

## Full Length Research

# Derivation and application of hydraulic equation for variable-rate contour-controlled sprinklers

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The variable-rate contour-controlled sprinkler (VRCS) for precision irrigation can throw water on a given shaped area and the flow rate is also varied with the throw distance of the sprinkler for the purpose of high uniformity irrigation. Much of past research work were concentrated on the mechanical availability of variable-rate application and the design of VRCS main construction without considering the theoretical operation principles of VRCS. This study aimed to develop the mathematic models describing the relationship between hydraulic parameters of VRCS and these models will be the theoretical guidance for the design of VRCS. The hydraulic operational equation that describes the internal connection of flow rate, rotating speed and throw distance of VRCS was derived using mathematical theory of limitation and double integral. The derived operational equation indicates that the flow rate of VRCS is proportional to the product of rotating speed and square throw distance. The square wetted area sprinklers were used to illustrate the application of the operational equation of VRCS. The theoretical throw distance equation for the square wetted area sprinkler was built. With the operational equation and theoretical throw distance equation, the theoretical flow rate and rotating speed equations of the square wetted area sprinkler were derived. These results of this study provide fundamental principles for the design of VRCS.

**Key words:** Variable-rate sprinkler, precision irrigation, contour-controlled wetted area, irrigation uniformity, square wetted area sprinkler, hydraulic equation.

## INTRODUCTION

Sprinklers are critical elements for a sprinkle irrigation system that have great influences on the irrigating quality. There are various types of sprinklers used for irrigating gardens, farms, lawns, or the like. Some such sprinklers utilize a rotating element which is spring-loaded and reacts to water emitting from a nozzle by bouncing away from the stream of water, thereby causing a counter

that the nozzle rotates in a 360° pattern and the irrigation of a substantially round area of land is obtained. These rotating sprinklers may be adjusted to rotate through only an arc and thereby irrigate only a sector of circle. Although, such rotating sprinklers have partial adjustability, they are very limited as to the shapes of the areas for which they can achieve precision irrigation. Indeed, these rotating sprinklers can only be programmed to irrigate a circular region or sector of a circular region. Unfortunately, these prior art rotating sprinklers waste a large amount of water when used to irrigate non-circular areas due to over-spraying the non-circular areas. This is particularly an undesirable result in regions suffering from drought. Furthermore, over-spraying may inconvenience

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**Abbreviation:** VRCS, Variable-rate contour-controlled sprinkler. weight to rotate the nozzle. This process is repeated such

pedestrians attempting to walk in the area over-sprayed. Indeed, when land owners are irrigating lawns or the like, nearby side walks may be subject to over-spray from a rotating sprinkler. Pedestrians attempting to traverse a wet area of a sidewalk (or other area) may be inadvertently drenched with water or may slip and fall on slippery surfaces caused by the overspray. Furthermore, such pedestrians may be at risk from being hit by passing automobiles if they attempt to leave the sidewalk and walk along the street to avoid the over-spray. Over-spray may also interfere with nearby electric lines causing shock hazards to passers by and/or users of the sprinkler. Accordingly, there is a need in the art for an automatic adjustable sprinkler for precision irrigation which can cover areas of many different shapes without over-spray. The variable-rate contour-controlled sprinkler (VRCS) for precision irrigation can throw water on a given shaped area and the flow rate is also varied with the throw distance of the sprinkler for the purpose of high uniformity irrigation.

A lot of research work has been done on the development of VRCS. However, much of these work were concentrated on the availability of variable-rate application and design of main construction. Benjamin (1981) designed a sprinkler for irrigating a square or other polygonally-shaped land area with the sprinkler including a unique structure which will accurately distribute an equal quantity of water on each increment of land area being irrigated. The sprinkler includes a rotatable nozzle in which the speed of the nozzle is varied by a gear drive mechanism which receives its power from a water powered turbine device. The rotational speed of the nozzle is slower when irrigating corner area of the land area being irrigated and faster when irrigating intermediate straight-sided portions of the land area with the nozzle including a diffuser which becomes operative to reduce the length of trajectory of the water being discharged when the sprinkler is irrigating the intermediate straight-sided portions of the land area being irrigated.

Edwin (1982) developed a sprinkler rotates and creates a predefined horizontal spray pattern around it. A rotating turbine drives a cylindrical nozzle assembly having a plurality of nozzle openings therein. The nozzle openings deliver streams of water of variable length and volume. A pattern defining disk has an aperture which was used to continuously and proportionately expose the nozzle openings to the source of pressurized water according to a preset function of the instantaneous rotational position of the nozzle opening as the cylindrical member turns to create the spray pattern. When the aperture was rectangular, a rectangular spray pattern was obtained. The nozzle openings supply a relatively constant volume of water to all portions of the pattern.

Ohayon (2000) designed a sprinkler for irrigating an area having a shape with an irrigating liquid includes a

base; an inlet disposed on the base for receiving the irrigating liquid from a source; and a nozzle having a central axis, the nozzle being rotatably coupled to the base about the central axis and operatively coupled with the inlet to receive the irrigating liquid, the nozzle being adapted to spray the irrigating liquid out onto the area. The sprinkler also includes an adjustable valve operatively coupled between the inlet and the nozzle for controlling a flow rate of the irrigating liquid through the nozzle; a plate being coupled to and rotatable with the nozzle and having a plate axis which is co-axial with the central axis, the plate including a cam surface having a contour which corresponds with the shape of the area; and a lever element having a cam engaging element which engages the cam surface, the lever element being operatively coupled to the valve such that the contour of the cam surface causes the valve to control the flow rate of the irrigating liquid.

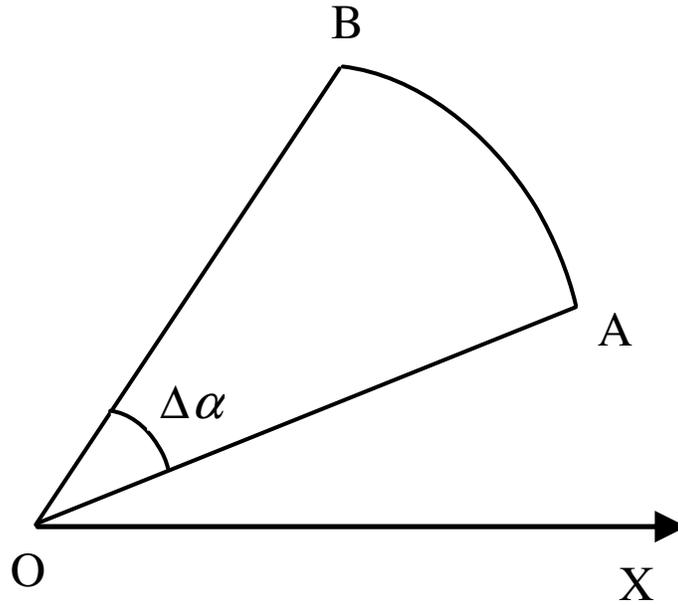
Han et al., (2004) designed state-plate-controlled, inter-groove-cam-controlled, and gear-rotating-plate-controlled three impacts VRCSs. Liu and Yuan et al. (2009) discussed a method for achieving irregular boundary area for complete fluidic sprinkler.

The purposes of this paper are to analyze theoretically on the high uniformity VRCSs for precision irrigation. The operational equation that describes the relationship between the flow rate, throw distance and rotating speed of VRCS for high uniformity irrigation was developed. The square wetted area sprinkler was taken as example to show the application of the operational equation.

## THE DERIVATION OF THE VRCS OPERATIONAL EQUATION

The control of contour for the VRCS is achieved by automatically adjusting the throw distance of the sprinkler. The wetted land area covered by the VRCS for each unit of rotation angle varies also with the throw distance. In order to make each unit of wetted land area to received same amount of irrigated water, the flow rate or rotation speed of the sprinkler should vary with the changing of throw distance. The operational equation that describes the instantaneous relationship between flow rate, rotation speed and throw distance of VRCS can be derived as follows:

As shown in Figure 1, the VRCS is at  $O$  point. Suppose the throw distance of the VRCS is  $OA$  at the time of  $t$  and  $OB$  at the time of  $t + \Delta t$ . The corner  $\Delta\alpha$  ( $\Delta\alpha = \omega\Delta t$ ,  $\omega$  is the rotation speed of the VRCS at the time of  $t$ ) is the rotation angle during the time of  $\Delta t$ . Let the throw distance of the VRCS  $R = OA$ . When  $\Delta t \rightarrow 0$ , there are  $\Delta\alpha \rightarrow 0$  and  $OB \rightarrow OA$ . The wetted area of the VRCS  $\Delta S$  during this very short



**Figure 1.** Variable-rate sprinkler instantaneous throw distance and wetting area.

period of time  $\Delta t$  can be regarded as the area of the sector  $OAB$  :

$$\Delta S = \pi R^2 \frac{\omega \Delta t}{2\pi} = \frac{1}{2} R^2 \omega \Delta t \tag{1}$$

Where,  $\omega$  is the rotation speed of the VRCS at the time of  $t$ ;  $R$  is the throw distance of the VRCS at the time of  $t$ .

In addition, the formula (1) can also be derived by using double integral mathematical model under polar coordinate system. The wetted area  $\Delta S$  by the VRCS within  $\Delta t$  is just the double integral to the area element  $d\sigma$  ( $d\sigma = RdRd\alpha$ ):

$$\Delta S = \iint_D d\sigma = \iint_D RdRd\alpha = \frac{1}{2} R^2 \omega \Delta t \tag{2}$$

Within  $\Delta t$ , the discharge of the VRCS is:

$$\Delta V = Q\Delta t \tag{3}$$

Where,  $\Delta V$  is the discharge of the VRCS within  $\Delta t$ ;  $Q$  is the average flow rate within  $\Delta t$ . When  $\Delta t \rightarrow 0$ ,  $Q$  is the instantaneous flow rate at the time of  $t$ .

High uniformity irrigation requires the amount of irrigated water received on each unit of area to be the same. Therefore, the relationship between the discharge and the area of wetted land within  $\Delta t$  can be described as following:

$$\Delta V = K\Delta S \tag{4}$$

Where,  $K$  is the constant.  $K$  can be regarded as the average amount of water received on each area of wetted land during each rings of the VRCS. It can be determined by the designed irrigating water quota.

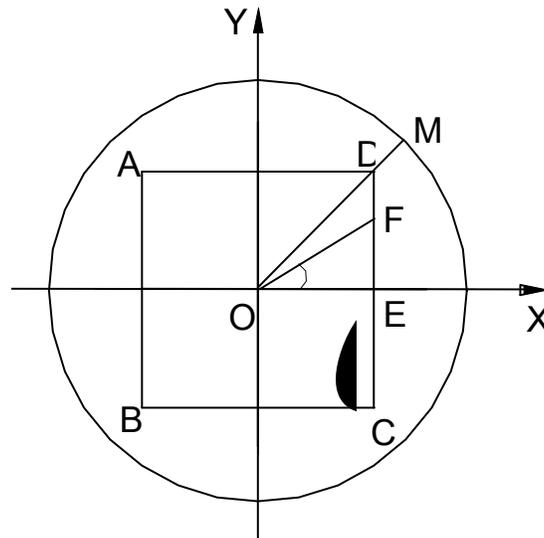
For an irrigation system that has only one sprinkler or that has no overlapping of wetted area between sprinklers, the constant  $K$  is calculated by the following formula:

$$K = \frac{HS}{TN} \tag{5}$$

Where,  $H$  is the designed irrigating water quota;  $S$  is the wetted land area of the VRCS;  $T$  is the operation duration for each VRCS and  $N$  is the number of rings within each unit of operation time of the VRCS.

For an irrigation system that has overlapping of wetted area between sprinklers, the calculation principle of the constant  $K$  is the same, but the derivation is very complicated and not discussed here.

From the formula (3) and (4), there is:



**Figure 2.** Square wetted area sprinkler theoretical throw distance.

$$Q\Delta t = K\Delta S \tag{6}$$

Put the formula (6) into (1) :

$$Q\Delta t = \frac{1}{2} KR^2 \omega \Delta t \tag{7}$$

After simplification:

$$Q = \frac{1}{2} K\omega R^2 \tag{8}$$

$K$  is a constant for a given sprinkler or sprinkle irrigation system. Therefore, it can be concluded from equation (8) that the instantaneous flow rate of the VRCS is proportional to the product of the square throw distance and rotation speed. This equation describes the relationship between the flow rates, throw distance and rotation speed of VRCS for high uniformity precision irrigation. So, this paper named it as “Operational equation of the VRCS”.

It can also be found from the operational equation of the VRCS that the flow rate will be a constant if the throw distance and rotation speed is kept unchanging. In this case, the equation describes what happens for the common circular wetted area sprinkler, which is a special case of the VRCS.

**THE APPLICATION OF THE VRCS OPERATIONAL EQUATION; TAKING THE SQUARE WETTED AREA SPRINKLE AS AN EXAMPLE**

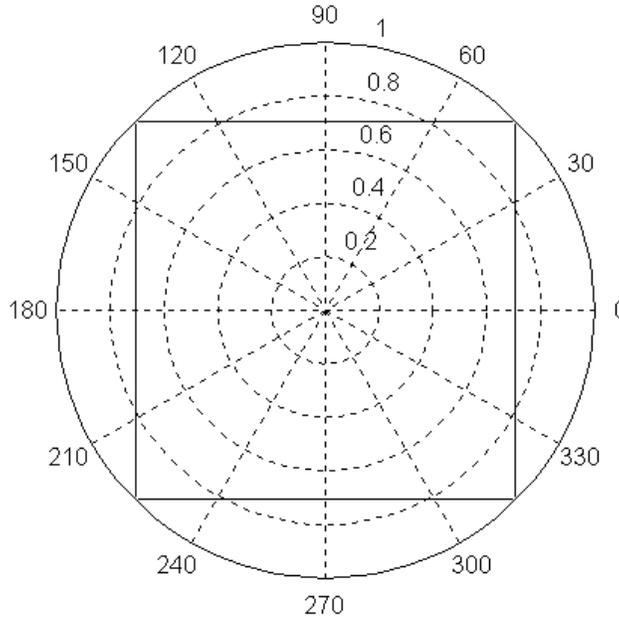
The square wetted area sprinkler is one of the common VRCS. Here, taking this sprinkler as an example to illustrate the application of the VRCS operational equation.

**Theoretical throw distance equation of the square wetted area sprinkler**

Firstly, the theoretical throw distance equation of the square wetted area sprinkler need to be built.

As shown in Figure 2, the sprinkler is at the zero point  $O$ . The irrigated area for a common circular wetted area sprinkler is the land within a circle whose semi-diameter is  $OM$ . This sprinkler can throw water on the square area  $ABCD$  by installing special accessory components to control the throw distance and flow rate automatically. The maximum throw distance of this square wetted area sprinkler is  $OD$ . In most cases,  $OD < OM$  is due to the lost of throw distance after installing special components. Therefore, the maximum throw distance of the square wetted area sprinkler is less than the one of the common circular wetted area sprinkler.

Considering the symmetry of the square, the section of one-eighth of the square  $ABCD$ , the triangle  $EOD$ , can be taken as an example to illustrate the derivation of the theoretical throw distance equation of the sprinkler. In Figure 2,  $\alpha$  is the rotation angle of the sprinkler rotating



**Figure 3.** Square wetted area sprinkler theoretical throw distance in polar coordinate system.

around the zero point  $O$ , which is in the range of  $0 \sim \pi/4$  in this figure.  $OE$  is the throw distance when  $\alpha$  equal to zero, that is the minimum throw distance of the sprinkler.  $OF$  is the throw distance at any angle, denoted by  $R$ .  $OD$  is the throw distance when  $\alpha$  equals to  $\pi/4$ , which is the maximum throw distance, denoted by  $R_0$ . The theoretical equation that shows the relationship between the throw distance and the angle can be derived from the geometry of the triangle  $EOD$ :

$$R = R_0 \frac{\sqrt{2}}{2 \cos \alpha} \quad ( 0 \leq \alpha \leq \pi/4 ) \tag{9}$$

Particularly, when  $\alpha = 0$ ,  $R = \sqrt{2}R_0 / 2$ .

The theoretical throw distance of the square wetted area sprinkler at the range of  $\pi/4 \leq \alpha \leq 2\pi$  can be obtained by the symmetry of the square. The Figures 3 and 4 show the theoretical throw distance in Cartesian coordinate system and polar coordinate system at all ranges that were drawn by the computer language MATLAB. Here suppose  $R_0 = 1$ .

It can be found from the theoretical throw distance equation of the square wetted area sprinkler that the

maximum spacing distance without un-irrigated land between the square wetted area sprinklers also equal to  $\sqrt{2}R_0$ , the same as the circular wetted area sprinklers. So, the square wetted area sprinklers cannot increase the spacing distance compared with the circular wetted area sprinklers.

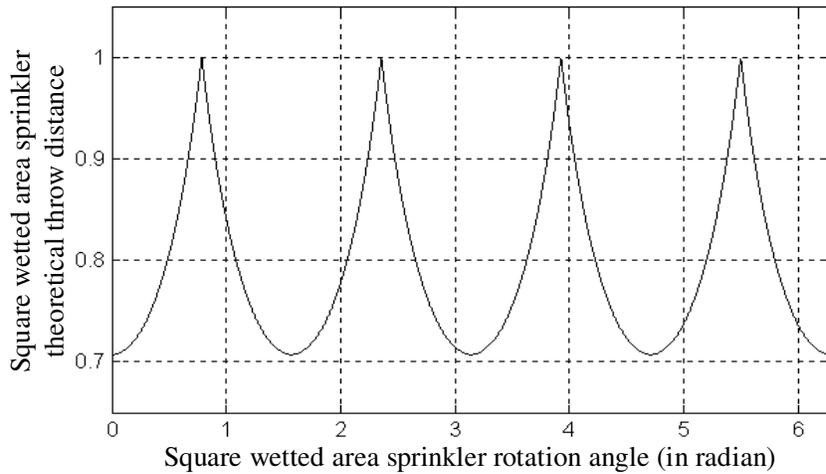
**Theoretical flow rate equation of the square wetted area sprinkler**

As shown in the VRCS operational equation of formula (8), the theoretical flow rate of the square wetted area sprinkler is proportional to the square throw distance when the rotation speed is a constant. That is:

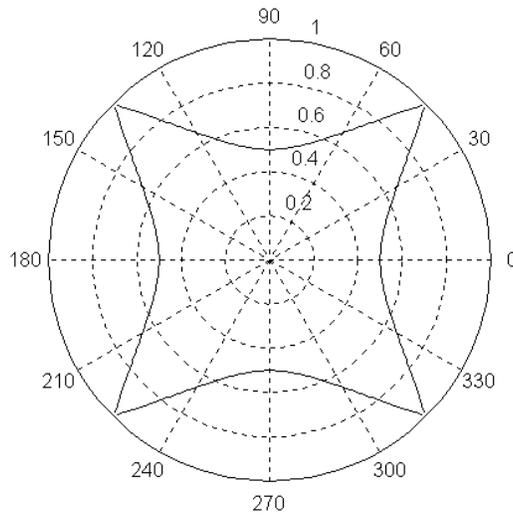
$$\frac{Q}{Q_0} = \frac{R^2}{R_0^2} \tag{10}$$

Where,  $Q$  is the theoretical flow rate of the square wetted area sprinkler and  $Q_0$  is the maximum flow rate of the square wetted area sprinkler (when  $\alpha = \pi/4$ ).

Putting formula (9) into (10), the theoretical equation



**Figure 4.** Square wetted area sprinkler theoretical throw distance in Cartesian coordinate system.



**Figure 5.** Square wetted area sprinkler theoretical discharge in polar coordinate system.

that shows the relationship between the flow rate and rotation angle of the square wetted area sprinkler is obtained:

$$Q = Q_0 \frac{1}{2 \cos^2 \alpha} \quad ( 0 \leq \alpha \leq \pi/4 ) \tag{11}$$

From formula (11), when  $\alpha = 0$ , the minimum flow rate

$$Q = Q_0 / 2.$$

The theoretical flow rate of the square wetted area sprinkler at the range of  $\pi/4 \leq \alpha \leq 2\pi$  can be obtained by the symmetry of the square. Figures 5 and 6 show the theoretical flow rate in Cartesian coordinate system and polar coordinate system at all ranges that were drawn by the computer language MATLAB. Here suppose  $Q_0 = 1$ .

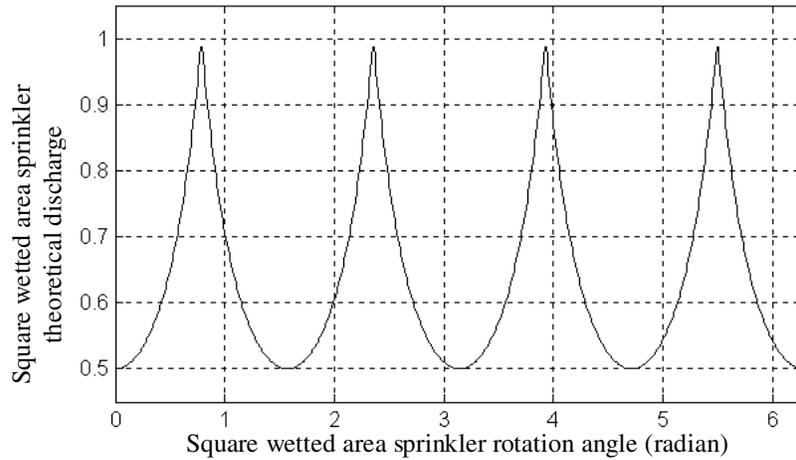


Figure 6. Square wetted area sprinkler theoretical discharge in Cartesian coordinate system

**Theoretical rotation speed equation of the square wetted area sprinkler**

The theoretical rotation speed of the square wetted area sprinkler can be derived from the VRCS operational equation of the formula (8):

$$\omega = \frac{2Q}{KR^2} \tag{12}$$

Formula (12) shows that the theoretical rotation speed of the square wetted area sprinkler is inversely proportional to the square throw distance when the flow rate is a constant. That is:

$$\frac{\omega}{\omega_0} = \frac{R_0^2}{R^2} \tag{13}$$

Where,  $\omega$  is the theoretical rotation speed of the square wetted area sprinkler and  $\omega_0$  is the minimum rotation speed of the square wetted area sprinkler (when  $\alpha = \pi/4$ ).

Putting formula (9) into (13), the theoretical equation that shows the relationship between the rotation speed and rotation angle of the square wetted area sprinkler is obtained:

$$\omega = 2\omega_0 \cos^2 \alpha \quad (0 \leq \alpha \leq \pi/4) \tag{14}$$

From formula (14), when  $\alpha = 0$ , the maximum rotation speed  $\omega = 2\omega_0$ .

The theoretical rotation speed of square wetted area sprinkler at the range of  $\pi/4 \leq \alpha \leq 2\pi$  can be obtained by the symmetry of square. Figures 7 and 8 show the theoretical rotation speed in Cartesian coordinate system and polar coordinate system at all ranges that were drawn by the computer language MATLAB. Here suppose  $\omega_0 = 1$ .

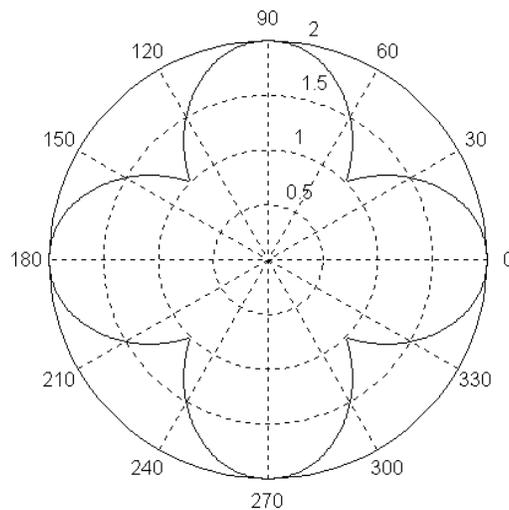
**CONCLUSION**

In order to achieve high uniformity irrigation, the variable-rate contour-controlled sprinkler operational equation was derived. This equation shows that the flow rate of the sprinkler is proportional to the product of the square throw distance and rotation speed.

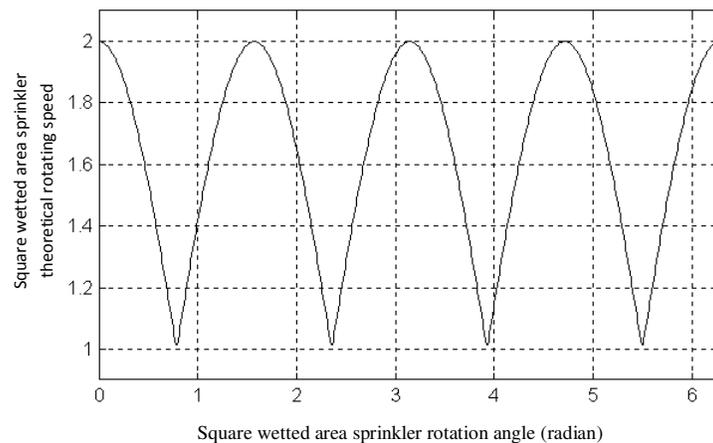
The square wetted area sprinkler was used to illustrate the application of variable-rate contour-controlled sprinkler operational equation. The theoretical equations of throw distance, flow rate and rotating speed of the variable-rate square wetted area sprinkler were derived. These equations provide fundamental principles for the design of variable-rate contour-controlled sprinklers and square wetted area sprinklers.

**ACKNOWLEDGEMENTS**

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**Figure 7.** Square wetted area sprinkler theoretical rotating speed in Cartesian coordinate system.



**Figure 8.** Square wetted area sprinkler theoretical rotating speed in Cartesian coordinate system

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