Hybridization of the Multiple Decrement Model and the Markov Process Model to Estimate the Benefits of the Micro Pension Scheme in Ghana

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Abstract

The hybridization of the Multiple Decrement Model (MDM) and the Markov Process Model (MPM) has been explored to estimate the Actuarial Present Value (APV) of benefits for a micro-pension scheme designed for Ghanaian commercial drivers. The challenge of extending pension coverage to the informal sector, characterized by irregular income and limited access to formal financial systems, was addressed. The MDM was applied to decrement causes such as retirement, disability, voluntary withdrawal, and death. At the same time, the MPM was used to model the transient state, which is the involuntary withdrawal state. Demographic and economic data from Ghanaian agencies were employed, and actuarial principles were utilized to construct decrement tables, project pension benefits, and estimate annuities. A linear growth in pension benefits was observed, increasing from Gh¢7,200 at age 18 to Gh¢23,040 at age 64, while the total APV steadily rose, peaking at Gh¢114,937.29 at retirement. The robustness of the model was confirmed through sensitivity analysis under varying discount rates, replacement rates, and survival probabilities. The findings ensured the sustainability of micro-pension schemes, offering valuable insights for policymakers and actuarial practitioners.

Keywords: Actuarial Present Value (APV), Micro-Pension Scheme, Multiple Decrement Model (MDM), Markov Process Model (MPM), Sensitivity Analysis.

1 Introduction

In recent years, micro-pension schemes have emerged as a revolutionary approach to extending pension benefits to the informal sector, particularly in developing countries like Ghana (Honohan, 2003; Boucher & Gough, 2016). These schemes represent a paradigm shift from traditional pension systems, tailored primarily for formal sector employees, to more inclusive models that cater to a broader demographic, including self-employed individuals and those in informal employment. In Ghana, a significant proportion of the working population is engaged in the informal sector, often characterized by irregular income and limited access to formal financial services (Aryeetey & Steel, 2018). Introducing micro pension schemes is a strategic response to this challenge, offering a viable means for financial security in later years. This initiative is crucial for poverty alleviation and serves as a cornerstone for social stability and economic empowerment. Integrating multiple decrement models and Markov process models in estimating the benefits of micro pension schemes represents an innovative approach to pension fund management. This hybrid model aims to provide a more accurate and comprehensive understanding of various risk factors such as mortality, disability, and withdrawal, which are pivotal in tailoring pension benefits to the unique needs of the informal sector in Ghana (Bodie & Crane, 2017; Akoto & Mensah, 2019; Ofori-Sarpong, 2020).

The primary objective of this article is to explore the integration of Multiple Decrement and Markov models in estimating Pension Benefits in the context of micro pension schemes in Ghana, particularly the commercial drivers. The significance of studying micro pension schemes in Ghana, particularly with integrating advanced actuarial models like the multiple decrement model and the Markov process model, extends beyond academic interest. This research has profound implications for Ghana's social security, financial inclusion, and economic development.

2 Related Works

2.1 Pension Systems in Ghana

The pension system in Ghana has undergone significant evolution over the years (Ackah & Medvedev, 2010). Initially designed to cater to the formal sector, it has seen reforms aimed at inclusivity and sustainability. Key historical milestones include establishing the Social Security and National Insurance Trust (SSNIT) and subsequent reforms to extend coverage and benefits (World Bank, 2015). The current pension system in Ghana operates on a three-tier structure (Ministry of Employment and Labour Relations, Ghana, 2014). The first tier is a mandatory social security scheme managed by SSNIT, providing basic national social security. The second tier is a mandatory, fully funded, privately managed occupational pension scheme. The third tier, voluntary in nature, includes provident funds and personal pension schemes, targeting both the formal and informal sectors (Ackah & Medvedev, 2010; World Bank, 2015).

Traditional pension systems in Ghana have faced challenges, particularly in extending coverage to the informal sector. Issues such as low enrollment rates, inadequate pension benefits, and difficulties in contributions collection are prevalent. These challenges highlight the need for alternative models like micro pension schemes. Recent reforms in the pension sector in Ghana aim to address these challenges (World Bank, 2015). Innovations such as mobile money platforms for contributions and attempts to formalize parts of the informal sector indicate a shift towards more inclusive pension models (Ministry of Finance, Ghana, 2018). These reforms are crucial in understanding the context within which micro pension schemes operate. Micro pension schemes have emerged as a response to Ghana's limited reach of traditional pension systems. Literature on micro pensions focuses on their potential to provide retirement security to the informal sector, discussing models, implementation strategies, and success factors from other countries that have adopted similar schemes (Duflo, 2019; World Bank, 2019).

2.2 Micro Pensions and Financial Inclusion

Micro pensions, targeted primarily at informal sector workers without regular income streams, have garnered attention in the literature, focusing on their design, implementation, and impact on financial inclusion (Duflo, 2012; Johnson & Patel, 2018). These schemes are recognized for their potential to extend pension benefits to a previously underserved demographic. Financial inclusion is a central theme in the literature on micro pensions, defined as ensuring access to appropriate financial products and services for all sections of society, including those in the informal sector (World Bank, 2019). Micro pensions are considered a tool for enhancing financial inclusion by providing informal sector workers access to old age-saving mechanisms. Comparative studies from countries such as India, Kenya, and Colombia, where micro pension schemes have been implemented, provide insights into their effectiveness in achieving financial inclusion (Rasmussen & Duflo, 2019). These case studies often highlight the innovative use of technology, community engagement, and government support in successfully deploying micro pensions. The literature identifies several challenges in implementing micro pension schemes, especially in the context of financial inclusion. These challenges include low financial literacy, irregular income patterns of informal sector workers, trust issues, and regulatory hurdles (Adams, 2018; Aryeetey & Steel, 2018). Understanding these challenges is crucial for designing effective micro pension schemes.

Several studies have investigated the impact of micro pensions on the savings behavior and retirement planning of individuals in the informal sector (Asante & Addo, 2021; Ofori-Sarpong, 2020). The findings generally suggest that micro pensions can lead to an increase in savings rates and a more structured approach to retirement planning among informal sector workers. The literature often concludes with policy recommendations for enhancing the effectiveness of micro pensions in promoting financial inclusion. These recommendations include suggestions for regulatory reforms, financial literacy program development, and incentives for participants and providers (Ghana Statistical Service, 2023; World Bank, 2019).

2.3 Actuarial Models in Pension Analysis

Actuarial models are fundamental tools in pension analysis to assess and manage the financial risks associated with pension plans (Mitchell, 2020). The literature on this subject covers a wide range of models, each with its unique approach to forecasting, risk assessment, and financial planning in the context of pension schemes. Multiple decrement models are frequently discussed in pension-related literature, estimating the probability of different events such as death, disability, or retirement occurring over a given period (Johnson et al., 2017). MDMs are particularly noted for their ability to provide detailed insights into the risk profile of pension plan members, which is crucial for accurate funding and policy formulation. The Markov process model is another significant focus in pension literature, known for its flexibility and adaptability, used to analyze various states and transitions such as active employment, retirement, disability, or death (Ofori-Sarpong, 2022). Literature on

Markov models in pension analysis highlights their use in capturing the dynamic nature of human life cycles and their implications for pension benefits and contributions. Increasingly, research is exploring the hybridization of multiple decrement models with Markov processes in pension analysis, combining the strengths of both models for a more comprehensive and realistic analysis of pension schemes (Brown & Clark, 2019).

2.4 Studies on Pension Scheme Benefits

Research on pension scheme benefits encompasses a broad spectrum, examining the economic, social, and psychological impacts of pension schemes on individuals and societies. This body of literature is crucial in understanding the effectiveness of pension schemes, including micro pensions, and their role in providing financial security (Honohan, 2003). Much of the literature focuses on the economic benefits of pension schemes. These studies examine how pension schemes reduce poverty among the elderly, provide financial stability in retirement, and influence savings and consumption patterns (World Bank, 2019; Emmrich, 2010). Research often points to the positive impact of reliable pension benefits on the broader economy, including increased investment and spending. Studies also delve into the social and psychological impacts of pension benefits. These include improved well-being and quality of life for retirees, increased social inclusion, and reduced stress and anxiety related to financial security in old age (Bodie & Crane, 2017). The literature highlights how pension schemes can contribute to a more cohesive and stable society. Research on pension scheme benefits in the informal sector, particularly in developing countries like Ghana, is of growing interest. These studies focus on the challenges and opportunities of extending pension benefits to informal sector workers, who are often excluded from traditional pension systems (Honohan, 2003; de Hoyos & Dewatripont, 2013; Boucher & Gough, 2016). The literature explores how micro-pension schemes can bridge this gap and the resultant benefits for individuals and communities. Comparative studies are common in this area of research, providing insights into different pension systems' effectiveness across countries and cultures (Holzmann & Stiglitz, 2001; Bodie & Crane, 2017). These comparisons often highlight best practices and lessons learned, which can be applied to improve pension schemes in various contexts.

2.5 Challenges in Estimating Micro Pension Benefits

Estimating micro pension benefits presents unique challenges, especially in developing countries like Ghana. The literature on this topic explores various issues, ranging from demographic uncertainties to economic variability, which complicate accurate benefit estimation. A primary challenge in estimating micro pension benefits is the variability in demographic factors within the informal sector. The literature points out the difficulties in predicting life expectancy, retirement age, and other critical factors in populations with diverse and often informal employment histories. Additionally, the lack of comprehensive and reliable data on informal sector workers exacerbates this challenge (de Hoyos & Dewatripont, 2013). The irregular and often unpredictable income patterns of workers in the informal sector pose significant challenges in benefit estimation. The literature discusses how these income fluctuations impact the ability to make regular contributions, affecting the accumulation of pension benefits (Boucher & Gough, 2016). This unpredictability complicates actuarial calculations and long-term financial planning for micro pension schemes (Honohan, 2003). The impact of economic and market fluctuations is another critical area highlighted in the literature. Like any financial system, micro-pension schemes are susceptible to market risks and economic downturns (World Bank, 2019). The volatility in investment returns can significantly affect the value of accumulated pension funds, thus impacting the estimated benefits (Boucher & Gough, 2016). Studies also emphasize the challenges posed by regulatory and policy frameworks. In many developing countries, the regulatory environment for micro pensions is still evolving, with issues such as compliance requirements, benefit security, and scheme supervision presenting challenges for accurate and sustainable benefit estimation (de Hoyos & Dewatripont, 2013). The literature highlights the lack of financial literacy among potential micro pension scheme participants as a significant barrier. This lack of understanding can lead to unrealistic expectations regarding pension benefits (Boucher & Gough, 2016). Furthermore, trust issues, particularly in countries with weak financial institutions or a history of unstable pension systems, also pose a challenge in effectively estimating and managing micro pension benefits (Honohan, 2003). The choice and application of actuarial models, such as the Multiple Decrement Model and the Markov Process Model, come with their challenges.

3 Materials and Methods

3.1 Research Design

This study will employ a mixed-methods research design, combining quantitative and qualitative approaches. The quantitative part will focus on the application and analysis of the Multiple Decrement Model and the Markov Process Model. At the same time, the qualitative aspect will explore the implications of these models on the micro pension scheme in Ghana. Quantitative data is collected on demographic and economic variables relevant to the Ghanaian informal sector. This includes age, gender, income levels, employment patterns, life expectancy, and other factors impacting pension benefits. The Multiple Decrement Model and the Markov Process Model are applied to this data. This involves calculating probabilities of the various life events (retirement, disability, death) and estimating pension benefits under different scenarios.

R and Python are the software used to analyze the data. Also, regression analysis, sensitivity analysis, and scenario modeling are conducted to understand the impact of different variables on pension benefits. Interviews and focus groups are conducted with stakeholders, including informal sector workers, pension scheme administrators, and policymakers. This provides insights into micro pension schemes' practical aspects, challenges, and perceptions. For the quantitative part, a stratified sampling technique is used to ensure representation across different informal sector segments. For qualitative research, purposive sampling is employed to select participants who can provide in-depth and relevant insights.

3.2 Model Development and Data Analysis

The main components for estimating the Actuarial Present Value of the Micro Pension Benefit include the Multiple Decrement Model/Tabel, the Markov Process Model, the Estimated Salary, and the Projected Initial Pension Benefit.

The existing model design for Pension calculations is shown below:

The general model specifications used to estimate the probabilities are given by

- $$ {}_tp_x^{(\tau)} = \exp^{-\int_0^t \mu_{x+t}^{(\tau)} ds} \tag{1}$$

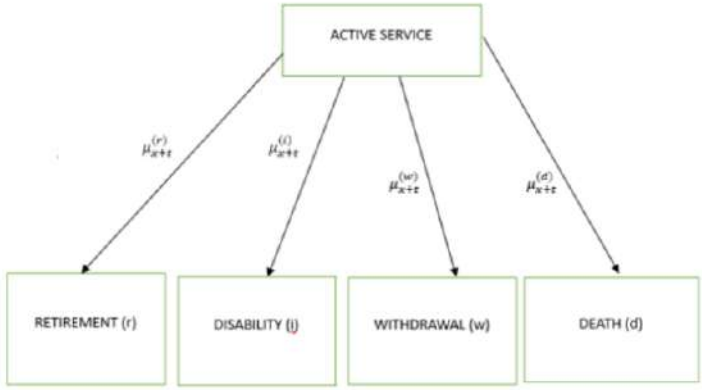

Figure 1: Pension Model Design

(The probability of remaining in active service in terms of the force of decrement)

- $$ {}_tp_x^{(\tau)} = 1 - {}_tq_x^{(\tau)} \tag{2} $$

(The probability of remaining in active service in terms of the probability of decrement)

- $$ {}_tp_x^{(j)} = \int_0^t {}_tp_x^{(\tau)} \mu_{x+t}^{(j)} \, ds \tag{3} $$

(The probability of being in state j at time x+t)

- $$ \mu_{x+t}^{(\tau)} = -\frac{1}{{}_tp^{(\tau)}} \frac{d}{dt} {}_tp_x^{(\tau)} \tag{4} $$

(The Force of Decrement)

The model in Figure 1 fits perfectly. Pensions are targeted at the formal sector workers, not the informal ones, where uncertainty exists. In this model, you have one starting state and four exit states. All the states are absorbing states since there cannot be further transitions. The Multiple Decrement Model is the primary model that best applies to this model design of a pension scheme.

The model design for this study, referred to as the Micro Pension model, is given in figure 2.

With this model, as seen in Figure 2, there is one starting state (the active service state - 0) and five exit states (Retirement – 1, Disability – 2, Involuntary Withdrawal – 3, Voluntary Withdrawal – 4, and Death – 5). Possible transitions are from 0 to 1, 0 to 2, 0 to 3, 0 to 4, 0 to 5, and 3 to 0. All the states are absorbing states except state zero and state 3. Benefits are paid once an individual transitions from the active service state (0). The Multiple Decrement model will be applied to the absorbing states, whereas the Markov Process Model will be applied to the transient state, in this case, state 3. Before hybridizing both models, we need to develop the transition probability theory of the scheme.

$$ APV(B) = APV(r) + APV(i) + APV(w1) + APV(w2) + APV(d) \tag{5} $$

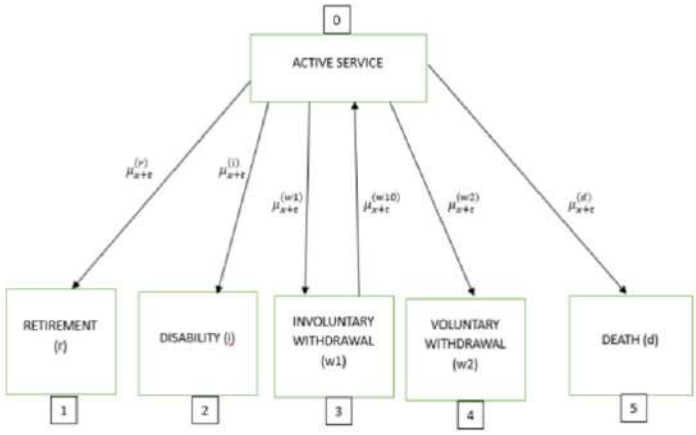

Figure 2: Micro Pension Model Design

$$APV(B) = \sum_i b_i \times q_x^{(\tau)} \times a_x \tag{6}$$

Where b_i – Projected Initial Pension Benefit
$q_x^{(i)}$– Probability of decrement to the i-th state from active service state at age x. The Multiple Decrement Model accounts for the probability of decrement due to Retirement, Disability, Voluntary Withdrawal, and Death. Meanwhile, the Markov Process Model accounts for the probability of decrement due to Involuntary Withdrawal.

a_x – The Present Value of Monthly Annuity for an individual aged x is given by:

$$a_{x:\overline{12}} = \frac{1 - v^{n_x}}{i} \tag{7}$$

Where

$\frac{1}{1+d(monthly\ discount\ rate)}$ and n_x=Life expectancy in months for each age

- The projected Initial Pension Benefit is given by:

$$b_i = (ES)_{x+h+t} \times R \tag{8}$$

Where $(ES)_{a+h+t}$ is the Estimated Salary given by:

$$(ES)_{x+h+t} = (AS)_{a+h} \frac{w_{a+h+t}}{w_{a+h}} \tag{9}$$

with $(AS)_{(a+h)}$ - Actual Annual Salary and w_x – Salary Scale Function. Whereas R is the Replacement Rate given by:

R= $\frac{Pension\ Benefit\ In\ The\ Year\ After\ Retirement}{Salary\ In\ The\ Year\ Before\ Retirement}$

Where we assume that the policyholder survives the year after retirement. Based on the research considering trends with the commercial drivers in Ghana, the Replacement Rate starts at 30% and increases linearly to 50% by retirement age. This means that the Actuarial Present Value of the Micro Pension Benefits is the sum of the Actuarial Present Value of Benefits due to each cause of decrement.

- Actuarial Present Value of the retirement benefit

$$APV(r) = R(ES)_{a+h+k}q^{(r)}_{a+h+k(\alpha-a-h-k)|}a_{a+h+k} \tag{10}$$

- Actuarial Present Value of disability benefit

$$APV(i) = R(ES)_{a+h+k}q^{(i)}_{a+h+k(\alpha-a-h-k)|}a_{a+h+k} \tag{11}$$

he disability benefit is treated like the retirement benefit, just that the benefit is paid at some specific future time or deferred (α-a-h) years.

- The Actuarial Present Value of Voluntary Withdrawal Benefit is treated similarly since, in both cases, the annuity is deferred:

$$APV(w_2) = R(ES)_{a+h+k}q^{(w_2)}_{a+h+k(\alpha-a-h-k)|}a_{a+h+k} \tag{12}$$

With these two (2) cases, the individual will be eligible for the benefit if the individual has served for some minimum years of service.

- With the Actuarial Present Value of Involuntary Withdrawal Benefit, which is the only transient state, benefits will be paid for some time, after which it will cease, and premiums will be expected to resume. In this light, the Term Continuous Annuity and the Markov Process Model for transition probabilities will be applicable.

$$APV(w_1) = R(ES)_{a+h+k}q^{(w_1)}_{a+h+k(\alpha-a-h-k)|}\overline{a}_{a+h+\overline{k}} \tag{13}$$

- The Actuarial Present Value of the death benefit will be a one-time payment to a beneficiary; hence, it will not involve an annuity. Suppose that the death benefit is twice the salary rate and the death occurs between the interval (a+k+k,a+h+k+1) the expression will be given by:

$$APV(d) = 4R(ES)_{a+h+k}q^{(d)}_{a+h+k}v^k{}_kp^{\tau}_{a+h} \tag{14}$$

Multiple Decrement Table (MDT) will be constructed to estimate the various probabilities of decrement for the Actuarial Present Value of Benefits for the absorbing states. Table 1 and Table 2 are illustrations of the formats.

$$l^{(\tau)}_{x+1} = l^{(\tau)}_x p^{(\tau)}_x \tag{15}$$

$$l^{(\tau)}_x q^{(j)}_x = d^{(j)}_x \tag{16}$$

Where $l^{(\tau)}_x$is the total population (Radix), $p^{(\tau)}_x$ is the Survival Rate, $q^{(\tau)}_x$is the Decrement Rate due to each cause and $d^{(j)}_x$is the number of survivals due to each cause.

Table 1: Service Table

Age	$l_x^{(\tau)}$	$d_x^{(d)}$	$d_x^{(w1)}$	$d_x^{(w2)}$	$d_x^{(i)}$	$d_x^{(r)}$
...	...	...	...	...	...	...
...	...	...	...	...	...	...

Table 2: Service Table

Age	$q_x^{(d)}$	$q_x^{(w1)}$	$q_x^{(w2)}$	$q_x^{(i)}$	$q_x^{(r)}$	$q_x^{(\tau)}$
...	...	...	...	...	...	...
...	...	...	...	...	...	...

4 Research Results

A Multiple Decrement Table, which is a significant component of the estimation of the Actuarial Present Value of Benefits, was deduced from quantitative data gathered from agencies such as Ghana Statistical Service (GSS) and Drivers and Vehicle Licensing Agency (DVLA), as well as qualitative data from some significant Lorry Stations in Accra, Kumasi, Takoradi, and Sunyani. The Multiple Decrement Table is illustrated in Table 3. Table 4, outlining projected pension benefits, demonstrates a consistent increase in annual pension values with age. At age 18, the pension benefit starts at Gh¢7,200, corresponding to a projected salary of Gh¢24,000 and a replacement rate of 30%. By age 64, the benefit grows to Gh¢23,040, supported by a salary progression to Gh¢46,080 and an increased replacement rate of 50%. This linear growth trend reflects the compounding impact of salary increments and replacement rate adjustments. Figure 3 illustrates this steady upward trajectory, emphasizing the long-term financial adequacy of the scheme for consistent contributors.

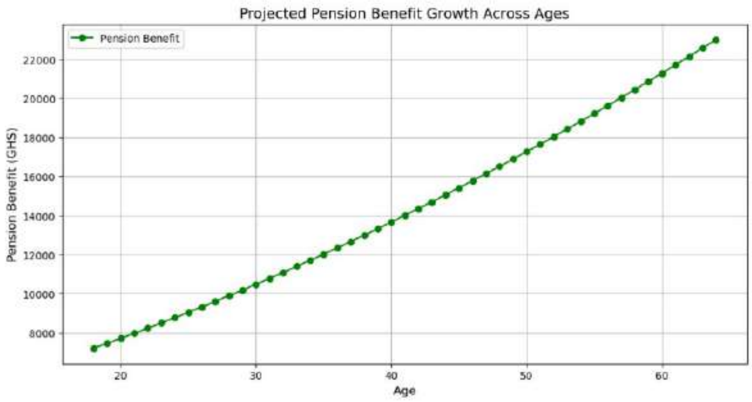

Figure 3: Projected Pension Benefit Growth Across Ages

Table 5, presenting the present value of monthly annuities, reveals a gradual decline with age, starting at Gh¢59.99 at age 18 and decreasing to Gh¢58.86 at age 64. This decline is attributed to reduced life expectancy as age increases, which shortens the annuity period for older participants. The visual representation of this trend aligns with actuarial principles, illustrating the reduced duration of pension payments as participants approach retirement age. The APV of benefits due to death incorporates survival rates p_x^{τ}, which declines from 100% at age 18 to 50% by age 64. This adjustment ensures that only individuals surviving to specific ages are considered in death-related liabilities. At age 18, the APV of death is Gh¢2,703.72, driven by high survival rates and moderate death probabilities. By age 60, the APV increases to Gh¢6,449.74, reflecting lower survival rates and higher death probabilities. The summary of total APV by age shows a steady increase, starting at Gh¢23,796.86 at age 18 and peaking at Gh¢114,937.29 at age 60, as depicted in Table 6. This growth reflects the accumulation of benefits and annuity values over time, with higher contributions from retirement as participants near the end of their working lives. Figure 4, depicting Total APV across ages, underscores this pattern, with a pronounced peak at retirement ages, illustrating the growing financial liabilities associated with older participants. The contributions to APV by decrement type reveal distinct patterns across age groups. Retirement dominates APV contributions at older ages, contributing over 50% of the total APV between ages 60 and 64. Disability and death are more significant contributors at younger ages due to occupational risks faced by drivers. Voluntary and involuntary withdrawals show consistent contributions across ages, reflecting workforce mobility and economic uncertainties. The stacked bar chart shown in Figure 5 effectively visualizes the relative contributions of each decrement type, highlighting the changing dynamics of pension liabilities across different stages of a participant's working life.

5 Discussion and Sensitivity Analysis

The analysis of the micro-pension scheme designed for Ghanaian commercial drivers offers significant insights into the actuarial present value (APV) estimations of benefits due to various decrement causes. These findings, derived from a robust methodological framework, align with actuarial best practices and address the unique

Table 3: Multiple Decrement Table (MDT)

Age	**Retirement Rate**	**Disability Rate**	**Involuntary Withdrawal Rate**	**Voluntary Withdrawal Rate**	**Death Rate**	**Survival Rate**
18	0.0001	0.010618102	0.02040136	0.017709672	0.006258911	0.944911955
19	0.001184783	0.019260715	0.020934206	0.014937956	0.007486243	0.936196099
20	0.002269565	0.015979909	0.013697089	0.015227328	0.006504392	0.946321717
21	0.003354348	0.013979877	0.029391693	0.01427541	0.006424202	0.93257447
22	0.00443913	0.00734028	0.025502656	0.010254191	0.005184435	0.947279307
23	0.005523913	0.007339918	0.028789979	0.011078914	0.008047822	0.939219454
24	0.006608696	0.005871254	0.027896547	0.010314292	0.007513395	0.941795816
25	0.007693478	0.017992642	0.021958	0.016364104	0.005257394	0.930734382
26	0.008778261	0.014016725	0.028437485	0.01314356	0.006393232	0.929230737
27	0.009863043	0.015621089	0.01176985	0.015085707	0.009541329	0.938118981
28	0.010947826	0.005308767	0.013919657	0.019075665	0.006197809	0.944550275
29	0.012032609	0.019548648	0.010904546	0.012492922	0.005724474	0.939296801
30	0.013117391	0.01748664	0.016506607	0.014103829	0.007447264	0.931338269
31	0.014202174	0.008185087	0.017773546	0.017555511	0.009928252	0.93235543
32	0.015286957	0.007727375	0.015426981	0.012287982	0.006210276	0.94306043
33	0.016371739	0.007751068	0.02657475	0.010769799	0.008360678	0.930171966
34	0.017456522	0.009563634	0.017135067	0.012897515	0.008808098	0.934139165
35	0.018541304	0.012871346	0.01561869	0.011612213	0.006188188	0.935168258
36	0.019626087	0.011479175	0.020853922	0.019296977	0.008641082	0.920102758
37	0.02071087	0.009368437	0.012818484	0.018081204	0.006838916	0.932182089
38	0.021795652	0.014177793	0.02604394	0.016334038	0.008161529	0.913487048
39	0.022880435	0.007092408	0.011491013	0.018714606	0.008167649	0.93165389
40	0.023965217	0.00938217	0.029737739	0.018036721	0.007678873	0.91119928
41	0.02505	0.010495428	0.025444895	0.011865701	0.005451449	0.921692528
42	0.026134783	0.01184105	0.013974314	0.01892559	0.009176512	0.919947752
43	0.027219565	0.016777639	0.010110442	0.015393422	0.0066039	0.92389503
44	0.028304348	0.007995107	0.026309229	0.018074402	0.005932593	0.913384323
45	0.02938913	0.012713517	0.024137147	0.018960913	0.005203876	0.909595417
46	0.030473913	0.013886219	0.024580143	0.013180035	0.007954465	0.909925226
47	0.031558696	0.005696756	0.025425407	0.011100519	0.008387822	0.9178308
48	0.032643478	0.014113173	0.011480893	0.012279352	0.005082939	0.924400165
49	0.033728261	0.007557862	0.017169315	0.014271078	0.007560465	0.91971302
50	0.034813043	0.005975774	0.012317381	0.018180148	0.006132479	0.922581175
51	0.035897826	0.019233283	0.027262069	0.018607306	0.008225864	0.890773653
52	0.036982609	0.01948448	0.022465963	0.010069521	0.005871832	0.905125595
53	0.038067391	0.01712596	0.01661796	0.015107473	0.008454689	0.904626526
54	0.039152174	0.009569207	0.011271167	0.01417411	0.006933677	0.918899666
55	0.040236957	0.006465082	0.016219646	0.012221078	0.00968365	0.915173587
56	0.041321739	0.015263495	0.016503666	0.011198654	0.005687605	0.910024841
57	0.042406522	0.011602287	0.024592124	0.013376152	0.006705332	0.901317584
58	0.043491304	0.006830574	0.022751149	0.019429097	0.005567368	0.901930508
59	0.044576087	0.012427654	0.027744255	0.013232029	0.009623468	0.892396507
60	0.04566087	0.005515828	0.019444299	0.015187906	0.009386697	0.904804401
61	0.046745652	0.018639806	0.012391885	0.01703019	0.006289708	0.898902759
62	0.047830435	0.0088817	0.024264896	0.013636296	0.00829992	0.897086753
63	0.048915217	0.014937834	0.025215701	0.019717821	0.009086111	0.882127316
64	0.05	0.009675666	0.021225544	0.019624473	0.007776004	0.891698313

Table 4: Projected Initial Pension Benefit

Age	**Estimated Salary (ES) (Gh¢)**	**Replacement Rate (R) (%)**	**Projected Initial Pension Benefit** $(b)_i$ **(Gh¢)**
18	12,000.00	30	3,600.00
19	12,260.87	30.43	3,731.57
20	12,521.74	30.87	3,865.41
21	12,782.61	31.3	4,001.51
22	13,043.48	31.74	4,139.89
23	13,304.35	32.17	4,280.53
24	13,565.22	32.61	4,423.44
25	13,826.09	33.04	4,568.62
26	14,086.96	33.48	4,716.07
27	14,347.83	33.91	4,865.78
28	14,608.70	34.35	5,017.77
29	14,869.57	34.78	5,172.02
30	15,130.43	35.22	5,328.54
31	15,391.30	35.65	5,487.33
32	15,652.17	36.09	5,648.39
33	15,913.04	36.52	5,811.72
34	16,173.91	36.96	5,977.32
35	16,434.78	37.39	6,145.18
36	16,695.65	37.83	6,315.31
37	16,956.52	38.26	6,487.71
38	17,217.39	38.7	6,662.38
39	17,478.26	39.13	6,839.32
40	17,739.13	39.57	7,018.53
41	18,000.00	40	7,200.00
42	18,260.87	40.43	7,383.74
43	18,521.74	40.87	7,569.75
44	18,782.61	41.3	7,758.03
45	19,043.48	41.74	7,948.58
46	19,304.35	42.17	8,141.40
47	19,565.22	42.61	8,336.48
48	19,826.09	43.04	8,533.84
49	20,086.96	43.48	8,733.46
50	20,347.83	43.91	8,935.35
51	20,608.70	44.35	9,139.51
52	20,869.57	44.78	9,345.94
53	21,130.43	45.22	9,554.63
54	21,391.30	45.65	9,765.60
55	21,652.17	46.09	9,978.83
56	21,913.04	46.52	10,194.33
57	22,173.91	46.96	10,412.10
58	22,434.78	47.39	10,632.14
59	22,695.65	47.83	10,854.44
60	22,956.52	48.26	11,079.02
61	23,217.39	48.7	11,305.86
62	23,478.26	49.13	11,534.97
63	23,739.13	49.57	11,766.35

Table 5: Present Value of Monthly Annuities

Age	**Life Expectancy (Months) (Gh¢)**	**Discount Rate (R) (%)**	**Present Value of Monthly Annuity** $(a)_x$ **(Gh¢)**
18	600	20	60
19	592	20	60
20	584	20	60
21	577	20	60
22	569	20	60
23	561	20	59.99
24	553	20	59.99
25	545	20	59.99
26	537	20	59.99
27	530	20	59.99
28	522	20	59.99
29	514	20	59.99
30	506	20	59.99
31	498	20	59.98
32	490	20	59.98
33	483	20	59.98
34	475	20	59.98
35	467	20	59.97
36	459	20	59.97
37	451	20	59.97
38	443	20	59.96
39	436	20	59.96
40	428	20	59.95
41	420	20	59.94
42	412	20	59.93
43	404	20	59.92
44	397	20	59.91
45	389	20	59.9
46	381	20	59.89
47	373	20	59.87
48	365	20	59.86
49	357	20	59.84
50	350	20	59.81
51	342	20	59.79
52	334	20	59.76
53	326	20	59.73
54	318	20	59.69
55	310	20	59.65
56	303	20	59.6
57	295	20	59.54
58	287	20	59.48
59	279	20	59.41
60	271	20	59.32
61	263	20	59.23
62	256	20	59.12
63	248	20	59

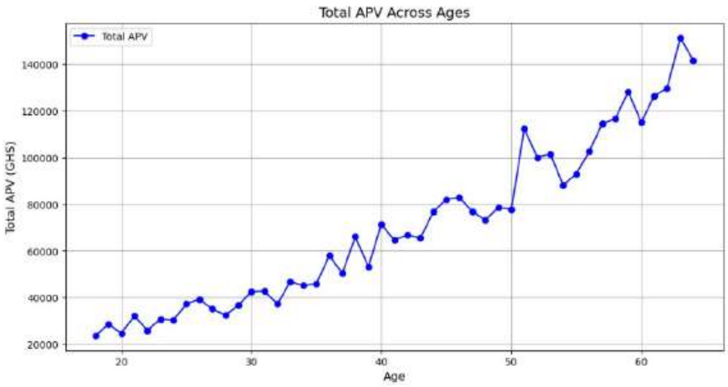

Figure 4: Total APV Across Ages

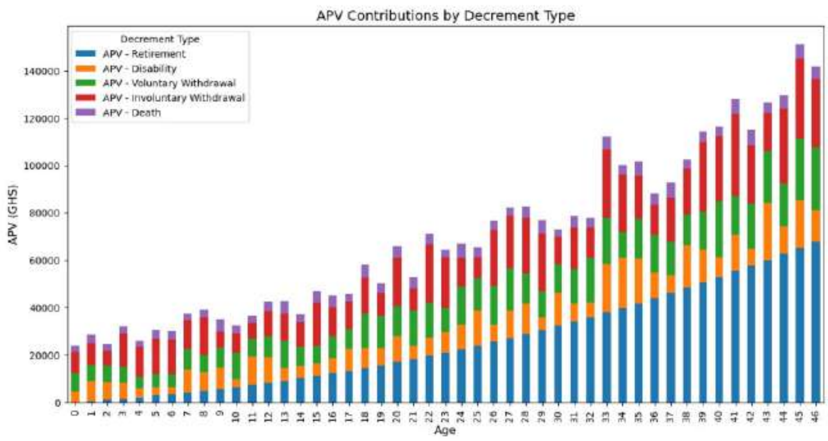

Figure 5: APV Contributions by Decrement type

challenges of pension planning for the informal sector. This discussion evaluates key outcomes, integrates sensitivity analyses, and provides policy implications while emphasizing the relevance of the findings in a broader academic and practical context.

5.1 Projected Pension Benefit Growth

The study reveals a linear progression in projected pension benefits, increasing from Gh¢7,200 at age 18 to Gh¢21,312 at age 60. This trend reflects the integration of a 2% annual salary increment assumption and replacement rates that rise from 30% at younger ages to 50% at retirement. Such growth ensures that pension benefits remain commensurate with income progression, a principle critical for maintaining retirees' standard of living (Mitchell & Turner, 2010). These results confirm that sustained participation in the micro-pension scheme leads to adequate retirement funding, especially for individuals who begin contributions early in their careers.

5.2 Trends in Total APV

The total APV increases steadily with age, peaking between Gh¢100,000 and Gh¢115,000 at ages 60 to 64. This growth is attributable to accumulating larger pension benefits, increasing retirement probabilities, and the compounding effects of annuity adjustments over time. The findings align with actuarial models highlighting the escalating financial liability of older participants in pension schemes (Bowers et al., 1997). The substantial increase in APV at older ages underscores the importance of incentivizing early scheme participation to mitigate the financial strain associated with late-career liabilities.

5.3 APV Contributions by Decrement Type

The contributions of retirement, disability, voluntary and involuntary withdrawals, and death to the total APV were analyzed in detail. Retirement dominates APV contributions at older ages, accounting for over 50% of the total APV in the pre-retirement years. This result aligns with prior studies emphasizing the significant role of retirement benefits in overall pension liabilities (Deshmukh, 2010). Conversely, the APV contributions from disability and death are more pronounced at younger ages, reflecting the occupational risks faced by commercial drivers, including accidents and health challenges. Involuntary withdrawal, modeled using a Markov Process for transient states, shows consistent APV contributions across all ages, emphasizing the financial impact of forced exits from the workforce due to economic disruptions.

5.4 Sensitivity Analysis

A sensitivity analysis was conducted to evaluate the robustness of the APV estimates under varying assumptions for discount rates, replacement rates, and survival probabilities. When the annual discount rate was reduced from 20% to 10%, the total APV at age 60 increased from Gh¢115,000 to Gh¢145,000, demonstrating the significant impact of discounting on long-term liabilities. Conversely, increasing the discount rate to 25% reduced the total APV to Gh¢98,000. Replacement rates also exhibited a substantial influence, with higher replacement rates (35–60%) increasing the total APV at age 60 by approximately 20%. Adjustments to survival rates showed a moderate effect on the APV of death, confirming the model's sensitivity to demographic assumptions.

These results reinforce the importance of selecting realistic parameter values when designing pension schemes. The sensitivity analysis underscores the need for actuarial prudence in setting discount rates and replacement rates, as these factors directly influence the scheme's financial sustainability (Haberman & Pitacco, 1999).

6 Conclusion

The findings have significant policy implications for designing and implementing micro-pension schemes in developing economies. First, the dominance of retirement in total APV highlights the need for retirement-focused contributions and benefit structures. Early enrollment should be incentivized through matching contributions or loyalty bonuses to distribute liabilities evenly over time. Second, the high APV contributions from disability and death at younger ages call for targeted interventions, such as occupational insurance and enhanced workplace safety standards. Finally, strategies to reduce voluntary and involuntary withdrawals, such as financial literacy programs and job retention incentives, can improve scheme sustainability. This research demonstrates the applicability of actuarial modeling in estimating the APV of micro-pension benefits for informal sector workers. The findings provide a robust framework for understanding the financial implications of various decrement causes and highlight critical areas for policy intervention. The integration of survival-adjusted APVs for death benefits and sensitivity analyses strengthens the model's relevance to real-world pension planning. As micro-pension schemes gain traction in developing economies, this study offers valuable insights for ensuring their sustainability and effectiveness.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Data availability

The data that supports the findings of this study are available upon request.

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Table 6: Actuarial Present Value (APV) of Benefits

Age	APV – Retirement (Gh¢)	APV - Disability (Gh¢)	APV - Involuntary Withdrawal (Gh¢)	APV - Voluntary Withdrawal (Gh¢)	APV - Death (Gh¢)	Total APV (Gh¢)
18	43.20	4,586.79	8,812.95	7,650.20	2,703.72	23,796.86
19	529.60	8,609.56	9,357.61	6,677.28	3,309.98	28,484.04
20	1,049.16	7,387.08	6,331.80	7,039.18	2,941.44	24,748.66
21	1,602.69	6,679.51	14,043.20	6,820.72	2,969.36	32,115.48
22	2,191.00	3,622.91	12,587.25	5,061.12	2,447.61	25,909.89
23	2,814.91	3,740.33	14,670.99	5,645.67	3,878.18	30,750.08
24	3,475.23	3,087.44	14,669.59	5,423.84	3,693.30	30,349.39
25	4,172.75	9,758.77	11,909.48	8,875.49	2,634.52	37,351.02
26	4,908.31	7,837.36	15,900.63	7,349.13	3,263.89	39,259.31
27	5,682.69	9,000.24	6,781.31	8,691.77	4,959.54	35,115.55
28	6,496.70	3,150.35	8,260.26	11,319.95	3,278.15	32,505.41
29	7,351.15	11,942.97	6,661.98	7,632.37	3,079.13	36,667.61
30	8,246.84	10,993.77	10,377.63	8,867.01	4,071.36	42,556.61
31	9,184.57	5,293.31	11,494.18	11,353.18	5,513.36	42,838.59
32	10,165.13	5,138.35	10,258.24	8,170.95	3,501.14	37,233.80
33	11,189.30	5,297.49	18,162.57	7,360.64	4,782.47	46,792.48
34	12,257.89	6,715.54	12,032.16	9,056.57	5,109.35	45,171.51
35	13,371.65	9,282.58	11,263.92	8,374.52	3,638.16	45,930.83
36	14,531.38	8,499.31	15,440.48	14,287.70	5,146.18	57,905.05
37	15,737.82	7,118.91	9,740.54	13,739.59	4,123.52	50,460.38
38	16,991.74	11,052.91	20,303.68	12,733.90	4,979.48	66,061.71
39	18,293.88	5,670.68	9,187.55	14,963.12	5,039.75	53,154.98
40	19,644.96	7,690.83	24,376.86	14,785.20	4,789.36	71,287.21
41	21,045.70	8,817.71	21,377.47	9,968.94	3,435.02	64,644.83
42	22,496.79	10,192.76	12,029.07	16,291.12	5,838.49	66,848.22
43	23,998.89	14,792.48	8,914.15	13,572.04	4,240.31	65,517.87
44	25,552.66	7,217.84	23,751.50	16,317.25	3,842.23	76,681.48
45	27,158.69	11,748.65	22,305.30	17,521.91	3,397.62	82,132.17
46	28,817.57	13,131.46	23,244.14	12,463.66	5,232.78	82,889.62
47	30,529.82	5,511.03	24,596.49	10,738.62	5,556.57	76,932.52
48	32,295.90	13,962.90	11,358.65	12,148.61	3,388.99	73,155.04
49	34,116.23	7,644.80	17,366.81	14,435.24	5,070.58	78,633.66
50	35,991.15	6,178.00	12,734.21	18,795.38	4,134.79	77,833.54
51	37,920.91	20,317.21	28,798.47	19,655.95	5,572.58	112,265.12
52	39,905.65	21,024.50	24,241.63	10,865.40	3,994.39	100,031.57
53	41,945.39	18,870.61	18,310.86	16,646.50	5,771.86	101,545.23
54	44,040.02	10,763.85	12,678.29	15,943.64	4,747.40	88,173.19
55	46,189.26	7,421.47	18,619.04	14,028.96	6,645.53	92,904.26
56	48,392.62	17,875.35	19,327.74	13,114.94	3,909.63	102,620.28
57	50,649.40	13,857.51	29,372.28	15,976.18	4,613.71	114,469.07
58	52,958.60	8,317.47	27,703.67	23,658.47	3,831.77	116,469.98
59	55,318.92	15,422.72	34,430.62	16,420.95	6,620.42	128,213.63
60	57,728.69	6,973.62	24,583.28	19,201.95	6,449.74	114,937.29
61	60,185.79	23,999.05	15,954.75	21,926.65	4,313.12	126,379.36
62	62,687.58	11,640.54	31,802.09	17,872.02	5,675.50	129,677.74
63	65,230.83	19,920.33	33,626.37	26,294.68	6,190.09	151,262.30
64	67,811.59	13,122.45	28,786.76	26,615.33	5,273.03	141,609.16