

Examining the Effects of Mathematical Modelling Instruction on Pre-Service Teachers' Geometry Problem Solving Abilities

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Abstract

The teaching of mathematics, particularly geometry, is vital in training future mathematics teachers. This study aimed to investigate the effectiveness of a modeling mathematics pedagogy compared to traditional methods for teaching geometry to pre-service teachers in Ghana. Utilising a convergent parallel mixed methods research model and a quasi-experimental pretest-posttest control group design, a sample of 191 students was analyzed across an experimental group (n=140), which received model-centered instruction, and a control group (n=51), taught using traditional methods. The instruction received by the experimental group was based on Polya's problem-solving stages and the control group was taught using the traditional method. The intervention lasted for 12 weeks and the results revealed that the geometry problem solving ability of the participants in the modeling instruction (or intervention) group was raised higher than that of the control group. Qualitative analysis of written work demonstrated improvement in how students in the modeling instruction group efficiently approached the geometry problems following the stages of Polya's. The findings strongly indicate that mathematical modelling is a more effective pedagogy for enhancing pre-service teachers' geometry problem-solving skills, underscoring its potential value in the education and training of teachers of senior high school mathematics.

Keywords mathematical modeling; pre-service teachers; geometry teaching; Polya's problem-solving model

Introduction

Problem-solving is fundamental to mathematical proficiency and crucial for life, permeating all aspects of mathematics education (Schoenfeld, 2007; Husna & Burais, 2019; Schoenfeld & Sloane, 2016). It cultivates analytical thinking, strategic reasoning, and creative application to unfamiliar problems (Polya, 1957). Historically, mathematics education, particularly in geometry, has relied on teacher-

dominated approaches emphasizing direct instruction, procedural knowledge, and rote learning (Atta & Bonyah, 2023; Idris, 2005). While these methods can develop short-term computational ability, they often fall short in fostering conceptual understanding and flexible problem-solving, potentially narrowing students' strategic repertoire (George et al., 2024; Yıldırım & Sidekli, 2018; Kiesmüller, 2009).

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In contrast, mathematical modeling offers an innovative pedagogical technique for developing advanced problem-solving skills (Hidayat et al., 2023; Tezer & Cumhur, 2017). Rooted in constructivist theories, it encourages students to engage with authentic problems, build mathematical representations, and iteratively refine their models through analysis and feedback (Abassian et al., 2020; Dele-Ajayi, 2016; Bada & Olusegun, 2015). This process involves identifying variables, forming hypotheses, translating them into mathematical objects, and validating the model against the original problem (Blum, 2011; Alsina & Salgado, 2022; Lesh & Harel, 2003). Such an approach enhances understanding of mathematical content and promotes flexible, creative, and reflective problem-solving (Suh et al., 2017; Lesh & Doerr, 2003). Hypothesis testing and iterative refinement are central to problem-solving within a modeling framework (Fensel & Motta, 2002; Guo et al., 2025).

Mathematical modeling is also vital for teacher education, preparing pre-service teachers with strong problem-solving skills and pedagogical tools (Zvoch, Holveck & Porter, 2021; Guerrero & Borromero, 2022). Preservice teachers educated through traditional methods may perpetuate these approaches, hindering their students' problem-solving abilities (Crespo, 2003; Lesh & Harel, 2003). Therefore, integrating innovative methods like mathematical modeling into teacher education is crucial for cultivating educators who can foster deep conceptual understanding and creative problem-solving in their students (Lee, 2007).

Recent studies confirm that mathematical modeling enhances pre-service teachers' mathematical knowledge, aids in designing relevant learning tasks, and improves their understanding of the link between

mathematics and reality (Dogan, 2020; Bukova-Güzel, 2011; Clarke & Roche, 2018; Meylani, 2025; Daher & Shahbari, 2015). These align with the 21st-century skills such as critical thinking, communication, and collaboration (Alsina & Salgado, 2022; Collins & Halverson, 2010).

Despite these advantages, the precise influence of mathematical modeling instruction on pre-service teachers' problem-solving processes, compared to traditional methods, remains underexplored. This study aims to address this gap by examining differences in problem-solving strategies in geometry between pre-service teachers taught with mathematical modeling and those taught traditionally. Geometry is a rich domain for problem-solving, connecting abstract ideas with visual and spatial reasoning (Atta & Bonyah, 2023; Borgersen, 2024).

The research is theoretically grounded in Polya's (1957) four-step problem-solving heuristics (understanding, devising a plan, executing, and reflecting) and Blum's (2011) iterative modeling cycle (understanding the real-world situation, simplifying and structuring, mathematizing, working mathematically, interpreting, validating, and de-mathematizing) (Blum & Ferri, 2009; Alsina & Salgado, 2022; Sul et al., 2017). Both frameworks emphasize iterative and reflective practices essential for mathematical learning. This study investigates how insights from Polya's process enacted within Blum's modeling cycle can illuminate the impact of mathematical modeling on pre-service teachers' problem-solving skills. The central research question is: "What is the statistically significant difference between pre-service teachers who are taught with mathematical modeling and those taught with traditional instruction in problem-solving strategies in geometry situations?" This empirical research

seeks to provide evidence on the effect of mathematical modeling instruction on problem-solving strategies, with potential implications for developing teacher education programs that better equip future educators to foster strong problem-solving skills in their students for a more mathematically literate citizenry.

Conceptual Framework

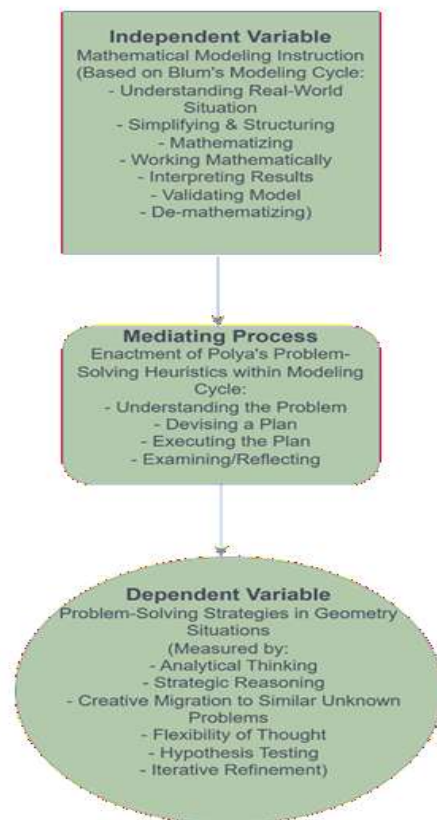
The conceptual framework posits that mathematical modeling instruction, by engaging pre-service teachers in authentic problem-solving scenarios, will lead to a statistically significant improvement in their problem-solving strategies compared to those taught with traditional instruction. This improvement is understood through the lens of how Polya's four problem-solving steps are enacted within the iterative stages of Blum's modeling cycle.

This study investigates the impact of mathematical modeling instruction, based on Blum's (2011) iterative modeling cycle, as its core independent variable. This approach, contrasting with traditional teacher-dominated methods, involves students actively understanding, structuring, mathematizing, working mathematically, interpreting, validating, and de-mathematizing real-world problems.

The study posits that the influence of mathematical modeling on problem-solving strategies is mediated by the enactment of Polya's (1957) heuristics—understanding, devising, executing, and reflecting—within Blum's cycle. These steps are iteratively applied in the practical context of modeling, with clear alignments between the two frameworks (e.g., Polya's "understanding the problem" maps to Blum's "understanding the real-world situation").

The dependent variable, problem-solving strategies in geometry situations, encompasses analytical thinking, strategic reasoning, creative migration, flexibility of thought, hypothesis testing, and iterative refinement. The research aims to identify statistically significant differences in these strategies between the experimental group (mathematical modeling instruction) and a control group receiving traditional instruction, characterized by direct instruction, procedural knowledge, and rote learning. This framework provides a robust foundation for assessing the pedagogical efficacy of mathematical modeling in

Figure 1: Mathematical Modeling and Problem-Solving Heuristics in Teacher Education



developing pre-service teachers' problem-solving skills in geometry.

Methodology

This work used a convergent parallel mixed-methods design (Creswell & Plano Clark, 2023) to provide a complete examination of the effect of a mathematical modeling-based intervention on pre-service teachers' problem solving in geometry. For the quantitative strand, Labordo (2024) quasi-experimental pre-test post-test control group design was employed, and for the qualitative strand, a multiple-case study design (Yin, 2017) was used. Framed within a realist-oriented epistemology (Kaushik & Walsh, 2019), this design supported the simultaneous generation and synthesis of numerical and narrative information and enabled a multi-layered interpretation of the impact of the intervention.

The study involved a total of 191 pre-service teachers from two colleges of education in the northern region of Ghana. The sample size was determined using Cochran's (1977) adjusted formula for finite populations, accounting for a population size of 350. A multi-stage sampling process was employed. First, one college was randomly selected for pilot testing and excluded from the main study. The remaining two colleges were then assigned to the experimental ($n = 140$) and control ($n = 51$) groups based on their mean pretest scores in geometry problem-solving, with the college exhibiting a lower mean assigned to the experimental group. Finally, stratified random sampling within each college ensured proportional representation of participants. Demographic data, including age, gender, and prior GPA, were collected and analyzed to ascertain baseline comparability between the groups.

The intervention, which spanned 12 weeks, delivered the same geometry content to both groups but differed in pedagogical approaches. The geometry content comprised of Polygons, Pythagorean theorem application in roofing and perimeter and area. The experimental group received instruction based on Blum's (2011) modeling cycle, emphasizing real-world problem-solving, collaborative group work, and the integration of GeoGebra software for dynamic visualization (Leong & Chick, 2011).

One of the tasks was on garden bed optimization, that is to determine the rectangular dimensions that minimize the amount of rope needed to enclose a fixed area of 36-meter square. Participants in their groups first analysed the problem to understand it by writing it in their own words, then identified keywords and made assumptions ($\text{length} = 2 \times \text{width}$). In the mathematizing stage the group expressed the perimeter as $P=2(L+W)$ and area $A=LW = 36$. The fourth stage which involve working the mathematics, the group used GeoGebra sliders, by varying the width to observe perimeter changes. During the fifth stage, they interpreted by relating the minima to efficient rope use. In the sixth stage, they then validated their results by checking it with the assumptions made in the second stage. In the final stage, the groups shared their solution with the entire class for further questions and answers. In contrast, the control group received traditional instruction characterized by a teacher-centered approach, direct lecturing, and handout-based exercises with limited contextual applications. Data collection occurred across four phases, utilizing a Geometry Achievement Test and the analysis of participants' written solutions to geometry problems.

The Geometry Achievement Test comprised two sections: one assessing problem-solving strategies aligned with Polya's (1957) stages and another evaluating modeling competency using rubrics based on Blum's (2011) modeling cycle. One section assessed problem-solving strategies using a rubric

involved applying rubric-based codes to each response, categorizing segments such as correct strategy identification, computational accuracy, or evidence of reflective verification. The Phase 3, Searching for Themes entailed grouping related codes under broader thematic categories corresponding to

Table 1- Participant Demographics

Characteristics	Experimental Group (n=140)	Control Group (n=51)	p-value
	Mean (Std. Dev.)	Mean (Std. Dev.)	
Age	21.30 (1.20)	21.50 (1.10)	.32
Prior GPA	3.40 (0.30)	3.50 (0.40)	.18
Percentage Females	32	38	.71

aligned with Polya's (1957) four stages. The stages were Understanding the Problem, Devising a Plan, Carrying Out the Plan, and Looking Back. This rubric rated responses from 0 to 5. While the other section of the instrument evaluated modeling competency using rubrics based on Blum's (2011) modeling cycle. Details of the rubrics can be found in Appendix A.

The written solutions of selected participants were analyzed qualitatively to complement and support the quantitative findings on problem-solving performance. A deductive thematic analysis was employed, following Braun and Clarke's (2006) six-phase framework, because the coding was guided by a pre-established problem-solving rubric based on Polya's four stages (Understanding the Problem, Devising a Plan, Carrying Out the Plan, and Looking Back). This rubric provided explicit descriptors for scoring and interpretation, ensuring that analysis remained anchored in the study's theoretical framework and research questions.

In Phase 1, that is familiarization, researchers repeatedly read the sampled solutions to become immersed in the data and noted preliminary impressions regarding participants' approaches and misconceptions. Generating Initial Codes which was phase 2

the four rubric stages, while also identifying cross-cutting patterns such as "misapplication of formulas" or "absence of verification."

During Phase 4, Reviewing Themes, these categories were refined by cross-checking coded excerpts against the entire dataset to ensure consistency and relevance. In Phase 5, Defining and Naming Themes involved articulating clear definitions for each and linking them to rubric descriptors. Finally, in Phase 6, that is Producing the Report, representative excerpts including both correct and incorrect solutions were selected to illustrate thematic patterns, and these qualitative insights were integrated with the statistical findings to provide a richer interpretation of the intervention's impact.

The research was conducted in strict accordance with the principles of research ethics, prioritizing the welfare and rights of all participants, consistent with the International Sociological Association's (ISA) Code of Ethics. Informed consent was obtained from all participants before their involvement in the study.

Results

Demographic Characteristics of Participants

The demographic information about the pre-service teacher participants in the current

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study is shown in Table 1. This table gives an overview of the crucial variables, age, gender ratio, and previous academic achievement represented by the Grade Point Average (GPA), in both the experimental and control groups. It is critical to consider these baseline demographics when understanding the sociodemographic makeup of each community and assessing comparability at baseline.

At baseline, the demographic profile of the experimental and control groups were generally comparable in characteristics. The mean age of participants did not differ significantly, $t(189) = -1.00, p = .32$. Prior GPA showed no significant difference, $t(189) = -1.34, p = .18$; and the percentage of females used in the study was also similar across the two groups, $\chi^2(1, N = 191) = 0.14, p = .71$. These results indicate that the groups were equivalent in age, academic performance and gender representation before the intervention.

Results of the mathematical modeling instruction experiment

The central research question of the study was "What is the statistically significant difference in problem-solving abilities in geometry between pre-service teachers taught with mathematical modeling instruction and their counterparts taught with traditional instruction?"

Table 2 Descriptive statistics of scores obtained by the experimental and control groups in the pre-test

Variable	N	M	SD
Control group	51	2.2157	2.17544
Experimental group	140	1.1286	1.27422

To answer the question, the pre-test scores of both experimental and control groups were compared to establish baseline differences. Table 2 shows the descriptive statistics,

including means (M) and standard deviations (SD), scores obtained on the problem-solving in geometry pre-test by the 191 participants.

A preliminary examination of the pre-test data indicated a significant difference in means between the experimental and control groups ($p < 0.001$), with the control group showing a mean pre-test score ($M = 2.2157, SD = 2.17544$) significantly higher than the experimental group ($M = 1.1286, SD = 1.27422$). These results show that the groups were not equivalent at the onset, which is an important fact to consider when making comparisons later on.

The descriptive statistics of the post-test

Rectangular Garden Bed (Minimizing Fencing)

Kofi wants to build a rectangular garden bed with an area of 200 m² but has limited fencing materials. He wants to determine the dimensions that will minimize the amount of fencing needed. Find the most efficient dimensions using Polya's problem-solving strategy.

Box I

scores of both experimental and control groups, presented in Table 3, were compared to establish differences in performance of the two groups at the end of the experiment.

The mean problem-solving score increased substantially from pre-test ($M = 1.42, SD = 1.63$) to post-test ($M = 7.94, SD = 5.78$), suggesting notable improvement after the intervention. The larger SD for the post-test indicates greater variability in participants' performance.

In addition to the quantitative aspect, an analysis of the students' sample works, from both the pre-test and post-test phases, revealed qualitative insights into the ways students solved problems. This analysis illustrates how students in both

experimental and control groups used Polya's four steps of problem-solving problems in geometry.

It was observed that in the pre-test both the experimental and control groups had difficulty at 'the understanding the problem' stage by failing to apply the relevant information to write down the mathematical relations required (or mathematizing) the problem in Box 1. Majority of the pre-service teacher participants had difficulty in 'devising a plan' or 'writing the relevant relation' between area and perimeter of a rectangle.

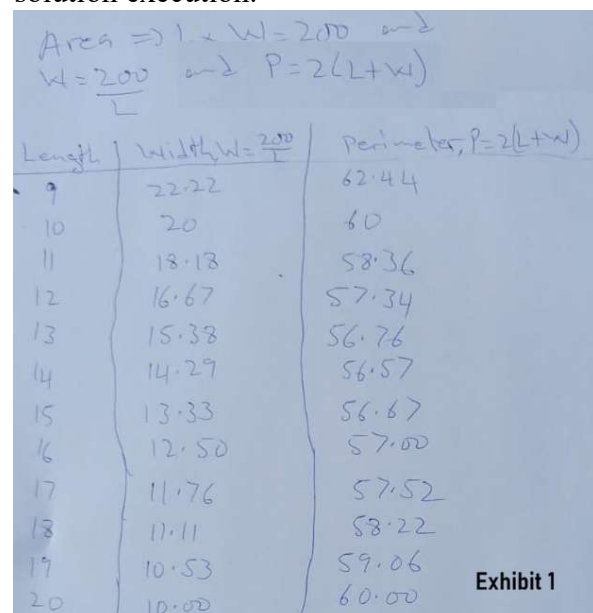
Their working showed vague or incomplete plans that are based on memorized formulas. But after the experiment, the treatment group exposed to the mathematical modeling instruction demonstrated better understanding and performance in problem-solving abilities. For the maximum perimeter of a rectangular garden problem, nearly all the experimental group participants were able to sketch and use the length (l) and width (w) of the rectangle to state the relevant relations between area and perimeter of a rectangle and apply the principle of minimum and maximum point in differentiation to solve the question. That is, many were able to express the two relations - $A = lw = 200\text{m}^2$ and $P = 2(l + w)$; find w in terms l and write an expression for perimeter in terms of l . That is, $w = \frac{200}{l}$ and $p = 2l + \frac{400}{l}$; then find the maximum point of $\frac{dP}{dl}$. A few of the participants used the trial and improvement method trying to find the dimensions (i.e., values for l and w) that will yield the product that is nearest to 200. See Exhibit 1, for example, is an excerpt from one of the participants that tried to use the trial and improvement method.

Discussion

The study's findings strongly support its theoretical frameworks, demonstrating that mathematical modeling instruction based on Blum's (2011) cycle effectively enhances pre-

service teachers' problem-solving skills. The pre-test revealed the control group's initial proficiency in structured problem-solving (Polya's 1957 heuristics), likely due to prior familiarity. However, the experimental group's significant post-test improvements, marked by higher scores and more structured approaches, underscore the efficacy of mathematical modeling.

The experimental group's development of diverse and complex strategies post-intervention suggests their engagement with Blum's iterative modeling cycle—simplifying, mathematizing, solving, interpreting, and validating. This shift towards real-world engagement and abstraction aligns with Blum's framework. The intervention, focused on complex, real-world geometric problems, fostered a move from rote memorization to a dynamic understanding of geometry's application in problem-solving. Qualitative findings further confirmed the experimental group's enhanced abilities in problem interpretation, plan devising, and solution execution.



Area $\Rightarrow l \times w = 200$ and
 $w = \frac{200}{l}$ and $P = 2(l + w)$

Length	Width, $w = \frac{200}{l}$	Perimeter, $P = 2(l + w)$
9	22.22	62.44
10	20	60
11	18.18	58.36
12	16.67	57.34
13	15.38	56.76
14	14.29	56.57
15	13.33	56.67
16	12.50	57.00
17	11.76	57.52
18	11.11	58.22
19	10.53	59.06
20	10.00	60.00

Exhibit 1

Ultimately, the study confirms that mathematical modeling instruction rooted in Blum's cycle improves pre-service teachers' problem-solving abilities, particularly in

geometry. The observed enhancements in problem comprehension, planning, and implementation in the experimental group validate the theoretical and conceptual frameworks' suitability for this context.

This research primarily investigated the statistically significant difference in problem-solving strategies between pre-service teachers instructed using mathematical modeling and those taught traditionally in geometry contexts. The mixed-methods analysis provides compelling evidence for the positive impact of mathematical modeling, particularly when viewed through Polya's (1957) problem-solving model.

The quantitative pre-test data revealed a significant initial disparity, with the control group showing higher baseline problem-solving scores ($\eta^2 = 0.087$). Qualitative pre-test analysis illuminated these differences: the experimental group struggled with 'Understanding the Problem' and 'Devising a Plan,' often relying on rote memorization and exhibiting more calculation errors and deviations in 'Carrying Out the Plan.' Their 'Looking Back' stage showed less engagement with solution verification. These observations align with criticisms of traditional teacher-centered methods for fostering rote learning and hindering conceptual understanding (Atta & Bonyah, 2023). Conversely, the control group's initial strengths in problem formulation, structured planning, precise execution, and thorough checks likely reflect their proficiency in traditional algorithmic approaches (Lockwood et al., 2016).

Qualitative analysis of the post-test provided rich insights into how mathematical modeling influenced problem-solving strategies. The experimental group exhibited organized thinking, logical planning, and accurate execution, contrasting sharply with the control group's limited understanding and general

planning on similar problems. This suggests mathematical modeling fostered a more structured, conceptually grounded, and effective approach, supporting claims that it encourages active engagement, mathematical representation, and iterative model refinement, thereby promoting higher-order thinking and deeper conceptual understanding (Tezer & Cumhur, 2017; Lesh & Doerr, 2003).

These improvements can be directly linked to the core principles of mathematical modeling instruction, which necessitated moving beyond rote memorization to identify variables, make assumptions, and iteratively refine models—processes inherently aligned with Polya's (1957) model. The modeling intervention fostered better problem understanding, plan construction, accurate execution, and reflective thinking. Conversely, the lack of positive development in the control group suggests traditional education, while building procedural fluency, may not stimulate the flexible and adaptable procedures needed for novel geometric tasks (Lockwood et al., 2016; George et al., 2024).

This study contributes new evidence to the literature on the specific effects of mathematical modeling instruction on pre-service teachers' problem-solving strategies in geometry, addressing a recognized research gap in teacher preparation (Blum & Ferri, 2009; Doerr & Lesh, 2003). The findings imply the necessity of expanding mathematical modeling in pre-service teacher training to develop diverse problem-solving abilities and guide future teaching towards student-centered, application-based instruction. This is crucial for supporting pre-service teachers who, having been trained traditionally, might perpetuate constrained problem-solving approaches (Zvoch, Holveck, & Porter, 2021; Berenger, 2018). The results highlight the urgency of

introducing new pedagogical strategies in initial teacher education to break this cycle.

Conclusion

This study provides compelling evidence that instruction through mathematical modeling has a positive direct effect on pre-service teachers' problem-solving strategies in geometry, outperforming traditional instruction. While the control group displayed a small initial advantage, the mathematical modeling group made substantial gains across all stages of Polya's widely known problem-solving process. The observed effectiveness of mathematical modeling in developing more flexible and advanced problem-solving abilities underscores its critical importance for incorporation into teacher education programs. By doing so, future educators will be better equipped to teach mathematics using tangible, real-life contexts, thereby enhancing mathematical learning outcomes for their students.

Despite these significant findings, the study acknowledges several limitations. The use of intact classes introduced uncontrolled pre-existing differences, and the disparate sample sizes (140 experimental vs. 51 control) could affect the robustness and generalizability of the results. Furthermore, the study's confinement to two colleges in northern Ghana limits its broader applicability. Practical challenges, including varying prior knowledge among pre-service teachers and the need for adequate instructor training in mathematical modeling, may also impact future implementation.

Nevertheless, this research contributes valuable insights into effective pedagogical approaches in teacher education. The findings transcend the specific context of teacher preparation, underscoring the broader need for educational institutions to shift away from rote memorization and algorithmic thinking towards fostering flexible problem-solving skills crucial for today's students. This study

addresses a key debate in teacher preparation, highlighting the vital role of innovative strategies and methodologies. Moving forward, teacher education programs must integrate novel teaching methods like mathematical modeling. Future research should explore larger, more diverse samples across different regions of Ghana and other countries, and conduct longitudinal studies to assess the long-term impact on both pre-service teachers' skills and their future students' learning outcomes. This will help cultivate a mathematically literate and competent populace prepared for the demands of 21st-century classrooms.

Declarations

Disclosure statement/ Conflicts of Interest

The author declared no potential conflict of interest with respect to the research, authorship, and/or publication of this article.

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Informed Consent Statement

Informed consent was obtained from all subjects involved in this study.

Data Availability Statement

The data presented in this study are available on request from the author since this research was conducted on students, for which their teachers gave consent.

Author Contribution

I, Evelyn Agyei, took the lead in conceptualizing the study, designing the methodology, conducting the investigation, performing the formal analysis, managing the data.

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Appendix A: Scoring Rubrics for Problem Solving

Criteria	No Attempt (Score 0)	Score 1	Score 2	Score 3	Score 4	Score 5
1. Understanding the Problem	No response provided or work is completely unrelated to the problem. Example: Student submits a blank page or writes about a different topic.	Identifies 1 or 2 key pieces of information but omits or misinterprets significant aspects of the problem. Example: Student correctly identifies the goal but fails to identify a key constraint.	Identifies most key information but struggles to clearly articulate the overall goal or the specific unknowns to be solved. Example: Student identifies the variables but cannot explain how they relate to each other.	Identifies all key information and articulates the problem and unknowns, but with some minor inaccuracies or lack of clarity. Example: Student provides a mostly accurate description but uses imprecise language or mixes up variables.	Clearly and accurately explains the problem, identifies all relevant information, and defines unknowns precisely. Example: Student provides a concise and accurate summary of the problem, clearly stating the goal and identifying all relevant variables and constraints.	Clearly and accurately explains the problem, identifies all relevant information, defines unknowns precisely, and explicitly considers all constraints or conditions. Example: Student not only identifies the constraints but also discusses their potential impact on the solution.
2. Devising a Plan	No attempt to devise a plan or suggests a plan that is completely irrelevant to the problem. Example: Student says "I don't know" or suggests a plan that	Suggests a vague or incomplete plan with no clear steps or strategies. Example: Student says "I'll use a formula" but doesn't specify which one.	Suggests a plan with some relevant steps but lacks specific details or justification. Example: Student outlines a general approach but doesn't explain why it's appropriate.	Suggests a plan with a logical sequence of steps, but with some gaps or inconsistencies in reasoning. Example: Student proposes a plan that mostly makes sense but overlooks a key step or includes an	Proposes a clear and detailed plan with a logical sequence of steps, clearly justified by the problem structure and mathematical knowledge. Example: Student presents a step-by-step plan, explaining the	Proposes a clear, well-developed plan with multiple potential approaches considered and evaluated for their suitability to the problem. Example: Student explores different solution strategies, comparing their advantages and

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Criteria	No Attempt (Score 0)	Score 1	Score 2	Score 3	Score 4	Score 5
	does not address the problem.			unnecessary step.	rationale behind each step and how it contributes to the solution.	disadvantages before selecting the most effective approach.
3. Carrying Out the Plan	No attempt to carry out the plan. Example: Student leaves the solution section blank.	Attempts to implement the plan but makes major errors or deviates significantly from the plan. Example: Student uses incorrect formulas or makes significant calculation mistakes.	Implements the plan with several errors or omissions, demonstrating a partial understanding of the concepts. Example: Student applies the correct method but makes minor calculation errors or omits important steps.	Implements the plan with minor errors but generally follows the chosen approach. Explanations may be incomplete or unclear. Example: Student arrives at an incorrect answer due to a small calculation error but shows a good understanding of the process.	Implements the plan accurately and efficiently with clear and concise explanations and justifications for each step. Example: Student shows all work, clearly explaining each step and how it leads to the solution.	Implements the plan accurately and efficiently, demonstrating flexibility and adaptability by considering alternative approaches or adjusting the plan as needed. Example: Student encounters an unexpected challenge during implementation, adapts the plan effectively, and explains the reasoning behind the adjustments.
4. Looking Back	No attempt to check the solution or interpret results. Example: Student does not revisit the solution after obtaining an answer.	Makes a minimal attempt to check the solution, but overlooks significant errors or misinterprets the results. Example: Student checks for calculation errors but fails to notice a wrong answer that is	Checks the solution for basic errors but does not connect the results back to the original problem or draw meaningful conclusions. Example: Student verifies calculations but does not explain what the solution means in the	Checks the solution for accuracy and attempts to interpret the results in the context of the problem, but with limited depth or insight. Example: Student provides a basic interpretation of the solution but does not explore its implications or	Checks the solution for accuracy, interprets the results within the problem context, and considers the reasonableness of the solution and alternative approaches. Example: Student verifies the solution, explains its meaning in relation to the problem, and discusses	Checks the solution thoroughly, interprets the results with depth and insight, reflects on the overall problem-solving process, and identifies potential improvements or alternative solutions. Example: Student evaluates the efficiency and effectiveness of

Criteria	No Attempt (Score 0)	Score 1	Score 2	Score 3	Score 4	Score 5
		clearly unreasonable.	context of the problem.	consider alternative explanations.	whether the answer makes sense.	the chosen strategy, explores alternative solutions, and identifies areas for improvement in their problem-solving approach.