THIN LAYER DRYING KINETICS OF AMARANTH (Amaranthus cruentus) GRAINS IN A NATURAL CONVECTION SOLAR TENT DRYER

Ronoh EK\textsuperscript{1*}, Kanali CL\textsuperscript{1}, Mailutha JT\textsuperscript{1} and D Shitanda\textsuperscript{1}

* Corresponding author, E-mail: eriqueron2002@yahoo.com

\textsuperscript{1}Biomechanical and Environmental Engineering Department, Faculty of Engineering, Jomo Kenyatta University of Agriculture and Technology, P.O. Box 62000-00200, Nairobi, Kenya.
ABSTRACT

An experimental solar tent dryer under natural convection was used to study thin layer drying kinetics of amaranth (*Amaranthus cruentus*) grains. Drying of grains in the dryer was carried out on a drying rack having two layers; top and bottom. The ambient temperature and relative humidity ranged from 22.6–30.4°C and 25–52%, respectively, while the inside temperature and relative humidity in the solar dryer ranged from 31.2–54.7°C and 22–34%, respectively. Freshly harvested amaranth grains with an average moisture content of 64% were dried under the solar tent dryer for seven hours to a final moisture content of 7% (dry basis). A non-linear regression analysis was used to evaluate six thin layer drying models (viz., Newton, Page, Modified Page, Henderson & Pabis, Logarithmic and Wang & Singh) for amaranth grains. The models were compared using coefficient of determination ($R^2$), root mean square error (RMSE), reduced chi-square ($\chi^2$) and prediction performance ($\eta_p$) in order to determine the one that best described thin layer drying of amaranth grains. The results show that the Page model satisfactorily described the drying of amaranth grains with $R^2$ of 0.9980, $\chi^2$ of 0.00016 and RMSE of 0.01175 for bottom layer and $R^2$ of 0.9996, $\chi^2$ of 0.00003 and RMSE of 0.00550 for top layer of the drying rack. Based on a ±5% residual error interval, the Page model attained the highest prediction performance ($\eta_p = 80\%$) when drying the grains in both layers of the dryer. This shows that there was a good agreement between the predicted and experimental moisture changes during solar drying of amaranth grains under natural convection. The transport of water during dehydration was described by applying the Fick’s diffusion model and the effective moisture diffusivity for solar tent drying of amaranth grains was found to be $5.88 \times 10^{-12}$ m$^2$s$^{-1}$ at the bottom layer and $6.20 \times 10^{-12}$ m$^2$s$^{-1}$ at the top layer. High temperatures developed at the top layer of the dryer led to high effective moisture diffusivity and this showed that temperature strongly influences the mechanism of moisture removal from the grains.

**Key words:** Amaranth, Models, Diffusivity, Thin layer
INTRODUCTION

Kenya is promoting the use of amaranth grains in alleviating hunger despite the fact that many communities are ignorant of its significance in health and food security. The crop, a native of South America, is mainly grown for its grain rather than for its leaves. Kenyan farmers in regions with marginal rainfall plant amaranth rather than maize because they believe there is less risk of a crop failure with amaranth [1]. Amaranth seed is small, nearly spherical and attrition-resistant with high nutritional value. Amaranth plants have thick, tough stems similar to sunflower and the tiny, lens-shaped grains are one millimeter in diameter. The leaves can be cooked like spinach while the grains are ground into flour, popped like popcorn and cooked into porridge. While amaranth is not a staple food in Kenya, it is still grown and sold as a health food.

The potential of both grain and vegetable amaranth as a food resource has been reviewed extensively by many researchers [2]. The increasing interest in the international community in its growth and use lies in its grains which contain between 16 and 18% proteins, with high lysine content. Amaranth grain, cultivated to family scale, is exposed to ambient air and naturally dried. When amaranth is cultivated on a large scale (25-800 acres), heavy field losses occur as the crop easily shatters the grains when dry. To reduce the field losses, amaranth can be harvested with a moisture content of about 30% (dry basis) or more with necessary artificial drying to reduce the moisture level to about 10% (dry basis) to assure good preservation [3]. Drying is one of the cheap and common preservation methods for biological products [4].

Solar drying is a good alternative for farmers in Kenya and other developing countries as the dryers can generate relatively high air temperatures and low relative humidity, both of which are conducive to improved drying rates. Solar drying is actually a form of convective drying in which the air is heated by solar energy obtained from the sun. However, it differs from open sun drying in that a simple structure, such as a flat plate collector is used to enhance the effect of insolation and minimize loss of collected sun energy to the surroundings. Open sun drying depends on weather, temperature and relative humidity of the ambient environment. While solar drying has many disadvantages over open sun drying, lack of control over weather is a problem in both methods. Solar energy and, in general, renewable energy sources are important and economical, particularly during energy crises, when the cost of fuel energy increases sharply [5].

Due to the elliptical orbiting of the earth around the sun, the distance between the earth and the sun fluctuates annually and this makes the amount of energy received on the earth’s surface \( I_{sc} \) to vary in a manner given by Eq. 1, where \( I_{sc} \) is the solar constant which is valued at 1367 W/m² and \( n \) is the day of the year.

\[
I_{sc} = I_{sc} \times \left(1 + 0.033 \cos \left(\frac{360n}{365}\right)\right)
\]
The direct solar radiation, \( I_d \), reaching a unit area of a horizontal surface in the absence of atmosphere can be expressed as in Eq. 2 [6], where \( \varphi \) is latitude (degrees), \( \beta \) is angle of inclination of surface from horizontal (degrees), \( \delta \) is angle of declination (degrees) and \( \omega \) is hour angle (degrees).

\[
I_d = I_\infty \left( \sin(\varphi - \beta) \sin\delta \cos\delta \cos\omega \cos(\varphi - \beta) \right) 
\] (2)

The angle \( \delta \) is evaluated from the Eq. 3 [7]. On the other hand, \( \omega \) is computed by Eq. 4 [8], where \( H_r \) is the hour of the day in 24 hour time.

\[
\delta = 23.45 \sin \left( \frac{360}{365} \left( \frac{284 + n}{365} \right) \right) 
\] (3)

\[
\omega = 15(12 - H_r) 
\] (4)

The diffuse radiation, \( I_d \), is that portion of solar radiation that is scattered downwards by the molecules in the atmosphere. During clear days, the magnitude of \( I_d \) is about 10 to 14% of the solar radiation received at the earth’s surface. \( I_d \) can be estimated as direct radiation incident at 60\(^\circ\) on the collector surface by Eq. 5 [8, 9], where \( C \) is the diffuse radiation factor.

\[
I_d = C I_b \cos 60^\circ = 0.5C I_b 
\] (5)

The total solar radiation, \( I_t \), incident on the horizontal surface is therefore given by adding the direct and diffused components of solar radiation as shown in Eq. 6. The total solar radiation is of great importance for solar dryers since it captures the required components of solar energy that is harnessed in the dryer.

\[
I_t = I_b (1+0.5C) 
\] (6)

The drying kinetics of food is a complex phenomenon and requires simple representations to predict the drying behaviour, and for optimizing the drying parameters. The prediction of drying rate of agricultural materials under various conditions is important for the design of drying systems. Many research projects on the mathematical modeling and experimental studies have been conducted on the thin layer drying processes of various agro-based products. However, little information is available on thin layer drying behaviour of amaranth grains [10]. The study was therefore undertaken to evaluate the developed drying models in describing thin layer drying kinetics of amaranth grains in a natural convection solar tent dryer.

MATERIALS AND METHODS

Study Area
The thin layer drying experiment was conducted in an open area near the Agricultural Processing Engineering Laboratory of Jomo Kenyatta University of Agriculture and Technology (JKUAT) in the month of December 2008. JKUAT is located in Juja (37.05\(^\circ\) E longitude, 1.19\(^\circ\) S latitude and at an altitude of 1550 m above sea level).
The mean annual temperature of Juja is 18.9°C with mean annual maximum and minimum temperatures of 26.1 and 13.6°C, respectively. The relative humidity ranges from 15 to 80% [11].

**Experimental Solar Dryer**

The schematic diagram of the natural convection solar tent dryer, used in this study, is shown in Figure 1(a). The dryer consisted of a chimney, the main structure with a door and a concrete base. The main structure measured 1.85 m wide, 2.73 m long and 2.55 m high. The top part of this structure was semi-circular in shape with a radius of 0.5 m and was entirely covered with a polyvinyl chloride (PVC) material. The size of the door in this structure is 0.6 m wide and 1.8 m high. A detailed diagram illustrating locations of trays in a drying rack is shown in Figure 1(b). Flat and angled iron bars were used to fabricate these trays, and a fine wire mesh placed at the top of each layer. The entire system was completely sealed from light in order to preserve light sensitive nutrients in the drying material. For air circulation purposes, a protruding chimney was provided at the top center of this structure. The design makes the solar dryer less costly and affordable in drying most agricultural materials such as amaranth grains.

![Figure 1: (a) Schematic diagram of a natural convection solar tent dryer. (b) Diagram showing the arrangement of drying trays in two layers.](image)

**Sample Preparation and Drying Conditions**

Light yellow *Amaranthus cruentus* seeds were planted in finely prepared soil and firmed to assure good seed-to-soil contact. The rains were not adequate during the period of October 2008 and irrigation was therefore necessary to ensure good germination. Germination took three to four days with no fertilizers applied and the weeding process between rows was done after two weeks. The plants were thinned after three weeks of germination in order to leave three plants per hole. This was followed by another thinning after two more weeks which left one plant per hole in order to provide sufficient air and sunlight to the crop. After about 90 days, fresh amaranth grains were harvested with a moisture content of approximately 64% (dry-basis). Grain samples were detached from the seed heads and hand cleaned to remove any foreign material before being dried. Figure 2 shows a sample of the cleaned amaranth grains.
A sample of approximately 50 g was evenly spread on a drying tray (0.25 m × 0.25 m) to form a single layer. Two layers (i.e. top and bottom layers) in the drying rack were used in the dryer. The data were recorded at 0.5 hour intervals from 9:00 a.m. to 16:00 p.m. The capacity and sensitivity of Shimadzu electronic balance (LIBROR EB-4300D, Japan) used were 600 and 0.01 g, respectively. The ambient and inside temperatures were taken using thermocouples which relayed data to a Thermodac electronic data-logger (ETO Denki E, Japan), while relative humidity was recorded using a digital thermo-hygrometer (HC-520, Hong Kong). The solar radiation data were evaluated from electronic world satellite solar maps.

**Prediction Accuracy of Thin Layer Models**

Thin layer drying is the process of removal of moisture from a porous media by evaporation, in which excess drying air is passed through a thin layer of the material until the equilibrium moisture content is reached. Numerous mathematical models have been developed by various researchers to describe the rate of moisture loss during the thin layer drying of agricultural products such as amaranth grains. The moisture ratio (MR) of the grains being dried is presented by Eq. 7, where M is the moisture content (% dry basis) of the grain at any drying time t (hours), M₀ is the initial moisture content (% dry basis) of the wet grain and Mₑ is the equilibrium moisture content (% dry basis). MR may be simplified to M/M₀ instead of (M-Mₑ)/(M₀-Mₑ) because the value of dynamic equilibrium moisture content Mₑ is very small compared to M and M₀ [12].

\[
MR = \frac{M - M_e}{M_0 - M_e}
\]  

(7)
Another important parameter that should be considered during drying is diffusivity which is used to indicate the flow of moisture out of the material being dried [5]. In the falling rate period of drying, moisture is transferred mainly by molecular diffusion. Moisture diffusivity is influenced mainly by moisture content and temperature of the material. For a drying process in which the absence of a constant rate is observed, the drying rate is limited by the diffusion of moisture from the inside to the surface layer, represented by Fick’s law of diffusion. Assuming that the amaranth grains can be approximated to spheres, the diffusion is expressed by Eq. 8 [13] where $D_e$ is the effective moisture diffusivity ($m^2s^{-1}$) and $r_a$ is the radius (m) of amaranth grain.

$$\frac{\partial M}{\partial t} = D_e \left( \frac{\partial^2 M}{\partial r_a^2} \right)$$  \hspace{1cm} (8)

For the transient diffusion in a sphere, assuming uniform initial moisture content and a constant effective diffusivity throughout the sample, the analytical solution of Eq. 8 yields Eq. 9.

$$MR = \frac{M - M_e}{M_0 - M_e} = \left( \frac{6}{\pi^2} \right) \exp \left[ -D_e \left( \frac{\pi^2}{r_a^2} \right) t \right]$$  \hspace{1cm} (9)

The effective moisture diffusivity ($D_e$) is determined by applying logarithms to Eq. 9 to obtain a linear relation of the form shown in Eq. 10. Therefore a plot of ln(MR) versus time yields a straight line, and the diffusivity is determined from the slope (slope = $-D_e \pi^2 / r_a^2$).

$$\ln(MR) = \ln \left( \frac{6}{\pi^2} \right) - \left( D_e \frac{\pi^2}{r_a^2} \right) t$$  \hspace{1cm} (10)

The collected moisture data were used to plot graphs of moisture content against drying time, and to evaluate Eq. 11, which is based on the theory of thin layer drying [14].

$$MR = e^{-kt}$$  \hspace{1cm} (11)

For mathematical modeling, the thin layer drying models in Table 1 were tested to select the best one for describing the drying behaviour of amaranth grains. Modeling the drying behaviour of different agricultural products often requires the statistical methods of regression and correlation analyses. Regression analyses were done using the GenStat (Discovery Edition 3) statistical tool. The coefficient of determination ($R^2$), reduced chi-square ($\chi^2$) and root mean square error (RMSE) were used to determine the quality of the fit. The higher the values of $R^2$, and lower the values of $\chi^2$ and RMSE, the better the goodness of fit [12, 15, 16]. These parameters were calculated using Eq. 12 and Eq. 13, where $MR_{exp,i}$ is the experimental moisture ratio found in any measurement and $MR_{pre,i}$ is the predicted moisture ratio for this
measurement. \( N \) and \( n_c \) are the number of observations and constants, respectively [17].

\[
\chi^2 = \frac{\sum_{i=1}^{N} (MR_{exp,i} - MR_{pre,i})^2}{N - n_c}
\]  
(12)

\[
RMSE = \left[ \frac{1}{N} \sum_{i=1}^{N} (MR_{pre,i} - MR_{exp,i})^2 \right]^{1/2}
\]  
(13)

The prediction performances (\( \eta_p \)) of the drying models were also compared. These were determined by Eq. 14, where \( N_c \) and \( N_t \) represent the number of correctly predicted and trial data, respectively [14]. The performance was based on a ±5% residual error interval. The absolute residual error (\( \varepsilon \)) was defined as shown in Eq. 15 [18].

\[
\eta_p(\%) = 100 \times \frac{N_c}{N_t}
\]  
(14)

\[
\varepsilon (\%) = \left| \frac{MR_{pre,i} - MR_{exp,i}}{MR_{exp,i}} \right| \times 100
\]  
(15)

RESULTS

Figure 3 compares the temperatures developed inside the solar tent dryer and the ambient air throughout the drying period. The inside temperatures corresponded to bottom and top layers spaced at 0.75 m from the ground surface in the solar tent dryer. The results show that the temperatures developed in the top layer were always higher than those developed in the bottom layer. This is perhaps due to the closeness to the solar energy harnessing surface of the cover material and it confirms that solar tent dryers can effectively be used to harness solar energy for drying of agricultural products such as amaranth grains [10]. The temperatures in the dryer varied from morning to evening with the highest temperatures developed between 12:30 and 13:30 p.m. The difference between the inside and ambient temperatures was also high during this time. Figure 3 also indicates that the solar radiation has a direct effect on ambient and inside temperatures. The mean value of ten-year (1996–2005) solar radiation data obtained from world satellite map was approximately 6 kW/m². This mean value was slightly lower than the sum of calculated hourly results of solar radiation (≈ 8 kW/m²) computed using Eqs. 1–6. Figure 4 presents the relative humidity values recorded in the solar tent dryer and the ambient air. The relative humidity values in the dryer were always lower than those in the ambient air during the drying period.
Figure 3: Comparison of temperature and total solar radiation with time on December 2008

Figure 4: Variation of relative humidity in the ambient air and inside the solar tent dryer with time

The drying curves of amaranth grains are shown in Figure 5 where the moisture content decreased continuously with drying time. As shown by these curves, the entire
thin layer drying process obeyed the falling rate period. Amaranth grains with initial moisture content in the range of 61.3–66.7% (dry basis) were dried to a final moisture content of 7% (dry basis). It took 3.5 and 4.5 hours to dry amaranth grains to the final moisture content for the top layer and bottom layer solar tent drying, respectively. Figure 6 shows that the drying rate decreases continuously with decreasing moisture content or increasing drying time. These results are in agreement with the observations of earlier researchers based on thin layer drying of amaranth grains [19].

Figure 5: Variation of moisture content of amaranth grains with drying time

Figure 6: Drying rate as a function of moisture content of amaranth grains
The regression analyses were done for six thin layer drying models by relating the drying time and dimensionless moisture ratio. The acceptability of the model was based on a value for the coefficient of determination ($R^2$) that should be close to one, and low values for the reduced chi-square ($\chi^2$) and root mean square error (RMSE). The model coefficients and parameters of error analysis are presented in Tables 2 and 3. From the regression analysis, it is seen that the Page model satisfactorily described the drying of amaranth grains with $R^2$ of 0.9980, $\chi^2$ of 0.00016 and RMSE of 0.01175 for solar tent drying at the bottom layer and $R^2$ of 0.9996, $\chi^2$ of 0.00003 and RMSE of 0.00550 for solar tent drying at the top layer. The prediction performances of the drying models based on a ±5% residual error interval are also shown in Tables 2 and 3. The Page model attained the highest prediction performances of 80% for both bottom and top layers of the solar tent dryer.

The continuous decrease in moisture ratio with increase in drying time shows that the results can be interpreted by using Fick’s diffusion model. Effective moisture diffusivity ($D_e$) was calculated using slopes derived from the linear regression of lnMR versus time data. The computed $D_e$ values of amaranth grains under solar tent drying at the bottom and top layers were found to be $5.88 \times 10^{-12}$ and $6.20 \times 10^{-12}$ $m^2 s^{-1}$, respectively.

**DISCUSSION**

In order to describe the thin layer kinetics of amaranth grains, temperature and relative humidity were monitored in the ambient air and the solar tent dryer. The temperatures inside the solar tent dryer were higher than the ambient temperatures throughout the drying period. The closeness of the solar energy harnessing surface of the PVC cover material led to high temperatures being developed at the top layer of the drying rack. The solar radiation has a direct effect on the temperature profile and this clearly indicates that the drying rate would be much higher in the solar tent dryer under natural convection than the open sun drying. In addition, the results obeyed the commonly observed behaviour that relative humidity decreases with increase in temperature during solar drying [19].

In the falling rate period the material surface is no longer saturated with water and the drying rate is therefore controlled by diffusion of moisture from the interior of solid to the surface [20]. As expected, the decrease of relative humidity in the solar dryer increased the drying rate of the grains because a higher driving force is developed. Although all models displayed good results, the Page model gave the best description of thin layer drying process of amaranth grains in the solar tent dryer. The Page model attained the highest $R^2$ and the lowest values of RMSE, $\chi^2$ and mean absolute residual errors. The residual errors and their corresponding standard deviations were close to zero; hence better prediction by the Page model [18]. In addition, the Page model achieved satisfactory prediction level ($\eta_p = 80\%$) as compared to the other models.
The effective moisture diffusivities estimated from the drying data represents an overall mass transport property of moisture in the material, which may include liquid diffusion, vapour diffusion or any other possible mass transfer mechanism. High temperatures developed at the top layer of the drying rack in the solar tent dryer led to the highest $D_e$ value and this proves the direct dependency of moisture removal on temperature.

CONCLUSION

Thin layer drying studies of *Amaranthus cruentus* grains were carried out at two levels (bottom and top layers) of the natural convection solar tent dryer. The entire drying process of amaranth grains occurred in the falling rate period. To explain the drying kinetics of amaranth grains, six mathematical drying models were fitted to the experimental data. Comparison of the coefficient of determination ($R^2$), reduced chi-square ($\chi^2$), root mean square error (RMSE) and prediction performance ($\eta_p$) showed that the Page model best described thin layer drying of amaranth grains in the solar tent dryer. The Page model attained the highest $R^2$ value and the lowest values of $\chi^2$ and RMSE in both bottom ($R^2$, 0.9980; $\chi^2$, 0.00016; RMSE, 0.01175) and top ($R^2$, 0.9996; $\chi^2$, 0.00003; RMSE, 0.00550) layers. High prediction performance (80%) of the Page model further confirmed its superiority over the other drying models. The effective moisture diffusivity during solar tent drying of amaranth grains was found to be $5.88 \times 10^{-12}$ m$^2$s$^{-1}$ at the bottom layer and $6.20 \times 10^{-12}$ m$^2$s$^{-1}$ at the top layer. The findings also demonstrate the potential of using natural convection solar tent dryers to enhance harnessing of solar energy for drying amaranth grains.

ACKNOWLEDGEMENTS

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### Table 1: Mathematical models widely used to describe the drying kinetics

<table>
<thead>
<tr>
<th>S/No.</th>
<th>Model*</th>
<th>Model name</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>MR = exp(-kt)</td>
<td>Newton</td>
<td>[21]</td>
</tr>
<tr>
<td>2</td>
<td>MR = exp(-kt(^n))</td>
<td>Page</td>
<td>[22]</td>
</tr>
<tr>
<td>3</td>
<td>MR = exp[-(kt)(^n)]</td>
<td>Modified Page</td>
<td>[23]</td>
</tr>
<tr>
<td>4</td>
<td>MR = aexp(-kt)</td>
<td>Henderson and Pabis</td>
<td>[24]</td>
</tr>
<tr>
<td>5</td>
<td>MR = aexp(-kt) + c</td>
<td>Logarithmic</td>
<td>[25]</td>
</tr>
<tr>
<td>6</td>
<td>MR = 1 + at + bt(^2)</td>
<td>Wang and Singh</td>
<td>[26]</td>
</tr>
</tbody>
</table>

* a, b, c and n are drying coefficients, t is drying time (hours) and k is drying constant (h\(^{-1}\))

### Table 2: Estimated parameters and comparison criteria of moisture ratio for the solar tent dryer (bottom layer)

<table>
<thead>
<tr>
<th>Model</th>
<th>Model coefficients and constants</th>
<th>R(^2)</th>
<th>RMSE</th>
<th>(\chi^2)</th>
<th>ε (%)</th>
<th>η(_p) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Newton</td>
<td>(k = 0.8341)</td>
<td>0.9479</td>
<td>0.05938</td>
<td>0.00378</td>
<td>9.9 ± 10.6</td>
<td>46.7</td>
</tr>
<tr>
<td>Page</td>
<td>(k = 1.1494, n = 0.8171)</td>
<td>0.9980</td>
<td>0.01175</td>
<td>0.00016</td>
<td>3.1 ± 2.3</td>
<td>80.0</td>
</tr>
<tr>
<td>Modified Page</td>
<td>(k = 1.1494, n = 0.8171)</td>
<td>0.9976</td>
<td>0.01278</td>
<td>0.00019</td>
<td>2.9 ± 2.4</td>
<td>73.3</td>
</tr>
<tr>
<td>Henderson &amp; Pabis</td>
<td>(a = 0.8105, k = 0.8341)</td>
<td>0.9600</td>
<td>0.05206</td>
<td>0.00313</td>
<td>4.6 ± 4.6</td>
<td>73.3</td>
</tr>
<tr>
<td></td>
<td>(a = 0.7333, k = 0.8446,)</td>
<td></td>
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<td></td>
</tr>
<tr>
<td></td>
<td>(c = 0.0053)</td>
<td></td>
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</tr>
<tr>
<td>Logarithmic</td>
<td>(a = -0.3249, b = 0.0334)</td>
<td>0.9320</td>
<td>0.06786</td>
<td>0.00576</td>
<td>4.4 ± 5.9</td>
<td>66.7</td>
</tr>
<tr>
<td>Wang &amp; Singh</td>
<td>(a = -0.3249, b = 0.0334)</td>
<td>0.8584</td>
<td>0.09792</td>
<td>0.01106</td>
<td>34.5 ± 22.5</td>
<td>6.7</td>
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</table>
### Table 3: Estimated parameters and comparison criteria of moisture ratio for the solar tent dryer (top layer)

<table>
<thead>
<tr>
<th>Model</th>
<th>Model coefficients and constants</th>
<th>$R^2$</th>
<th>RMSE</th>
<th>$\chi^2$</th>
<th>$\varepsilon$ (%)</th>
<th>$\eta_p$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Newton</td>
<td>$k = 0.8816$</td>
<td>0.9360</td>
<td>0.06619</td>
<td>0.00469</td>
<td>15.9 ± 14.7</td>
<td>46.7</td>
</tr>
<tr>
<td>Page</td>
<td>$k = 1.2969$, $n = 0.8219$</td>
<td>0.9996</td>
<td>0.00550</td>
<td>0.00003</td>
<td>2.9 ± 2.0</td>
<td>80.0</td>
</tr>
<tr>
<td>Modified Page</td>
<td>$k = 1.2969$, $n = 0.8219$</td>
<td>0.9986</td>
<td>0.00967</td>
<td>0.00011</td>
<td>4.0 ± 3.8</td>
<td>66.7</td>
</tr>
<tr>
<td>Henderson &amp; Pabis</td>
<td>$a = 0.6337$, $k = 0.8816$</td>
<td>0.9638</td>
<td>0.09655</td>
<td>0.01076</td>
<td>6.0 ± 8.4</td>
<td>66.7</td>
</tr>
<tr>
<td>Logarithmic</td>
<td>$a = 0.7277$, $k = 0.9044$, $c = -0.0049$</td>
<td>0.9274</td>
<td>0.07048</td>
<td>0.00621</td>
<td>8.4 ± 6.3</td>
<td>33.3</td>
</tr>
<tr>
<td>Wang &amp; Singh</td>
<td>$a = -0.3347$, $b = 0.0353$</td>
<td>0.8343</td>
<td>0.10650</td>
<td>0.01309</td>
<td>40.8 ± 24.6</td>
<td>6.7</td>
</tr>
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REFERENCES


