

Learning to teach for mathematical proficiency: Behavioural changes for pre-service teachers on teaching placement

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Abstract

This study reports on an investigation of how field experiences in teaching prepares pre-service teachers (PSTs) to effectively deal with the challenges and complexities of teaching mathematics in Zimbabwean secondary schools. The study was premised on the view that improvement in learning secondary school mathematics in the classroom is related to practitioner development in teaching. However, despite overwhelming research on mathematics knowledge for teachers, the problem of mathematics failure in high schools has not been addressed. A mixed methods approach, based on the perspectives of PSTs on teaching practice (TP) was employed to depict how their practices impact on their knowledge development for teaching mathematics. The study, found that a proficient mathematics PST is considered one who embraces knowledge of mathematics content and the ability to teach it. Hence, the development of mathematics knowledge for PSTs requires them to be mentored and workshopped by mathematics subject specialists who are well positioned in terms of mathematics pedagogy and mathematics content knowledge.

Keywords: learning to teach, mathematics knowledge, pre-service teachers, teaching knowledge, teaching practice

Introduction

Teachers' conceptual understanding and knowledge of a subject is critically important at any level (Walshaw, 2012) and the importance and relevance of teaching mathematics cannot be overstated (Kusmaryono, 2014). Hence, according to Hollins, Luna and Lopez (2014), the quality of teacher preparation is fundamental because it influences teaching competence that impacts on the quality of learning opportunities for learners. The quality of learning opportunities for learners, therefore, determines learning outcomes. Similarly, Lipton and Wellman (2014) assert that the quality of the teacher mostly determines the variation in students' learning achievements and that quality teaching matters for successful student learning. To this end, the fragile knowledge of teachers in mathematics is likely to place boundaries around the development of the students' understanding of the subject (Walshaw, 2012). Hollins et al. (2014) and Walshaw's (2012) assertions suggest that significant improvement in academic performance in schools for learners is unlikely to take place without significant improvement in the preparation of PSTs. This means that a review of the mathematics teacher education curriculum design is paramount. Since it is the teacher who filters the curriculum through to the learners (Jegede, Taplin & Chan 2000), teachers must be adequately and comprehensively developed so that their sound knowledge of the subject makes good sense of mathematical ideas. This paper, therefore, examines how the knowledge of

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mathematics for pre-service teachers (PSTs) on teaching practice (TP) is developed and how it impacts on the PSTs' classroom practices/behaviour.

According to Lipton and Wellman (2014), teachers' knowledge of how learners acquire the knowledge of mathematics content being taught, the process the learners use, and the successes and difficulties that are likely to occur, may form part of a teacher's knowledge of teaching mathematics. What is not clear, however, is the exact appropriate type and level of subject knowledge for teaching school mathematics. In addition, despite overwhelming research on mathematics knowledge for teachers (Dreher, Lindmeier, Heinze & Niemand 2018), the problem of mathematics failure in high schools globally has not been addressed. Hence, based on the Zimbabwean PSTs' perspectives, the study sought to establish how mathematics PSTs' knowledge can be developed to improve their understanding and performance in teaching mathematics. The following research question (RQ) sought to achieve the purpose of the study:

How do PSTs reportedly learn about mathematics and mathematics teaching during teaching practice?

Literature Review

Learning to Teach and Teaching Practice (TP)

TP is a component of learning to teach in teacher education, which provides a transition from theory to real teaching contexts (Saban & Cocklar, 2013). Since learning to teach can be defined as a cognitive and/or behavioural change process (Haser, 2010), TP therefore needs to involve the changing of existing knowledge relating to teaching and learning. The purpose of TP therefore, according to Altintas and Gorgen (2014), is to ensure that PSTs are well prepared for the teaching profession. This is the time when PSTs portray their creativity, ability and talent to marry college-acquired knowledge with practice, thereby comprehending the real world of teaching (Hamaidi, Al-Shara, Arouri & Abu Awwad, 2014). Their capability to transfer knowledge and skills into action enhances their competence in the real teaching environment. This idea reflects that measuring experience and expertise in terms of the passage of time may not suffice if it does not portray the extent of behavioural change in PSTs. Hence, TP, coupled with a reflection of classroom practices and the enactment of identified changes can equate to teaching expertise (Kleickmann, Richter, Kunter, Elsner, Besser, Krauss, & Baumert, 2013).

Precisely because of the presumed significance of TP in the education of prospective teachers, the government of Zimbabwe has, in recent years, prioritised TP. This was done, especially in the teaching of, science, technology and mathematics (STEM) as the gaze on this component of teacher education was intensified (National Report of Zimbabwe, 2004). To date, however, there has been no large-scale research of learning to teach secondary school mathematics from practice in Zimbabwe to justify the increased focus. This study thus, sought to provide insights regarding the contribution of TP to mathematics knowledge development for PSTs, in part because of the consistently disappointing mathematics results at "O" level in Zimbabwe (Kusure & Basira, 2012, ZIMSEC Results Analysis, 2015). With reference to the poor performance in "O" level examinations of 2011 and 2012 (Majongwe, 2013) in Zimbabwe, Mukeredzi (2013) poses that poorly educated teachers produce poorly educated learners. Hence, special attention needs to be directed towards the development of mathematics teacher knowledge to enhance PSTs' proficiency in teaching mathematics.

Koehler and Grouws (1992) proposed a model which establishes that teacher behaviour in the classroom is influenced by the teachers’ knowledge of the content to be taught, knowledge of the learners and knowledge of the methods to teach specific content, in addition to beliefs about teaching mathematics and beliefs about mathematics as a subject. Rozenszajn and Yarden, (2014) also surmise that learning to teach should not be perceived in parts, that is, considering different types of teacher knowledge like CK and PCK, as distinct entities which work independent of each other. Hence, the link between various components of mathematics teacher knowledge, mathematics beliefs and attitudes in relation to classroom behaviour, is illustrated in the model (Figure 1) below (a modified extract from Koehler and Grouw’s (1992) model). The various components from the model are subsequently discussed.

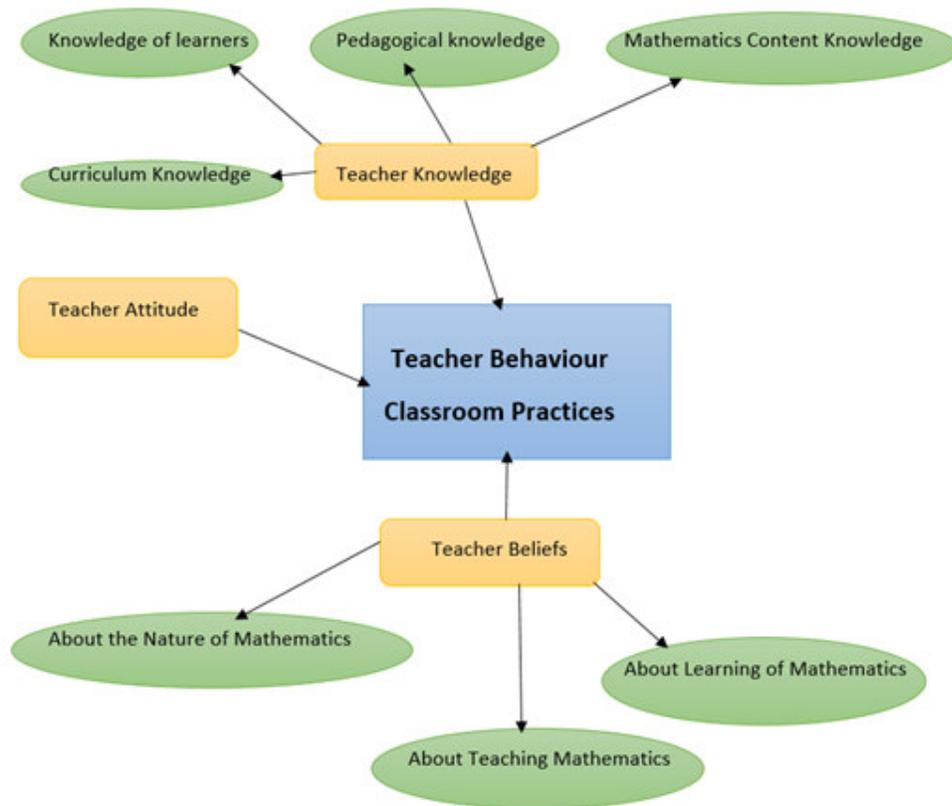


Figure 1 Koehler and Grouws (1992) Research model modified

Teacher knowledge

Pedagogical Knowledge

According to Ball, Hill and Bass (2005), many studies show that mathematics knowledge for teachers is weak and thin and this has impeded effective teaching. To this end, some researchers advocate for an overhaul of teacher preparatory programme curricula to include classroom mathematics and to do away with mathematics methods and professional development. The National Commission on Teaching and America’s Future (NCTAF) (1996), provides compelling evidence that teachers’ subject matter content, has a strong influence on what learners learn. They further argue that teachers need to be conversant with mathematics content thoroughly in order to make mathematical ideas accessible to learners explicitly.

However, content knowledge alone may fail to satisfy the demands of teaching at a particular moment. For this reason, Ball, *et al.* (2001), contend that knowing mathematics content is not synonymous with teaching it, although teaching depends on knowledge. Some researchers argue that PSTs need to learn skills and routines that can be applied to any situation at any time, despite the subject matter. Others contend that PSTs need to learn particular theories about teaching (McDiarmid & Ball, 1998). This suggests that scholars are more concerned about what and how the learners learn, rather than the subject matter knowledge PSTs possess. On the contrary, others assert that subject matter knowledge does not only include mathematical concepts and theories but also an understanding of how knowledge is discovered, tested and organised. This means that the researchers emphasise knowledge on active learning rather than mere memorisation of theories, facts and ideas, which learners cannot sustain.

According to Ball, *et al.* (2005), the volume and complexity of knowledge that teachers have do not assist in disentangling the learners' challenges in the subject and they rather emphasise specifically the question of what to teach and how to teach mathematics. Similarly, Tatto and Senk (2011) argue that, mathematics teachers need to be acquainted with the knowledge of mathematics content, and mathematical pedagogy, if they are to be successful. Shulman (1986), in support of this idea, suggests the amalgam of content and pedagogy (pedagogical content knowledge) to promote effective teaching. This means, without pedagogical content knowledge (PCK), the boundary between a teacher and a subject specialist becomes blurry. The overriding point here is that an understanding of the subject matter (or knowledge thereof), is not essential to a mathematics teacher if it cannot be communicated to the learners. According to Mosvold and Fauskanger (2014), the debate on which component of teacher knowledge is more essential than the other, is not beneficial, though research on a specific component is still relevant. In the same vein, Diko and Feza (2014) suggest that emphasising one component of teacher knowledge at the expense of another is risky. It is not only theoretical knowledge that determines successful teaching but transformation of concepts through meaningful practicum also (Hamaidi *et al.*, 2014). Hence, Shulman (1986) emphasises the balance between subject content knowledge and PCK during learning to teach. However, Darling-Hammond (2006) stresses that teachers need certain levels of ability to be able to teach learners from diverse backgrounds. This suggests the need for intertwining teacher training components for effectiveness (Diko & Feza, 2014).

While several studies acknowledge that much research has been done on teachers' mathematical knowledge, they doubt whether this has achieved the purpose of enhancing learners' mathematics achievement (Ball *et al.*, 2001). Carter and Gonzalez (1993) reiterated that research on teacher knowledge has largely been unproductive. This is partly because, despite the overwhelming research on PSTs' mathematical knowledge, the problem of poor achievement in mathematics continues unabated (Ball *et al.*, 2005). The present study thus seeks to contribute insights on the development of PSTs' knowledge of mathematics by examining one component of the education of prospective mathematics teachers, viz. the field-based experiences". Field experiences, as the basis of teacher training, is considered the blind spot in educational research because it has not found a place in the discussion and research on the components of PSTs' knowledge (Oonk, 2009). Hence, this study has explored the development of teacher knowledge from practice during learning to teach.

Content and Curriculum Knowledge

Leikin and Levav-Waynberg (2009) identified two types of mathematical understanding that teachers sometimes enforce in the classroom to enhance internalisation of mathematics

concepts. Each one of them has pros and cons that PSTs may take note of, for them to make appropriate decisions during teaching.

(i) *Instrumental understanding*

As described by Leikin and Levav-Waynberg (2009), this type of understanding involves the application of certain procedures to solve mathematical problems without understanding why and how the procedures work. Though the implementation of practical situations may be difficult, Leikin and Levav-Waynberg (2009) contend that this form of understanding assists learners in cases where the result of an examination takes precedence. However, consideration of examination results only, defeats the purpose of education for sustainable development. According to Dambudzo (2015), education that is only confined to the classroom and detached from the environment is not sustainable. Dambudzo calls this irrelevant education that fails to equip learners with basic skills and work ethics. In the same vein, an analysis of this kind of understanding reveals that learners may lack critical thinking, making the learning of mathematics devoid of any value to its beneficiaries. Instead of having learners grapple with memorising concepts, for example, the “**Total Surface Area (TSA)**” of a cylinder as $2\pi r(h + r)$, the teachers may involve learners in classroom activities where they start with an open rectangular sheet of paper which can be folded into a cylinder. That way, learners are in a position to identify how the dimensions of the two shapes are related, hence finding the curved surface area of the cylinder could be understandable.

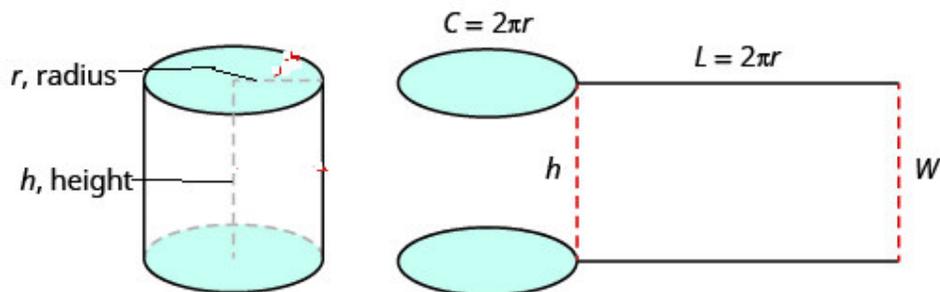


Figure 2 Shapes used in determining the curved surface area of the cylinder

In this instance, area of rectangle = Length (l) × Width (w). But when the rectangle is folded into a cylinder, the Length becomes the circumference of the cylinder ($2\pi r$) and the width of the rectangle is now the height (h) of the cylinder but the area remains the same. Hence, the Curved surface area of the cylinder is $2\pi r \times h$. To find the TSA, the two bases (circles) ($2 \times \pi r^2$) are added to get $2\pi rh + 2\pi r^2 = 2\pi r (h + r)$.

This is a learner centred approach, where learners are actively involved in the learning process, and are privy to how the formula is deduced, hence, evoking motivation to learn the subject (Makamure, 2018). Shulman (1986) advised that instruction in mathematics should focus on learning to reason and to construct proofs as part of mathematics understanding. PSTs therefore, need to be able to promote ability to investigate conjectures, develop and evaluate mathematical arguments among learners for successful mathematics teaching.

(ii) *Relational understanding*

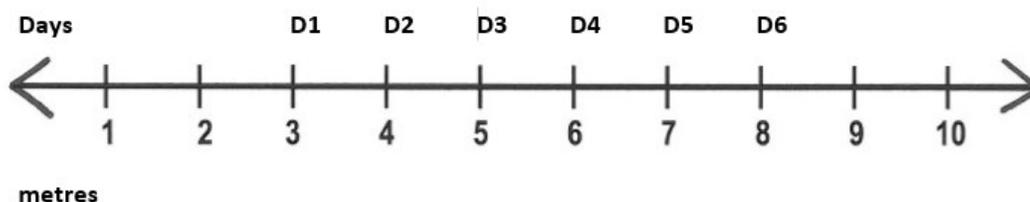
For relational understanding, Leikin and Levav-Waynberg (2009) posited that the learners develop mathematical knowledge from previously learnt concepts. Learners are able to link ideas to solve challenging tasks in mathematics and these ideas can then be applied to new and related ideas. Similarly, Guzman Gurat (2018) added that teachers must breed the ability to present a problem and develop skills in learners, needed to solve the problem. This is more

motivational than teaching the skill without context. Approaching Mathematics this way creates a context which prompts solution of real-life problems. Problem Solving skills in the learners according to Guzman Gurat (2018), enable them to adapt to changes and unexpected problems in their careers and other aspects of their lives. To embrace problem solving skills, learners may be required to solve some practical life problems such as the one in Box 1.

A tortoise is at the bottom of an 8-metre tree. Each day, it crawls up 3 metres and at night, it slips back 2 metres. How many days will the tortoise take to get to the top of the tree?

Box 1

If one looks at this problem without much thinking, he/she thinks that the tortoise effectively crawls 1 metre a day, therefore it will take 8 days to get to the top. To understand this problem clearly, learners need to think critically and draw a diagram to solve the problem practically as it is given below:



NB: The tortoise gets to the top on the 6th day, and escapes the following night

Figure 3 A strategy for solving the tortoise problem

For this type of understanding, using different approaches is the major tool to developing the connectedness in mathematical knowledge to solve problems. This may promote quality teaching and learning and it also teaches tenacity among the learners (Leikin & Levav-Waynberg, 2009).

Knowledge of Learners

A teacher's pedagogical knowledge should also allow him/her to assess the learner's background environment, for example, before teaching a particular topic like "algebra". The PSTs should be aware that if a mathematical word problem involves "alien" terms, that these terms may be meaningless for learners who are not familiar with them (Makamure, 2016). Such problems may hide the conceptual foundation of algebra for these students and are liable to hindering their understanding of the topic. The use of indigenous games for rural learners, as an example, could be a prudent idea. Borko *et al.* (2000) suggest that mathematics tasks should convey the message that the tasks must connect with the children's real world. Gredler (1997) explains the importance of the learners' background and culture in learning mathematics. For example; teaching conical shapes can be introduced by making reference to conical thatched roofs where the learners (in rural settings) come from. That way, mathematics becomes real. The method encourages the learner to reach his own type of truth, related to his background, culture or his own conception of the world. Gredler mentions that the learners build up their

own understanding which is not a mere mirror to reflect what they read. TP is therefore, a time when PSTs learn to inquire into these practices with the assistance of the mentors.

Teacher Beliefs About Mathematics and Mathematics Teaching

Koehler and Grouws (1992), developed a theoretical framework of mathematical beliefs for PSTs. These consists of the **beliefs about the nature of mathematics** and **beliefs about the general conception of mathematics**. These beliefs can have a substantial influence on the PSTs' knowledge of teaching mathematics. Teachers' beliefs about teaching play a powerful role in learning to teach (Lee, 2003). Teacher educators therefore need to pay attention to how these beliefs assist or hinder PSTs' knowledge development. PSTs' beliefs shape their decisions about TP and their future actions. Some of these beliefs include the usefulness and essence of mathematics (Grouws et al, 1996). The beliefs point to the question of whether mathematics taught has any value in real life. Teaching styles, teaching approaches and PSTs' practices are therefore influenced by their beliefs and their theories about the knowledge of mathematics

a) Beliefs About the Nature of Mathematics

This category involves beliefs about the composition, structure and status of mathematical knowledge. For example, mathematics can be viewed as a collection of unrelated, isolated facts or as coherent concepts (Grouws *et al.*, 1996). This view can determine the way PSTs approach certain topics in mathematics. However, according to Kim, Ham and Paine (2011), it is difficult to establish the nature of mathematics knowledge that suits all PSTs because teacher knowledge is largely determined by cultural dynamics. The three philosophies or views about the nature of mathematics are therefore distinguished by Van der Sandt (2007) due to their occurrence in mathematics teaching.

(i) Problem Solving View:

According to Van der Sandt (2007), a pre-service teacher with this view, views mathematics as a continuously expanding field of human inquiry. Mathematics is not seen as a finished product and its results are always open for revision (Van der Sandt, 2007). With this view, the teacher approaches a lesson as a facilitator whilst learners are autonomous in the process. This view resonates with the constructivist view (OECD, 2009) about teaching. The constructivist view of teaching is underpinned by the belief that knowledge is tentative and changeable, which explains that the teacher's ability to teach is not innate. The pre-service teacher's learning to teach therefore, depends on study rather than skill (Yilmaz & Sahin, 2011). In this context, a constructivist teacher is keen to learn various ways to involve students actively in the learning process. According to Chan and Elliot (2004), the more students are actively involved in the learning process, the more they become engaged and this is likely to bring about learner achievement. A pre-service teacher with constructivist beliefs is therefore more likely to be positive about teaching and is potentially prepared to face challenges.

(ii) Platonistic View

Mathematics is viewed as a static or fixed body of knowledge and procedures consisting of interconnecting structures which are to be discovered and not created (Ernest, 1989). The teacher in this case is the explainer. The OECD (2009) calls this view the direct transmission conceptions. PSTs with direct transmissions (traditional) conceptions are likely to hold beliefs that knowledge is certain and unchanging (OECD, 2009). The role of the pre-service teacher with such beliefs is to disseminate knowledge to the learners with the teacher being the source of information during the "learning to teach" process, based on the assumption that the teacher knows everything already.

(iii) *Instrumentalist View*

Mathematics is considered useful and consists of unrelated collection of facts and rules, skills and processes to be memorised (Leung, 1995). According to McDiarmid (1998), PSTs think that “good” mathematics learners are those who are able to remember formulae and procedures and as a result, failure to memorise these implies poor performance. In the same context, Peressin *et al.* (2004) also assert that PSTs believe that doing mathematics means finding correct answers quickly and learning mathematics means mastering procedures. PSTs with this view thus may consider themselves the sole suppliers of information, which is likely to result in teacher domination in the classroom.

b) *Beliefs about Mathematics Teaching*

Kuhs and Ball (1986) identified four dominant and distinctive views teachers hold of how mathematics should be taught.

(i) *Learner Focused View*

According to Manouchehri and Enderson (2003), teachers with a “learner focused view”, focus on learners’ construction of mathematical knowledge during teaching. This is consistent with the constructivist view of teaching mathematics. The learner is actively involved in constructing meaning from experiences by doing mathematics through exploration (Van der Sandt, 2007). Van der Sandt (2007) contends that this belief is normally advocated by those teachers with a *problem-solving view* of mathematics. During teaching, inquiry-based methods of teaching are used, that is, dealing with self-generated ideas. If PSTs learn to teach that way, they are likely to produce learners who are relevant to the real world of industry and technology because of their critical thinking.

(ii) *Content Focused (With emphasis on conceptual understanding)*

This is related to the *platonistic* view of mathematics content. According to Van der Sandt (2007), content is made the focus of the classroom activity and emphasis is on learners’ understanding of the procedures of solving mathematical tasks. This concurs with Grouws *et al.*’s (1996) model about the character of mathematics. The character of mathematics is doing mathematics that involves recalling and obeying the appropriate rules and procedures (Lampert, 1990). Mathematical ideas are validated and this may be perceived as implementing procedures or as making sense of concepts (Van der Sandt, 2007).

(iii) *Content Focused (With emphasis on performance)*

According to Van der Sandt (2007), emphasis in this view of teaching mathematics is focused on the Mastery of mathematics rules and procedures and use of exact mathematical language (instrumentalist view). Knowledge of mathematics is demonstrated by correctly answering and solving problems using the learned rules without understanding and taking note of the source of errors made. This type of learning hence, encourages the regurgitation of concepts without understanding how they came to be.

(iv) *Classroom Focussed (With mathematical teaching based on knowledge about effective classroom)*

Van der Sandt (2007) emphasises that the classroom activities must be well organised and structured, hence the teacher is considered effective. It is clear that if teacher educators have this view about mathematics teaching, then the development of knowledge for teaching mathematics among PSTs may be compromised.

Models of teaching Mathematics

According to Steinbring (1998) and Simon (1997), a teacher's subject matter knowledge and knowledge of learners determines the tasks assigned to the students, the learning setting, learning process perception and the adjustment of the initial plans to suit reality. Depending on the models of understanding mathematics, some models of teaching were developed to match students' understanding. Leikin and Levav-Waynberg (2009) thus identified the cyclic models of teaching as follows:

(i) *Steinbring's (1998) model*

In Steinbring's model, the teachers use the content knowledge they possess and their knowledge of the learners to design tasks for the students. The students then use their knowledge to interpret the given tasks. The *Steinbring's model* resonates with the problem-solving philosophy by Koehler and Grouws (1992). The role of the teacher in this case is to provide a more conducive environment for the learners to approach the tasks, reflect on them and then construct their own knowledge of mathematics autonomously. The teacher observes the learning process, adjusting the tasks according to the needs of the class. The needs analysis is significant in teaching mathematics because the entire process of teaching becomes relevant to the learner, hence, promoting motivation among the learners (Author, 2018).

(ii) *Simon's (1997) model of teaching*

This is a model of teaching in which the role of the teacher is to design a learning trajectory that includes learning plans, objectives, the learning process and activities and ensures that they are strictly followed. The trajectory can be adjusted in the process of interacting with learners, and this creates new ideas for subsequent lessons.

Generally, according to Leikin and Zazkis (2010), the models include goals, choosing instructional tasks and teachers' interaction with the learners. The duty of the teacher in the two models is to adapt the planned learning trajectory and to be privy to the mathematical understanding required (Leikin & Levav-Waynberg, 2009). The view of these two models is that TP has a significant potential for generating PSTs' learning to teach experiences, whilst dismissing the idea that it is the sole composition of teacher knowledge. Teachers' knowledge is therefore developed during the process of planning, working on the learners' tasks and interacting with them. During active participation of the PSTs and the learners, meanings are constructed and practices are formulated (Barnard & Torres-Guzman, 2008;). The intention of most lessons is that learners learn by design and PSTs, while supporting the learning also unintentionally learn from this (Leikin & Zazkis, 2010).

Teaching Mathematics for Proficiency

McDiarmid and Ball (1988) identified examples of subject matter knowledge which PSTs need to be aware of. They suggested that "*substantive knowledge*" of the field of mathematics should include content concepts as well as knowledge of the school curriculum. These are actual concepts that students learn according to the school syllabus. They also identified "*knowledge of the syntax*" which involves testing the viability of a conjecture or proof of a theorem. For example, proving that the sum of exterior angles in a polygon is equal to 360° . In presenting the solution to the proof of this conjecture, learners should understand all the procedures to be followed. However, despite the acquired subject matter knowledge by PSTs, Ball *et al.* (2005) emphasise that PSTs need to connect the mathematics they have with the mathematics they will teach. The connection ensures that whatever they teach remains relevant and continues to serve its purpose. Bauersfeld (1995), explains the difference between a mere teacher and a facilitator as shown in Table 1.

Table 1 **Attributes of Teachers VS Facilitators (Extracted from**

Teacher	Facilitator
Says	Interrogates
Reads in front of the classroom	Waits for the answer from the class
Answers according to the given curriculum	Guides and creates environment for student to reach his own conclusion
Soliloquizes	Continually uses dialogue with learners

Table 1 suggests that PSTs should be facilitators in the classroom rather than imposing content into the learners.

Method

Research paradigm and design

In order to achieve the purpose of the study, the mixed methods approach explored the PSTs' practices, activities and experiences that develop their mathematics knowledge and how this knowledge directs and shapes the mathematics PSTs' classroom behaviour during TP. The explanatory sequential design was opted for, where data were collected from PSTs using quantitative methods first followed by a qualitative methods approach. Questionnaire data were clarified and/or refuted or supported with data from the interviews to ensure the trustworthiness and credibility of the data (Terrell, 2012).

Participants

Participants in this study were PSTs on TP. PSTs in Zimbabwe take two or three years to finish the teacher training course, depending on their entry qualifications. Ordinary level holders spend 12 months on TP whilst Advanced level holders spend 8 months. Participants were selected from both programs. PSTs for both programs graduate with a diploma in mathematics education and are qualified to teach mathematics up to 'Ordinary' level.

Sampling and data collection procedures

120 PSTs on TP were sampled purposively from two secondary teachers' colleges, A and B (pseudonyms) in Zimbabwe. 105 PSTs answered the questionnaire about their classroom experiences during TP. The questionnaire examined PSTs' classroom experiences to determine the development of mathematics knowledge during TP and how that impacted on their performance in the classroom. Separate follow up focus group interviews were conducted with 22 PSTs from the same groups that answered questionnaires. The interviews focused on the PSTs' beliefs about mathematics, TP experiences and activities for all the participants. The focus of the instruments employed was to establish the development of mathematics knowledge on PSTs through their experiences, practices and beliefs about mathematics during TP.

Data analysis procedures

(i) Descriptive statistics

The analysis of quantitative data was done using the SPSS program. The descriptive statistics included the frequencies, means, percentages and standard deviations to analyse the data from the questionnaire. The responses to the questionnaire were presented on a 5-point Likert scale.

SA (strongly agree) took the highest score of 5 and SD (Strongly Disagree) had the lowest score of 1. Open-ended questions were grouped into related categories and explained. The PST questionnaire was tested for reliability using Cronbach’s alpha coefficient and the coefficient was 0.850 This implies that the instrument had a relatively high (>0.7) internal consistency (Field, 2006).

(ii) *Interpretive analysis*

Data from interviews were grouped into related categories and explained. Audio recordings that were transcribed into textual data were used to meet the criteria of trustworthiness and credibility of the qualitative data. Pseudonyms were used in place of participants’ actual names.

Results, Findings and Discussion

This section discusses how PSTs reportedly learn about mathematics and mathematics teaching in the field, *knowledge of teaching mathematics*. Table 2 outlines PSTs’ classroom experiences and practices during field placement.

Table 2: Classroom experiences during teaching practice (PSTs Questionnaire)

Experiences during teaching practice	N	Percent Responses			Mean	Std. Dev.
		Disagree	Neutral	Agree		
7. The college has done well to prepare me for the classroom	105	8.6	8.6	82.9	4.16	0.911
8. I am confident to teach mathematics	105		3.8	96.2	4.69	0.543
9. My classroom management skills are quite appropriate	105	2.9	12.4	84.7	4.1	0.714
10. I have an understanding of how students learn mathematics	100	31.0	22	75.0	3.97	0.771
11. I can apply different teaching approaches during lessons at the appropriate time	103	3.9	8.7	87.4	4.21	0.762
12. Using a variety of approaches to teach a mathematical concept may confuse students	105	58.1	12.4	29.5	2.64	1.381
13. Knowing about different approaches means I can use them for teaching	101	17.8	7.9	74.2	3.78	1.18
14. I use the textbook quite often during my lessons	104	13.5	23.1	63.5	3.75	1.068
15. I can select appropriate teaching resources that enhance my teaching approaches for a mathematics lesson	104		9.6	90.4	4.28	0.63
16. Teaching practice has given me an opportunity to experiment with teaching approaches covered theoretically at college	105	2.9	1.0	96.2	4.52	0.708
17. I got a lot of insight on how students learn mathematics during teaching practice	105	1.0	6.7	92.4	4.39	0.658
18. It is quite easy to utilise the skills and	103	16.5	22.3	61.1	3.62	1.021

Experiences during teaching practice	N	Percent Responses			Mean	Std. Dev.
		Disagree	Neutral	Agree		
techniques gained in college during teaching practice						
19. I can motivate students who lack the desire to do mathematics	105	3.9	6.7	89.5	4.24	0.779
20. There is a sound relationship between myself and my students	105	3.8	8.6	87.7	4.3	0.786
21. I am concerned about my ability to meet the needs of slow learners	103	5.8	13.6	80.4	4.11	0.917
22. I can adjust my styles of teaching to suit various learners	102	1.0	12.7	86.3	4.31	0.731
23. I give remedial work every time students have difficulties in grasping a concept	104	11.5	20.2	68.3	3.9	1
24. I respect and accept students' thoughts and suggestions	105		9.5	90.5	4.41	0.661
25. I allow students to use their own methods of learning	105	15.3	21.0	63.8	3.63	1.002
26. I can assess and evaluate my students' performance in the classroom	105	2.9	4.8	92.4	4.34	0.782
27. The school is doing enough to assist me during teaching practice	104	19.3	17.3	63.5	3.69	1.278
28. The college is doing enough to assist me during teaching practice	104	10.5	16.3	73.1	3.91	1.053
51. Knowing mathematics involves the ability to remember formulas and procedures	104	24.1	22.1	53.9	3.33	1.186
52. The textbook is the best resource to use when teaching mathematics	104	31.7	30.8	37.5	3.03	1.092
53. The role of the mathematics teacher is to transmit knowledge and ensure that the learners have received this knowledge	104	16.4	7.7	76	3.85	1.147
54. Correct answers are more important than the method used to obtain them	105	83.8	8.6	7.6	1.81	0.955
Total average					3.88	0.912

The results of Table 2 indicate that most of the PSTs were satisfied with their classroom experiences and practices during TP. The highest mean was 4.69 with a count of 96.2% and a standard deviation of 0.543. This was given by the probe "I am confident to teach mathematics". Confidence is unlikely to develop where a student teacher is struggling with

lesson presentations as well as mathematics content. Borko *et al.* (2000) allude to several studies that have suggested that teachers with good subject matter knowledge and skills emphasise conceptual problem solving and enquiry aspects compared to PSTs with less content knowledge. Those with less content knowledge, Borko *et al.* (2000) argue, emphasise facts, rules and procedures.

Shulman (1986) asserts that a pre-service teacher therefore needs to be empowered with knowledge and skills to be effective during teaching. Effectiveness produces confidence and self-efficacy of the teacher. The standard deviation of 0.543 implied that the responses were fairly homogeneous. Item 12 has a negative mean of 2.64 (29.5%) but yielded a positive response which indicated that PSTs were using a variety of approaches in their teaching of mathematics concepts.

Item 54 has a low mean (mean=1.81) and standard deviation 0.955, showing that PSTs refuted the idea that answers are more important than the method, although they still believed that knowing mathematics involves the ability to memorise formulae (item 51). The means in table 2 show positive responses by PSTs to the items (51, 52, 53) as indicated by the mean scores above three. This is consistent with the study by Nicol and Crespo (2003) that most PSTs see teaching as a simple transfer of information to pupils and that teaching is largely based on the teacher and the textbook. Of particular focus is item 53 where 76% of the participants are agreeable that the role of the mathematics teacher is to transmit knowledge and ensure that the learners have received this knowledge. According to Grouws *et al* (1996), such PSTs have a *Platonistic View*, where Mathematics is viewed as a fixed body of knowledge and the role of the teacher is to fill in the learners with information. The OECD (2009) also calls this belief about teaching mathematics as the direct transmission conceptions. PSTs with direct transmissions conceptions hold beliefs that knowledge is unchanging. In this case, the pre-service teacher is the sole disseminator of information to the students during learning to teach (OECD, 2009). Guzman Gurat (2018) posited that teachers must have the ability to present a problem and breed skills in learners that are needed to solve the problem. This is more captivating than teaching a skill without substance. Although the participants indicated that they are able to motivate the learners who do not have the desire to learn mathematics (item 19), that they respect learners' suggestions and that they allow learning to take place autonomously (items 24 & 25), this view contradicts their views in 51,52, 53. By sticking to the book and encouraging learners to memorise formula, creativity and critical thinking may be stifled. Learners become receptors of knowledge which could be boring to them, hence, Fried (2011) asserts that when a learner is stressed, the major part of the brain shuts down and reverts to survival needs such as attention seeking. Interviews with PSTs confirm that they intended to just transmit knowledge to the learners without even considering the nature of the classes they were teaching. This classroom behaviour was rampant among the PSTs. One of the PSTs, A5 had this to say in the interviews:

A5: ...Or else you go with a chart, you think they will quickly understand what is on the chart, then you realise sometimes you need concrete things like when you choose media. Say I am doing area, I say, side by side. Sometimes you need to go with something like a tile then you show them this is the dimension, and this is another dimension, instead of just doing charts.

From the reflections above, A5 quickly caught up to the importance of using concrete artefacts to improve learners' understanding. Clearly, many of the PSTs seemed to have a repertoire of strategies for teaching mathematics but still needed to learn the details of when and how to apply them during the actual teaching process. Otherwise, the strategies become irrelevant. Shulman (1986) emphasises on mathematics activities in the classroom that focus on reasoning as part of mathematics understanding.

P2 also disclosed the method that he uses in order to get better results in the classroom. This is what he said;

P2: Sometimes I remember I gave them a test, and I said the highest here I give you a prize. It was the second test. They had failed the first test, the second test the highest was about 16/20. Given the overall, I think it's about motivation.

Although the participants were agreeable that they valued the method most than the final answer in a mathematical task (item 54, Table 2), it seems P2 did not mind whether the students had to memorise the answers or formulas in order to pass. His concern for high scores resonates with Leikin and Levav-Waynberg (2009)'s instrumental type of understanding where the result of an examination takes precedence. Dambudzo (2015) denounces this thinking as unsustainable learning. P2 raised the point of the use of persuasion and extrinsic rewards to control his classes. A popular theme in literature asserts that extrinsic rewards diminish intrinsic motivation (Ledford, Gerhart & Fang, 2013). Hence the method of giving prizes might not be sustainable. Guzman Gurat (2018) thus, recommends that the use of problem solving approaches in teaching mathematics creates a context which stimulates solution of real life problems and therefore, justifies the learning of Mathematics rather than treating it as an end in itself. Below are other examples of how the PSTs responded when they were asked how they approach different topics in mathematics.

A2: First you do the introduction. The introductions should not empty the whole lesson in an introduction. Maybe I use something tangible or linked to their everyday life. For example, you are teaching equations, if you want to solve an equation, if you subtract both sides, what you give to this one, you do the same to the other side or give an example of a child living in a polygamous family. If I give this wife's child, I also give the other wife's child. Once they have an idea of what is happening, you work out an example, you demonstrate the first example, one of the pupils then demonstrates the second example, then maybe pair work before individual work. We demonstrate pair work and discuss as a class, criticising where s/he went wrong, here and there, because of this and this, then after that we give individual work.

A2 takes the approach of teaching mathematics for understanding by using practical life examples in addition to the demonstration method. However, there is a possibility that not everyone knew or had a polygamous experience, hence the example might not suffice. In this instance, practical life examples, like the use of indigenous games, that involve everyone would serve the purpose.

Another pre-service teacher remarked as follows,

A3: As for me, I find demonstration, group work and individual work; they are working very much with the students. They usually understand demonstration within group work and pair work because they share ideas. I usually use group work, related to the media with work cards and at the same time I use group work only when it's a double period. It helps so much because students do not remain idle every day. They are always filled up with something to do.

A3 suggests that in addition to the "demonstration approach", he uses group work in every double lesson to keep students occupied and focused on the work. Both A2 and A3 are aware of the student-centred approaches that attract the attention of the learners, but the question that remains is, "Do the methods create a context which stimulates solution of real life problems? Guzman Gurat (2018) recommends that learners should be taught problem Solving skills that enable learners to adapt to changes and unexpected problems in their careers and other aspects of their lives. Using different approaches to teach mathematics is the major tool to developing

the connectedness in mathematical knowledge to solve problems, which may promote quality learning and teaches tenacity among the learners (Leikin & Levav-Waynberg, 2009).

In addition to this, PSTs R1 and R3 mentioned activity-based teaching and question and answer methods respectively as some of the methods they used for teaching. The responses show that PSTs are privy to the student-centred approaches that capture the students’ attention in a class, which Blumberg (2005) says promote student engagement with the content. The responses on teaching approaches confirm the quantitative data result in table 2 where the majority of the PSTs (76%) had an understanding that their role was to transmit knowledge to the students for understanding. However, it is the “how” part that seems to give them some challenges.

When the participants were asked about their experience in the job of teaching, they indicated that they had challenges on how to plan their work before conducting a lesson. P2 had this to say:

P2: I think the skill of ordering the content, because sometimes you discover you are now covering a topic, maybe sketching graphs then you discover that these pupils don’t know something, what do I do, that’s why I have to reteach. So I said let me just cancel this one, and do substitution first because the students could not draw the table of values. So sometimes you discover that. I think what you need to do is to be able to tell that these students did this, did this, they did this, to test them, pre-test or something before you start teaching a topic.

The sentiments by P2 add to improper planning, which could have been aggravated by a lack of experience. The ‘teaching and re-teaching’ may therefore be considered as part of learning to teach hence, developing their knowledge for teaching mathematics.

Pre-service Teachers Experiences with Mentors

According to Maphosa et al (2007), mentors need to supervise, guide and instruct the mentees. In addition, Maphosa et al added that mentors contribute to the means and structures through which knowledge is shared with PSTs during learning to teach. In other words, PSTs learn through interactions with their supervisors, which makes it necessary to look at their practicum experiences with school and college supervisors. The PSTs’ experiences in schools, on the job interactions and the identification of supportive antecedents has the greatest potential to change or endorse the PSTs’ pre-existing knowledge about mathematics during training (De Neve *et al.*, 2015). The table below focuses on how mathematics teaching knowledge is shared with the PSTs by their supervisors (school-based).

Table 3 Pre-service teachers’ experiences with mentors

Experiences with mentors	N	Mean	Std. Dev.
29. A mentor is an expert in teaching mathematics	104	3.29	1.297
30. Mentors are always motivated and enthusiastic about teaching mathematics	104	2.96	1.222
31. My mentor helps me to plan for the lessons	104	2.64	1.307
32. My mentor helps me to decide on the media to use for developing concepts	103	2.47	1.178
33. My mentor helps me to decide on which teaching approaches to use for my lessons	104	2.71	1.220
34. My mentor let me sit in a lesson she was teaching during my TP	104	2.74	1.400

Experiences with mentors	N	Mean	Std. Dev.
35. My mentor demonstrated some of the teaching approaches before asking me to teach a lesson	104	2.45	1.238
36. My mentor coached me on how to teach	103	2.90	1.287
37. My mentor regularly sits in on lessons that I teach	101	3.07	1.283
38. My mentor allows me to use the teaching methods I feel will be useful	104	4.03	1.038

The results in table 3 above illustrate that PSTs’ responses were less than positive about the assistance they received from their mentors, as shown by the several mean item scores below three. In this regard, Rakicioglu-Soylemez and Eroz-Tuga (2014) concur that there are differences between the definitions of mentoring and the actual practices. If this practice is not monitored, the objectives of TP may not be attained. The highest mean on this theme was 4.03 (82.7% agreed and 8.7% disagreed) given by the statement “My mentor allows me to use the teaching methods I feel will be useful”. Such a response could be expected because, according to Kiggundu and Nayimuli (2009) and Maphosa *et al.* (2007), some PSTs are normally on their own in the classrooms during TP. This occurs because some mentors tend to place the burden of teaching on the PSTs without assistance and therefore the pre-service teachers were likely to employ the approaches they wanted. This interpretation was confirmed in the interviews with PSTs in which they affirmed that they were teaching on their own most of the time. If supervision is limited, mathematics knowledge attainment is also stifled. Here is what some of the participants said:

A1: We taught the very first day. I was even asked to teach on the first day without a lesson plan. I requested that she teaches first whilst I am watching but refused. She just thought the first form one topics are easy and I should be able to teach them easily even without preparation. You are told on the first day that do you know that you have a lesson?

Similarly, R3 had this view

R3: I am not getting enough assistance from the mentor and I’m being told to attend to HOD classes when they are having a meeting. I had to ask for help from other maths teachers in the department.

A2 I was thinking that when I go on TP, I would have a mentor, and I don’t own any class, it’s the mentor’s class. I expected the mentor to tell me that I am going to teach this and this. Now I am the owner and the teacher of those classes, throughout the term. So I am just a student at my college but here I am a teacher.

The study exposed that PSTs had challenges related to the quality of mentorship they received. This is in agreement with the results of table 3 of the quantitative data where most of the mean responses for mentorship assistance were below three, implying that the PSTs’ views about mentorship were less than positive. The PSTs indicated that the mentoring was limited and for others, was completely absent. The responses are consistent with the report by Gulamhussein (2013) that if problems such as a lack of support from supervisors are not addressed, they are likely to affect PSTs’ performance in the classroom. Gulamhussein (2013) contends that PSTs

need even more support during implementation (TP) in order to address the challenges of the classroom practices.

The study also established through PSTs' interviews that colleges sometimes send supervisors who are not mathematics specialists to oversee PSTs' work during TP, which seems to be unhelpful for the PSTs in terms of content knowledge feedback. Regarding this issue, R3 had this to say:

The thing is, so far, what happens is, lecturers come from different departments and they have different requirements. The lecturer who comes to see us is from a different department. So to tell us what to expect exactly in terms of content, I don't think it works....

Concerning the issue of specialists in the subject, Evans *et al.* (2014) report that subject specialists are better positioned to perform supervision activities compared to non-specialists. They add that non-specialist supervisors may not be able to give appropriate feedback on the subject content of observed lessons. Evans *et al.* (2014) also asserts that non-specialists lack expertise and confidence to assist mathematics PSTs on the subject content since they are insufficiently equipped to offer advice.

Conclusion and recommendations

The purpose of the study was to explore the development of teacher knowledge for PSTs during TP. Although PSTs had relatively adequate mathematics content knowledge for the levels that they were teaching (table 2), it is not only the content knowledge that develops them into effective teachers in the classroom. The content knowledge works well with proper guidance from the supervisors as far as classroom and pedagogical practices are concerned. If supervisors fail to offer proper mentorship and guidance to the PSTs (as stipulated above) and do not develop the desired and effective teaching skills in them during TP, they will not be able to demonstrate the kind of teaching approaches that take cognisance of the interaction within and between social and cultural interests of learners for understanding (McDiarmid & Ball, 1988).

The results of the quantitative survey data show that the ability to teach mathematics cannot be separated from the subject content knowledge (see table 2). Similarly, Ball *et al.* (2001) emphasise that teaching depends on subject content knowledge but subject content knowledge is not synonymous with teaching. This means that for PSTs to be able to teach mathematics effectively, they need to demonstrate adequate mathematics content knowledge. However, having mathematics content knowledge alone does not imply the ability to teach that content. Hine (2015) agrees with Ball *et al.* (2001) on the interplay that exists between mathematics content and pedagogy when they argue that without mathematics content knowledge, the pedagogical processes may be impeded. An effective teacher in this study is therefore considered as one who embraces knowledge of the subject and the ability to use appropriate teaching strategies to make information accessible to learners. Failure by PSTs and supervisors to realise this might result in one component being weak. If one component is lacking or weak, the pre-service teacher may not acquire the adequate skills and competences expected of them after training. The importance of understanding the interplay between mathematics teacher preparation, pedagogy, content knowledge and student achievement is likely to motivate a new line of research based on teacher knowledge (Marshall and Sorto 2018). Based on the results of the study, Figure 4 below is a proposed model designed to improve the teaching of mathematics during field placement.

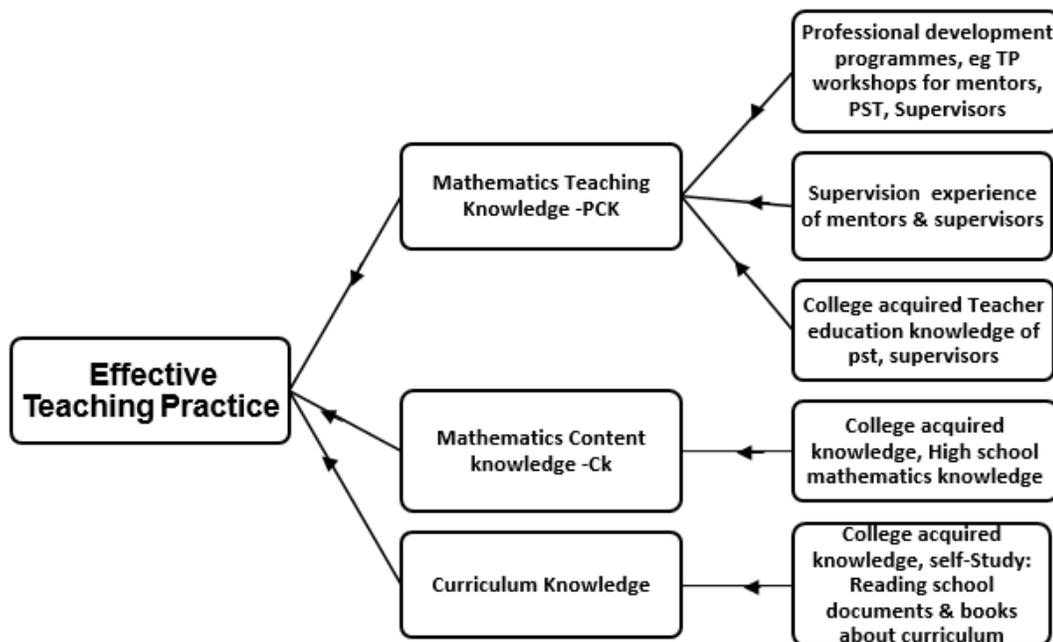


Figure 4: Proposed model for TP composite

(Extracted from Makamure, 2016)

Figure 4 shows that effective mathematics teaching during TP needs to be approached in a holistic manner. This means that viewing only one component of teaching mathematics (among pedagogical content knowledge, content knowledge and curriculum knowledge) as the only contributing factor of TP effectiveness, may fail to achieve the goals of effective mathematics teaching during practice. Consequently, there is a need for teacher educators to focus on adhering to the demands of teaching to improve PSTs' classroom practices by adding value, not only to the mathematics content that PSTs will teach but also to enhancing how they plan for the effective teaching of mathematics.

Based on the findings of the research, it is also recommended that mathematics PSTs on TP may be supervised, mentored and workshopped by mathematics subject specialists who are well positioned in terms of mathematics pedagogy and mathematics content related feedback in order to enhance PSTs' knowledge for mathematics teaching.

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