Afrika Statistika

Afrika Statistika
Vol. 14 (1), 2019, pages 1891 - 1902.
DOI: http://dx.doi.org/10.16929/as/2019.1891.140

ISSN 2316-090X

Bootstrap Bartlett Adjustment on Decomposed Variance-Covariance Matrix of Seemingly Unrelated Regression Model

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Received November 14, 2018. Accepted February 23, 2019

Abstract. We investigated hypothesis testing in Seemingly Unrelated Regression (SUR) using Log Likelihood Ratio (LLR) test. The asymptotic distribution of this statistic is well documented in literature to have substantial inaccuracy by an order of magnitude leading to the rejection of too many true null hypotheses. Bartlett adjustment of Barndorff and Blaesild and Efron's bootstrap methods were considered to provide more accurate significance level to the distribution. Simulation results from the partitioned variance-covariance matrix showed that the lower triangular matrix performed better than the upper triangular matrix. The Bartlett method of Barndorff and Blaesild provided better significance value than the bootstrap method.

Key words: Bartlett Adjustment, Bootstrap, Generalised Least Squares, Likelihood Ratio Test, Maximum Likelihood, Triangular Matrices, Seemingly Unrelated Regression.
AMS 2010 Mathematics Subject Classification : 62J05, 62F03.

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Résumé (French) Ici, nous étudions des tests d'hypothèses dans une regression avec vraisemblance de non-correlation, basée le rapport du logarithme de la vraisemblance. La distribution asymptotique de la statistique utilisée est connue pour avoir une grande efficacité. Pour rémédier à cette situation, deux types d'ajustement sont considerés : un premier basé sur la méthode de Bartlett et un seconde basé sur la méthode de Barndorff et Blaesid. Une étude de simulation montre l'éfficacité des méthodes d'ajustement et la superiorité du second ajustement sur le premier.

1. Introduction

Seemingly Unrelated Regression (SUR) is a system of multivariate regression equations with different explanatory variables which satisfies the assumptions of classical regression.

The joint estimation procedure of SUR is more efficient than the equation by equation estimation procedure of the Ordinary Least Squares (OLS). The gain in efficiency would be magnified if the contemporaneous correlation between each pair of the disturbance in the SUR system of equations is very high and the explanatory variables in different equations are uncorrelated (Zellner (1962)). Several situations of SUR estimators on different system of equations have been investigated.

An efficient estimator for SUR system in share equations with random coefficient that have additive heteroscedastic contemporaneous correlation was proposed by Mandy (1993). In multi-party elections, SUR estimators was used to model election returns Jackson (2002). Zellner (1962)'s basic recommendation was maintained by different SUR situation procedures developed by different scholars on high contemporaneous correlation between the error vectors but with uncorrelated explanatory variables.

Ebukuyo (2013) estimated SUR models at varying degrees of autocorrelated disturbance using bootstrap approach. The effects of atypical observations on the estimation of SUR model was proposed by Adepoju (2017). Several other inference approaches have become available and accessible, such as the Bartlett corrected likelihood, which provides a substantial increase in distributional accuracy for small and medium sized samples known to have the potential for substantial inaccuracy. Bootstrap is an alternative to asymptotic approximation for carrying out inference. The idea is to mimic the variation from drawing different samples from a population by the variation from redrawing samples from samples. The cholesky decomposition provide analytic simplicity and computational convenience necessary for computationally intensive matrix inversion; partitions the variance-covariance matrix into a unique lower triangular matrix with positive diagonal entries (Agarwal (2014)).

In this paper, we examined the effects of the partitioned variance-covariance on the bootstrap Bartlett adjustment of multi-equation model. The rest of the paper is organised as follows: In Section 2, the structural framework of SUR system and the Bootstrap Bartlett method adopted are presented, the simulation experiment carried out in the work is discussed in Section 3. Results are presented in Section 4, discussion of the results in Section 5, while Section 6 provides some concluding remarks.

2. Material and Methods

2.1. The Model

The model contains system of regression equations with several response variables with T observations. Each equation is assumed to satisfy the basic assumption of Gauss Markov properties. The system can be represented by:

$$y_{it} = X_{it}\beta_{it} + \varepsilon_{it} \tag{1}$$

for $i = 1, \cdots, M$ is the number of equations and $t = 1, \cdots, T$ is the number of observations. That is

$$\begin{array}{ccccc} y_1 & = & X_1\beta_1 & + & \varepsilon_1 \\ y_2 & = & X_2\beta_2 & + & \varepsilon_2 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ y_m & = & X_m\beta_m & + & \varepsilon_m \end{array} \tag{2}$$

The Seemingly Unrelated Regression (SUR) model in (2) in stacked form is:

$$y = X\beta + \varepsilon \tag{3}$$

where

$$y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_M \end{bmatrix} = \begin{bmatrix} X_1 & 0 & \cdots & 0 \\ 0 & X_2 & \cdots & 0 \\ \vdots & \cdots & \ddots & \vdots \\ 0 & 0 & \cdots & X_M \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_M \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_M \end{bmatrix} \tag{4}$$

where y is MT $\times$ 1 vector of response variables, X is MT $\times$ K matrix of explanatory variables, β is K $\times$ 1 vector of parameters and ε is MT $\times$ 1 vector of the departure term. We assume that a total of T observations are used in estimating the parameters of the M equations. The equations in (3) are linked through their (mean-zero) error structure.

$$E(\varepsilon\varepsilon^1) = \Omega = \Sigma \otimes I_T = \begin{bmatrix} \sigma_{11}I_T & \sigma_{12}I_T & \cdots & \sigma_{1M}I_T \\ \sigma_{21}I_T & \sigma_{22}I_T & \cdots & \sigma_{1M}I_T \\ \vdots & \vdots & \vdots & \vdots \\ \sigma_{M1}I_T & \sigma_{M2}I_T & \cdots & \sigma_{MM}I_T \end{bmatrix} \tag{5}$$

where

$$\Sigma = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \cdots & \sigma_{1M} \\ \sigma_{21} & \sigma_{22} & \cdots & \sigma_{2M} \\ \vdots & \vdots & \vdots & \vdots \\ \sigma_{M1} & \sigma_{M2} & \cdots & \sigma_{MM} \end{bmatrix} \tag{6}$$

is the variance-covariance matrix for each t = 1, $\cdots$, T error vector.

2.2. Hypothesis Testing in SUR

We addressed testing whether a restriction $R\beta = r$ was contradicted by the simulated data. The restriction was that some of the coefficients were 0, so the null hypothesis is always true. In this work, the GLS estimate and the likelihood ratio test were considered. Minimising the departure term in (3), the GLS minimizes:

$$(y - X\beta)'\Sigma^{-1}(y - X\beta) \tag{7}$$

which is solved by

$$\hat{\beta} = (X'\Sigma^{-1}X)^{-1}X'\Sigma^{-1}y \tag{8}$$

If Ω is known, the GLS estimator for the coefficients in this model is

$$\hat{\beta} = \left(X^1\Omega^{-1}X\right)^{-1}\left(X^1\Omega^{-1}Y\right) = \left[X^1\left(\Sigma^{-1}\otimes I\right)X\right]^{-1}\left[X^1\left(\Sigma^{-1}\otimes I\right)Y\right] \tag{9}$$

Assuming multivariate normally distributed errors, the log likelihood function is :

$$LogL = \sum_{t=1}^{T} logL_t = -\frac{MT}{2}log(2\pi) - \frac{T}{2}log \mid \Sigma \mid -\frac{1}{2}\sum_{t=1}^{T}[(y - X\beta)'\Sigma^{-1}(y - X\beta)] \tag{10}$$

2.3. The Bartlett Adjustment

Given (3), to test that the null hypothesis is a true model, the likelihood ratio statistic is computed as:

$$w = 2[\ell(\hat{\theta}) - \ell(\hat{\theta^o})] \tag{11}$$

with $\ell(\hat{\theta})$ representing the log likelihood of the unrestricted maximum likelihood estimators and $\ell(\hat{\theta^o})$ represent the log likelihood of the restricted likelihood estimators. The general form for the Bartlett Correction considered in this work is given by Barndorff-Nielson and Blaesild (1986) as:

$$w\prime = \frac{w}{b} \tag{12}$$

The term b is obtained from

$$b = \frac{[n + \frac{1}{2}(T+1) + (k + k^0)]}{n} \tag{13}$$

where n is the sample size, T is the number of response variables, k and k^0 are the number of columns of unrestricted and restricted regression parameters respectively.

As an alternative to Barndorff-Nielson and Blaesild (1986) adjustment, the LLR approximation would be determined using the Bootstrap method of Efron (1979, 1982). In bootstrap method, a simulation study is run using β (the constrained estimate of β) as the true value and the value of X as observed. Errors of the t^{th} data point are sampled (with replacement) from the set of T residual vectors from the constrained estimation.

3. Design of Simulation Experiment

Considering a system of SUR equations having three distinct linear equations with each of them being contemporaneously and serially correlated with the structural form specified as follows:

$$\begin{gathered} y_1 = \beta_{10} + \beta_{11}X_{11} + \beta_{12}X_{12} + \beta_{13}X_{13} + \varepsilon_1 \\ y_2 = \beta_{20} + \beta_{21}X_{21} + \beta_{22}X_{22} + \varepsilon_2 \\ y_3 = \beta_{30} + \beta_{31}X_{31} + \varepsilon_3 \end{gathered} \tag{14}$$

The positive definite 3×3 variance-covariance matrix is given as:

$$\Sigma_{3\times 3} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix} = \begin{bmatrix} 1.0 & 0.6 & 0.9 \\ 0.6 & 1.0 & 0.7 \\ 0.9 & 0.7 & 1.0 \end{bmatrix} \tag{15}$$

Since Σ is a positive definite matrix, we decomposed the matrix such that $\Sigma = PP'$. Decomposing the variance-covariance matrix gave

$$P = \begin{bmatrix} 1 & 0.6 & 0.9 \\ 0 & 0.8 & 0.2 \\ 0 & 0 & 0.3873 \end{bmatrix}, \; P' = \begin{bmatrix} 1 & 0 & 0 \\ 0.6 & 0.8 & 0 \\ 0.9 & 0.2 & 0.3873 \end{bmatrix} \tag{16}$$

The random disturbance series for the upper triangular matrix is

$$\begin{bmatrix} u_{1i} \\ u_{2i} \\ u_{3i} \end{bmatrix} = \begin{bmatrix} 1 & 0.6 & 0.9 \\ 0 & 0.8 & 0.2 \\ 0 & 0 & 0.3873 \end{bmatrix} \begin{bmatrix} \varepsilon_{1i} \\ \varepsilon_{2i} \\ \varepsilon_{3i} \end{bmatrix} \tag{17}$$

$$\begin{gathered} u_{1i} = \varepsilon_{1i} + 0.6\varepsilon_{2i} + 0.9\varepsilon_{3i} \\ u_{2i} = 0.8\varepsilon_{2i} + 0.2\varepsilon_{3i} \\ u_{3i} = 0.3873\varepsilon_{3i} \end{gathered} \tag{18}$$

While the random disturbance series for the lower triangular matrix is

$$\begin{bmatrix} u_{1i} \\ u_{2i} \\ u_{3i} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0.6 & 0.8 & 0 \\ 0.9 & 0.2 & 0.3873 \end{bmatrix} \begin{bmatrix} \varepsilon_{1i} \\ \varepsilon_{2i} \\ \varepsilon_{3i} \end{bmatrix} \tag{19}$$

$$
\begin{aligned}
u_{1i} &= \varepsilon_{1i} \\
u_{2i} &= 0.6\varepsilon_{1i} + 0.8\varepsilon_{2i} \\
u_{3i} &= 0.9\varepsilon_{1i} + 0.2\varepsilon_{2i} + 0.3873\varepsilon_{3i}
\end{aligned}
\tag{20}
$$

In this pattern, the desired error terms were obtained.

Step 1. The vectors of explanatory variables were generated from uniform distribution, U(-3, 3)

Step 2. Mutually independent N(0, 1) sequences ε_{i1}, ε_{i2}, ε_{i3} were generated respectively and transformed to ensure that the disturbance terms are contemporaneously correlated and distributed as $N(0, \Sigma)$.

Step 3. Specific values were assigned to the structural parameters. The coefficients were predetermined in the Monte Carlo experiment.

$$
\begin{aligned}
y_1 &= 4 + 5X_{11} + 7X_{12} + 1.0X_{13} + \varepsilon_1 \\
y_2 &= 5 + 7X_{21} - 2X_{22} + \varepsilon_2 \\
y_3 &= -1.1 + 9X_{31} + \varepsilon_3
\end{aligned}
\tag{21}
$$

Step 4. For sample sizes n = 5, 10, 15, 20, 25 and 30. The experiment was replicated 10000 times in turn.
Step 5. For each Monte Carlo trial, data set were generated in which the LLR statistic observed significance were calculated using the asymptotic distribution.

4. Results

The performance of each estimator (SUR and OLS) was examined on upper and lower triangular matrix by their RMSE. The results of the simulation experiment described above when the variance-covariance matrix was partitioned for sample size $n \in \{5, 10, 15, 20, 25, 30\}$ are presented in the tables below. The performance of the Bartlett adjustment of Barndorff-Nielson and Blaesild (1986)) Bootstrap adjustment on partitioned variance-covariance matrix are adjudged using the probability value at each sample sizes.

Tables 1 and 6 present the simulation results for SUR and OLS estimators and their RMSE for each partitioned matrix at varying degree of sample sizes. The simulation results for absolute bias of the parameter estimates are presented in Tables 3 and 8 while the standard error of the parameter estimates are presented in Tables 4 and 9.

The simulation results of RMSE values of lower and upper triangular matrices for SUR when n = 5, are; 0.9671 and 1.8533, 0.4680 and 0.1555, 1.0026 and 0.3810 while OLS estimators are; 1.9939 and 3.8401, 0.7299 and 0.2447, 1.2116 and 0.4657 respectively. When n = 30, the SUR values are; 0.9009 and 1.4393, 1.1095 and 0.9227, 0.9489 and 0.3749, while the OLS estimators are; 0.9288 and 1.4980, 1.1573 and 0.9640, 0.9454 and 0.3835 respectively.

The RMSE of the SUR estimators are lower in value compared to the OLS estimators, except for model 3 when n = 30, where we had a reverse order in the lower triangular matrix.

The standard errors presented in Tables 4 and 8 for the lower triangular matrix tends to decrease as the sample size increases in most cases except at β_{10} the standard error tends to increase at sample size 15, 20 and 25. More so, β_{11}, β_{12} and β_{13}, the standard error of the parameter estimates tends to increase at sample sizes 25 and 30 respectively. But at the upper triangular matrix, the standard error of the parameter estimates increased in most cases as the sample size increases in model 2 (y_2).

In Table 5, the LLR accuracy tends to be better as the sample size increases. The Bartlett adjustment performance significantly improved the approximation on several cases of the sample sizes considered. The bootstrap adjustment performance at sample sizes 5, 10 and 15 was not better than the LLR approximation. But as the sample size increases from n = 15, the adjustment were significantly unaltered with the LLR approximation.

In Table 10, the validity of the LLR for the upper triangular matrix was presented. As the sample sizes increases, the LLR asymptotic distribution bias tends to reduce. The Bartlett adjustment performed better as the sample sizes increases. Although the bootstrap adjustment performed better when the sample size is 5, at sample sizes 10 and 15, the performance was not better than the LLR statistic approximation. The significant performance became unaltered as the sample sizes increase from 15 to 30.

Table 1. Simulation Results for SUR and OLS RMSE (Lower Triangular Matrix)

Model	Estimators	Sample sizes					
		5	10	15	20	25	30
y_1	SUR	0.9671	0.5029	0.6954	0.9100	1.1265	0.9009
	OLS	1.9939	0.6104	0.7665	0.9992	1.2259	0.9288
y_2	SUR	0.4680	0.7386	0.6955	0.8220	1.0211	1.1095
	OLS	0.7299	0.8815	0.7633	0.8619	1.0672	1.1573
y_3	SUR	1.0026	0.8849	0.8560	0.9248	1.0437	0.9489
	OLS	1.2116	0.9848	0.9102	0.9596	1.0876	0.9454

Table 2. Simulation Results of Models for SUR Parameter Estimate (Lower Triangular Matrix)

n	y_1				y_2			y_3	
	β_{10}	β_{11}	β_{12}	β_{13}	β_{20}	β_{21}	β_{22}	β_{30}	β_{31}
5	4.3013	5.5167	6.7263	0.9644	4.3589	6.7718	-2.0594	0.0039	8.2955
10	4.4668	4.8173	7.0050	0.9221	5.9052	7.3413	-2.0501	-0.5922	9.0723
15	4.3053	4.9138	6.9643	1.0139	5.3716	6.9788	-1.9970	-0.7980	9.1145
20	3.9736	4.9804	6.9774	0.9923	4.9526	7.1615	-1.8871	-1.1017	8.9867
25	3.8251	5.0070	6.9666	1.0199	4.8981	6.9117	-2.0231	-1.3273	9.0312
30	4.3049	5.0134	7.0140	0.9574	5.2006	6.9212	-1.9110	-0.8642	9.0623

Table 3. Bias of the Parameter Estimate (Lower Triangular Matrix)

n	y_1				y_2			y_3	
	β_{10}	β_{11}	β_{12}	β_{13}	β_{20}	β_{21}	β_{22}	β_{30}	β_{31}
5	-0.3013	-0.5167	0.2737	0.0356	0.6411	0.2282	0.0594	-1.0961	0.7045
10	-0.4668	0.1827	-0.0050	0.0779	-0.9052	-0.3413	0.0501	-0.5078	-0.0723
15	-0.3053	0.0862	0.0357	-0.0139	-0.3716	0.0212	-0.0030	-0.3020	-0.1145
20	0.0264	0.0196	0.0226	0.0077	0.0474	-0.1615	-0.1129	0.0017	0.0133
25	0.1749	0.0070	0.0334	-0.0199	0.1019	0.0883	0.0231	0.2273	-0.0312
30	0.3049	0.0134	-0.0140	0.0426	-0.2006	0.0788	-0.0890	-0.2358	-0.0623

5. Discussion of Results

From the tables, the *RMSE* of the SUR and OLS estimators for both upper and lower triangular matrices showed that the SUR estimator is better and efficient than the *OLS* estimator for multi equation model (Alaba (2013)). It was observed that the RMSE of the lower triangular matrix are lower than that of the upper triangular matrix except at equation 2 and 3 where we had a reverse order.

It was also observed that the standard error of the lower triangular matrix reduced as the sample size increased in most cases.

The asymptotic χ^2 reference distribution for the *LLR* statistic for testing in *SUR* results is biased on significance levels in which many true null hypothesis would

Table 4. Simulation Results of Standard Error of Parameter Estimates (Lower Triangular Matrix)

n	y_1				y_2			y_3	
	β_{10}	β_{11}	β_{12}	β_{13}	β_{20}	β_{21}	β_{22}	β_{30}	β_{31}
5	1.2111	0.8614	0.4188	0.2167	0.4017	0.1399	0.1321	0.8377	0.5375
10	0.2006	0.0816	0.0932	0.0823	0.3066	0.1291	0.1155	0.3180	0.1328
15	0.2069	0.0764	0.0709	0.0763	0.1975	0.0967	0.1108	0.2689	0.1167
20	0.2048	0.0571	0.0604	0.0548	0.1966	0.0958	0.1100	0.2164	0.0521
25	0.2488	0.0803	0.0786	0.0657	0.2135	0.0707	0.0784	0.2106	0.0462
30	0.1714	0.0683	0.0652	0.0609	0.2137	0.0676	0.0749	0.1751	0.0312

Table 5. Validity of Methods of Assessing the Significance of Likelihood Ratio (Lower Triangular Matrix).
Abbreviation: (BA)=Bartlett Adjustment

n	LLR	P-value	Bartlett Adjustment	P-value	(BA)	P-value
5	42.267	7.964e-11	20.1270	3.792e-11	25.282	4.954e-7
10	71.852	2.2e-16	46.356	1.496e-17	53.146	3.096e-13
15	87.986	2.26e-16	64.3797	4.824e-21	60.667	6.761e-15
20	133.01	2.2e-16	104.319	7.066e-31	98.385	2.2e-16
25	148.27	2.2e-16	121.533	3.394e-34	80.231	2.2e-16
30	185.81	2.2e-16	157.021	2.209e-42	123.61	2.2e-16

Table 6. Simulation Results for SUR and OLS RMSE (Upper Triangular Matrix)

Model	Estimators	Sample sizes					
		5	10	15	20	25	30
y_1	SUR	1.8533	1.2257	1.1758	1.2744	1.4522	1.4393
	OLS	3.8401	1.5401	1.3322	1.4244	1.5777	1.498
y_2	SUR	0.1555	0.6338	0.5534	0.7203	0.7517	0.9227
	OLS	0.2447	0.7343	0.6171	0.7636	0.7971	0.964
y_3	SUR	0.3810	0.3309	0.3507	0.2899	0.3750	0.3749
	OLS	0.4657	0.3621	0.3765	0.3038	0.3910	0.3835

Table 7. Simulation Results of Models for SUR Parameter Estimate (Upper Triangular Matrix)

n	y_1				y_2			y_3	
	β_{10}	β_{11}	β_{12}	β_{13}	β_{20}	β_{21}	β_{22}	β_{30}	β_{31}
5	4.0628	5.6629	6.3631	1.1417	4.2534	6.8999	-2.0232	-0.8300	8.7725
10	5.0807	4.6843	7.0922	1.0177	5.6457	7.2145	-1.9687	-1.0126	9.0115
15	4.2756	4.8715	6.8208	1.0321	5.1828	6.9272	-2.0125	-1.1552	9.0290
20	3.9918	4.9668	6.8669	0.9398	4.9155	6.9963	-1.9235	-1.0586	9.0078
25	3.6405	5.0017	6.9611	0.9724	4.9505	6.9212	-1.9956	-1.1809	9.0220
30	4.0876	4.9910	7.1758	1.0524	4.9677	6.9222	-1.9036	-1.1808	9.0042

Table 8. Bias of the Parameter Estimate (Upper Triangular Matrix)

n	y_1				y_2			y_3	
	β_{10}	β_{11}	β_{12}	β_{13}	β_{20}	β_{21}	β_{22}	β_{30}	β_{31}
5	-0.0628	-0.6629	0.6369	-0.1417	0.7466	0.1001	-0.0232	-0.2700	0.2275
10	-1.0807	0.3157	-0.0922	-0.0177	-0.6457	-0.2145	0.0313	-0.0874	-0.0115
15	-0.2756	0.1285	0.1792	-0.0321	-0.1828	0.0728	-0.0125	0.0552	-0.0290
20	0.0082	0.0332	0.1331	0.0602	0.0845	0.0037	0.0765	-0.0414	-0.0078
25	0.3595	-0.0017	0.0389	0.0276	0.0495	0.0788	0.0044	0.0809	-0.0220
30	-0.0876	0.0009	-0.1758	-0.0524	0.0323	0.0778	0.0964	0.0808	-0.0042

Table 9. Simulation Results of Standard Error of Parameter Estimates (Upper Triangular Matrix)

n	y_1				y_2			y_3	
	β_{10}	β_{11}	β_{12}	β_{13}	β_{20}	β_{21}	β_{22}	β_{30}	β_{31}
5	2.4069	1.7195	0.9069	0.5156	0.1556	0.0612	0.0649	0.3225	0.2071
10	0.5009	0.1766	0.2011	0.1754	0.2681	0.1776	0.1227	0.1176	0.0555
15	0.3641	0.1493	0.1482	0.1517	0.1596	0.0734	0.0841	0.1158	0.0706
20	0.3444	0.1404	0.1478	0.1340	0.1770	0.0979	0.1103	0.0701	0.0321
25	0.3307	0.1891	0.1751	0.1553	0.1595	0.0721	0.0707	0.0783	0.0376
30	0.2774	0.1274	0.1181	0.1120	0.1785	0.0808	0.0900	0.0725	0.0388

Table 10. Validity of Methods of Assessing the Significance of Likelihood Ratio (Upper Triangular Matrix).
Abbreviation : BAA : Bartlett Adjustment, BOA : Bootstrap Adjustment

n	LLR	P-value	(BAA)	P-value	(BOA)	P-value
5	37.672	8.37e-10	17.9391	3.986e-10	46.535	9e-12
10	56.07	6.993e-14	36.174	4.512e-14	34.428	4.424e-09
15	62.336	2.896e-15	45.6114	2.1173e-15	31.725	1.776e-08
20	90.117	2.2e-16	70.6802	1.7605e-21	92.231	2.26e-16
25	91.017	2.2e-16	74.6043	1.1674e-21	79.726	2.2e-16
30	134.91	2.2e-16	114.011	2.915e-31	103.59	2.2e-16

be rejected, the accuracy of the distribution is still not satisfactory when the sample size is small relative to the number of equations in the adjustment for the partitioned matrix (Rocke (1989)).

Comparing the validity of significance of likelihood ratio statistic for lower and upper triangular matrices, it was observed that the significance level of lower triangular matrix is better than the upper triangular matrix. It was also observed that the Bartlett adjustment of Barndorff-Nielson and Blaesild provide better significance level than the bootstrap Bartlett adjustment method.

6. Conclusion

The study found that the Barndorff-Nielson and Blaesild (1986) Bartlett adjustment significance value is better than the significance value of the bootstrap adjustment approach for both lower triangular matrix and upper triangular matrix. The decomposed variance-covariance matrix results justify that the lower triangular matrix performed better than the upper triangular matrix.

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