given by

$$Var(\tilde{\sigma_2}) = (\zeta'\mathbf{H}^{-1}\zeta)^{-1}\sigma_2^2. \tag{26}$$

From (25), we have $\tilde{\sigma_2}$ is a linear functions of the concomitants of upper records $R^*_{[r]}, r = 1, 2, \cdots, n$ and hence $\tilde{\sigma_2}$ can be written as

$$\tilde{\sigma_2} = \sum_{r=1}^{n} a_r R^*_{[r]}, \quad \text{where} \quad a_r, r = 1, 2, \cdots, n \quad \text{are constants.}$$

For numerical illustration, we derived an unbiased estimator of σ_2, namely $\hat{\sigma}_2$ based on the largest concomitants of upper record value, which is obtained as

$$\hat{\sigma}_2 = \frac{R^*_{[n]}}{\frac{3}{2} + \frac{11}{16}\phi\left(1 - 2^{1-n}\right)}, n \geq 1$$

and its variance is obtained as

$$Var(\hat{\sigma}_2) = \frac{\left[\frac{7}{4} + \frac{3}{4}\phi\left(1 - 2^{1-n}\right) - \frac{121}{256}\phi^2\left(1 - 2^{1-n}\right)^2\right]}{\left[\frac{3}{2} + \frac{11}{16}\phi\left(1 - 2^{1-n}\right)\right]^2}\sigma_2^2, n \geq 1.$$

We have evaluated the numerical values of means, variances and covariances using the expressions (14), (15) and (16) respectively for $\phi = 0.25(0.25)0.75$ and for $n = 2(1)9$. Using these values we have evaluated the coefficients of $\mathbf{R}^*_{[n]}$ in the BLUE $\tilde{\sigma_2}$ and its variances for $\phi = 0.25(0.25)0.75$ and for $n = 2(1)9$ and are given in Tables 1 and 2. Also we have computed the ratio $\frac{Var(\tilde{\sigma}_2)}{Var(\hat{\sigma}_2)}$ to measure the efficiency of our estimator $\tilde{\sigma}_2$ relative to $\hat{\sigma}_2$ for $n = 2(1)10$ and $\phi = 0.25(0.25)0.75$ and are also presented in Tables 1 and 2. From those tables, it is clear that, BLUE of σ_2 performs well compared to the unbiased estimator of σ_2, namely $\hat{\sigma}_2$.

Remark 2.1. *In the case of Morgenstern type bivariate Lindley distribution (MTBLDD), our assumption about the dependence parameter ϕ is, it will be known. But sometimes it is not possible. In this situation, our recommendation is to consider a moment type estimator of ϕ. For that, we have evaluated the correlation coefficient between the two variables of MTBLDD and is given by $\rho = \frac{121}{448}\phi$. Also we take the sample correlation coefficient between R_i and $R^*_{[i]}, i = 1, 2, \cdots, n$ be τ. Now, we have evaluated the moment type estimator of ϕ namely $\hat{\phi}$ which is obtained by equating the population correlation coefficient and the sample correlation coefficient, and is given by*

$$\hat{\phi} = \begin{cases} -1, & if \quad \tau < \frac{-121}{448} \\ \frac{448}{121}\tau, & if \quad \frac{-121}{448} \leq \tau \leq \frac{121}{448} \\ 1, & if \quad \tau > \frac{121}{448}. \end{cases} \tag{27}$$

Table 1. Coefficients of $R^*_{[r]}$ in the best linear unbiased estimator $\tilde{\sigma}_2$

n	ϕ	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}
2	0.25	0.32277	0.32527								
	0.50	0.31137	0.31877								
	0.75	0.29906	0.31369								
3	0.25	0.21043	0.21203	0.21370							
	0.50	0.19769	0.20233	0.20776							
	0.75	0.18400	0.19301	0.20391							
4	0.25	0.15535	0.15652	0.15777	0.15860						
	0.50	0.14313	0.14647	0.15048	0.15322						
	0.75	0.13102	0.13650	0.14440	0.14988						
5	0.25	0.12287	0.12379	0.12479	0.12546	0.12585					
	0.50	0.11160	0.11420	0.11735	0.11953	0.12081					
	0.75	0.09972	0.10461	0.11073	0.11502	0.11758					
6	0.25	0.10154	0.10229	0.10312	0.10368	0.10400	0.10418				
	0.50	0.09125	0.09337	0.09597	0.09776	0.09882	0.09940				
	0.75	0.08050	0.08445	0.08943	0.09292	0.09501	0.09618				
7	0.25	0.08648	0.08713	0.08783	0.08831	0.08859	0.08874	0.08882			
	0.50	0.07711	0.07890	0.08109	0.08261	0.08352	0.08401	0.08428			
	0.75	0.06738	0.07068	0.07486	0.07780	0.07956	0.08055	0.08108			
8	0.25	0.07530	0.07586	0.07648	0.07689	0.07714	0.07727	0.07734	0.07738		
	0.50	0.06673	0.06828	0.07018	0.07150	0.07228	0.07272	0.07295	0.07307		
	0.75	0.05789	0.06073	0.06432	0.06685	0.06838	0.06923	0.06969	0.06993		
9	0.25	0.06668	0.06717	0.06772	0.06809	0.06830	0.06842	0.06848	0.06852	0.06853	
	0.50	0.05880	0.06017	0.06185	0.06301	0.06370	0.06408	0.06429	0.06439	0.06445	
	0.75	0.05073	0.05322	0.05637	0.05859	0.05993	0.06067	0.06108	0.06129	0.06139	
10	0.25	0.05982	0.06027	0.06076	0.06109	0.06128	0.06139	0.06144	0.06147	0.06149	0.06150
	0.50	0.05256	0.05378	0.05528	0.05632	0.05693	0.05728	0.05746	0.05756	0.05760	0.05763
	0.75	0.04514	0.04735	0.05016	0.05213	0.05332	0.05399	0.05435	0.05454	0.05463	0.05468

Table 2. Legend : $V_1 = \frac{Var(\tilde{\sigma}_2)}{\sigma_2^2}$, $V_2 = \frac{Var(\hat{\sigma}_2)}{\sigma_2^2}$ and the efficiency $e = \frac{V_2}{V_1}$

n	ϕ	V_1	V_2	e
2	0.25	0.37763	0.73011	1.93340
	0.50	0.36745	0.68259	1.85764
	0.75	0.35817	0.63587	1.77533
3	0.25	0.24655	0.70628	2.86465
	0.50	0.23465	0.63587	2.70987
	0.75	0.22337	0.56812	2.54340
4	0.25	0.18214	0.69442	3.81256
	0.50	0.17040	0.61294	3.59707
	0.75	0.15908	0.53549	3.36617
5	0.25	0.14411	0.68850	4.77760
	0.50	0.13306	0.60160	4.52127
	0.75	0.12235	0.51951	4.24610
6	0.25	0.11911	0.68554	5.75552
	0.50	0.10888	0.59596	5.47355
	0.75	0.09895	0.51160	5.17029
7	0.25	0.10146	0.68407	6.74226
	0.50	0.09204	0.59315	6.44448
	0.75	0.08289	0.50767	6.12462
8	0.25	0.08835	0.68333	7.73435
	0.50	0.07967	0.59175	7.42751
	0.75	0.07125	0.50571	7.09768
9	0.25	0.07823	0.68296	8.73015
	0.50	0.07021	0.59105	8.41832
	0.75	0.06245	0.50473	8.08215
10	0.25	0.07019	0.68278	9.72760
	0.50	0.06275	0.59070	9.41355
	0.75	0.05557	0.50424	9.07396

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