# EVALUATION OF A CLASS OF POSSIBLE SIMPLE INTERIM MANAGEMENT PROCEDURES FOR THE NAMIBIAN HAKE FISHERY 

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#### Abstract

During 1997, considerable scientific differences arose about the status of the Namibian hake (Merluccius capensis and M. paradoxus) resource, and as to whether the hake Total Allowable Catch (TAC) should be substantially decreased or increased. These differences revolved primarily around whether or not abundance estimates from the swept-area trawl surveys by the Norwegian research vessel Dr Fridtjof Nansen should be considered as reliable measures of biomass in absolute (as distinct from relative) terms. The paper relates the computations underlying the Interim Management Procedure (IMP) approach that was put forward at that time as a basis to resolve this impasse. The anticipated performance, in terms of catch and risk of resource depletion, of a number of simple candidate IMPs for the Namibian hake resource is evaluated. The IMPs depend on two parameters, whose values are to be chosen by decision-makers, and adjust the TAC up or down from one year to the next according to whether trends in recent commercial catch rate and survey indices of abundance are positive or negative. Performances are evaluated across the then current wide range argued for resource abundance and status. Trade-offs in performance across the candidates considered are discussed. One of the candidates was subsequently selected by a joint meeting of scientists, industry and Ministry officials in February 1998 and served as the basis for scientific recommendations for the TAC for the hake resource for the following three years.


Key words: abundance, Cape hake, Management Procedure, Namibia, performance, surveys, trade-off

The two species of Cape hake, Merluccius capensis and M. paradoxus, constitute Namibia's most valuable marine resource (Boyer and Hampton 2001). The hake fishery off Namibia, which began in the mid 1960s, was primarily international and managed under the auspices of ICSEAF (the International Commission for the Southeast Atlantic Fisheries), prior to Namibian Independence in 1990. Then, Namibian authorities immediately placed heavy restrictions on foreign access, and commenced the expansion of what until that time had been a comparatively small local fishery for hake. The hake Total Allowable Catch (TAC) was severely cut to little more than 50000 tons (see Appendix Table App.I), and a new research programme commenced with the assistance of the Norwegian aid agency NORAD.

That research programme incorporated scientific surveys of the hake resource by the research vessel Dr Fridtjof Nansen. Survey results were analysed to give biomass estimates in absolute terms, based upon the trawl swept-area method with hydroacoustics-based corrections to make allowance for hake in midwater. Hake TACs were recommended as approximately $20 \%$ (an ad hoc choice) of the estimates of biomass of fish longer than 35 cm (corresponding approximately to the mature component of the resource).

By 1997, the hake TAC had risen to some 120000 tons, but both the Dr Fridtjof Nansen survey results and the commercial catch per unit effort (срие) had dropped
markedly over the three preceding years. This led Namibian scientists to recommend a substantial TAC reduction (of about $50 \%$ ), an action which would have had severe implications for the Namibian economy.

A major international scientific workshop on the hake resource was held in October 1997 (Anon. 1997). Two opposing views of the status of the resource crystalized from the discussions there, distinguished essentially by whether or not the Dr Fridtjof Nansen survey results were to be regarded as reliable in absolute (as distinct from only relative) terms in assessments. At the one extreme, if estimates of abundance from the Dr Fridtjof Nansen surveys were taken to provide reliable values for the resource biomass in absolute terms, the resource was evaluated as heavily depleted, with large reductions in the TAC needed if further depletion was to be avoided. At the other extreme, if the Dr Fridtjof Nansen results were meaningful only as indices of relative abundance, the resource was estimated to be above its maximum sustainable yield level, with large increases in TAC being possible without endangering the future of the fishery (Fig. 1).

To attempt to provide guidance to management authorities on how to proceed given this impasse, the authors (with financial support from the Namibian Ministry of Fisheries and Marine Resources) commenced work towards developing a simple feedback "Management Procedure" as a basis for recommending Namibian hake TACs in the short term. The "Manage-

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Fig. 1: Age-structured production model estimates of age $2+$ biomass $\left(B^{2+}\right)$ for the Namibian hake resource for two scenarios. One treats the Dr Fridtjof Nansen research survey results as an index of relative abundance and the other as an index of absolute abundance. The Dr Fridtjof Nansen summer and winter biomass estimates and the reported historical catches are also shown
ment Procedure" approach to fisheries management involves the use of computer simulation to check that the basis proposed for setting TACs for a fishery will perform adequately, in terms both of securing a reasonable level of catch and of not exposing the resource to undue risk of substantial depletion (Butterworth and Punt 1999, Cooke 1999). Particular emphasis is laid in the approach on ensuring that such performance remains adequate across the range of uncertainties that exist about a resource's status and dynamics. The large uncertainties associated with such perceptions in 1997 for the Namibian hake resource rendered it an obvious candidate to which to attempt to apply the approach. The approach has also been applied to a number of other fisheries off southern Africa, as summarized in Geromont et al. (1999).

The account that follows reflects the report on this initiative that was presented at a meeting between scientists and industry in February 1998 (see concluding Afterword). Few changes have been made, as the intent is to convey accurately the basis upon which that meeting discussed and reached agreement on using such an approach for recommending TACs for Namibian hake for the following few years. As such it constituted a watershed in the approach to the management of Namibia's major renewable marine resource.

It should be noted that the report and its proposals were presented at a time when strong and widely diverging views were held by different parties involved
in the management of Namibian hake, as summarized above. To have any chance of facilitating progress, the report had to reflect an even-handed consideration of both sets of views. Accordingly, key issues, such as whether or not the abundance estimates from the Dr Fridtjof Nansen surveys were to be regarded as reliable in absolute terms, are addressed in a neutral fashion in what follows, but this should not be interpreted as necessarily reflecting the authors' personal views on such issues.

## BACKGROUND

From discussions held at the International Workshop on Research and Management of Hake in Namibian Waters that took place in Swakopmund in October 1997 (Anon. 1997), it had become clear that management of the Nambian hake resource at that time was confounded by the wide range of the results of scientific assessments of the status of the resource, for the reasons summarized above. Resolution of the debate about the reliability or otherwise of estimates of absolute abundance from the Dr Fridtjof Nansen surveys was seen as unlikely to be immediate. In the meantime, therefore, an interim approach for recommending annual $T A C$ s was required. This on the one hand had to be such as not to waste the resource, if the more optimistic appraisals of its status were correct. On the other hand it had to provide recommendations for timeous reductions in catch levels to avoid further substantial depletion of stock, should future data indicate that it was the then current more pessimistic appraisals that reflected reality.

The analyses that follow evaluate some possibilities for such an approach. Their restriction, late in 1997, to relatively simple approaches was deliberate, because a defensible basis for deciding upon a TAC for 1998, in the circumstances of the then large degree of uncertainty about resource status, was urgently required. The approaches considered were not necessarily the best possible; such would have taken more time to develop and test than was then available. However, these simple approaches were seen to have the potential, indeed were required, to ensure that the resource was safeguarded against heavy depletion in the short term, while still allowing reasonable levels of catch.

The discussion reported here of the results of the analyses does not include a recommended choice from among the possible approaches (candidate "Interim Management Procedures", or IMPs) considered. Rather, its intent was to illustrate the different trade-offs between expected catch and risk of heavy depletion for each, so as to provide decision-makers with a sound basis for choosing between the alternatives.

## CANDIDATE IMPs

The idea underlying the candidate IMPs considered is that the TAC each year be determined early in that year by adjusting the previous year's TAC up or down depending on the rate of increase or decrease in the size of the resource as indicated by two indices, for each of which a new data point would just have become available at that time. This approach is known as "derivative control" and is similar to that proposed for whale stock management by Magnusson and Stefánsson (1989). The two indices are:

- the index of relative abundance provided by a generalized linear modelling (GLM) analysis used to standardize commercial cpue data, which would be available for years up to and including that just completed; and
- the abundance index from a scientific survey that is assumed to continue to take place at the start of each year (i.e. continuing the Dr Fridtjof Nansen summer survey series - see Appendix Table App.I).

The specific formula that provides the TAC recommendations for these candidate IMPs is

$$
\begin{equation*}
T A C_{y}=T A C_{y-1}\left[1+\lambda s_{y}\right], \tag{1}
\end{equation*}
$$

where $T A C_{y}$ is the $T A C$ in year $y, \lambda$ a control parameter whose value is pre-chosen, and $s_{y}$ is a measure of the trend in the abundance indices at the start of year $y$.

This trend measure is computed as follows from the abundance indices derived from commercial cpиe data $\left(I_{y}^{\text {cpue }}\right)$ and scientific surveys $\left(I_{y}^{\text {sur }}\right)$ :

- linearly regress $\ell n I_{y^{\prime}}^{\text {cpue }}$ against year $y^{\prime}$ for $y^{\prime}=y-p$, $y-p+1, \ldots, y$ to yield a regression slope value $s_{y}^{\text {cpue }}$;
- linearly regress $\ell n I_{y^{\prime}}^{s u r}$ against year $y^{\prime}$ for $y^{\prime}=y-p+1$, $y-p+2, \ldots, y$, to yield a regression slope value $s_{y}^{\text {sur }}$; then

$$
\begin{equation*}
s_{y}=\left[s_{y}^{\text {cpue }}+s_{y}^{\text {sur }}\right] / 2 . \tag{2}
\end{equation*}
$$

Note that the lengths of the periods considered for these regressions are the same ( $p$ years), but the period pertinent for the surveys is displaced one year forward from that considered for the cpue. This is because shortly after the start of year $y$, the result of the survey for that year ( $\left.I_{v}^{s u r}\right)$ will be available, but the cpue index will be available only up to the preceding year $\left(I_{y-1}^{\text {cpue }}\right)$.

The choice of a value for $p$ involves a trade-off. If $p$ is chosen too small (e.g. $p=2$ ), trend estimates will fluctuate markedly from one year to the next because of noise (observation error) in the data, leading to undesirably large variations in the TAC from one year to the next under Equation 1. On the other hand,
if $p$ is chosen too large (e.g. $p=10$ ), although such variations will be damped down, the $s_{y}$ values will not be very sensitive to possible recent changes in resource abundance trends, and thus may not adjust the TAC sufficiently in response. For the evaluations within this document, the compromise choice $p=5$ was made throughout.

The different candidate IMPs described by Equation 1 are distinguished by two "control parameters", each of which was to be chosen by decision-makers on the basis of achieving what they considered to be an optimum trade-off between the values expected for various measures of procedure performance, as described below. The first of these parameters $(\lambda)$ reflects how strongly the TAC is to be adjusted in response to a trend in resource size. Thus, for example, if the $s_{y}$ value calculated is 0.05 , suggesting that the resource is increasing at $5 \%$ per annum, the choice $\lambda=1$ would see the TAC increased by $5 \%$, whereas $\lambda=3$ would result in a $15 \%$ increase.

The second control parameter reflects the "starting level" for the Equation 1 formula. To calculate the TAC for 1998, that formula requires an input " $T A C_{1997}$ " referenced below as $C_{97}^{*}$ - whose value the decisionmakers would need to choose (i.e. it might differ from, and is not to be confused with, the actual TAC for 1997 of 120000 tons).

The different candidate IMPs considered below are therefore characterized by the values of $\lambda$ and $C_{97}^{*}$. Note that the choice of these two values (particularly that of $C_{97}^{*}$ ) did not immediately determine the TAC recommendation for 1998 (TAC 1998 in Equation 1). This is because $s_{1998}$ could be calculated only once срие results for 1997 (to give $I_{97}^{\text {cpue }}$ ) and survey results for the summer Dr Fridtjof Nansen survey in 1998 (to give $I_{98}^{\text {sur }}$ ) became available (they were not available at the time the analyses were developed). Depending on whether these results were higher or lower, indicating a resource status that was better or worse than anticipated, the $T A C_{1998}$ recommendation would be greater or smaller, as appropriate.

## PROCESS USED TO TEST CANDIDATE IMPs

The basic process consists of the following steps.

1. From an assessment of the resource, determine estimates for the numbers-at-age vector at the start of 1998 ( $N_{1998, a}: a=1, \ldots m$ ), as well as for the parameters of the stock-recruitment relationship $(\alpha, \beta)$ - see Appendix Equations App.1-App.4, which in turn permit evaluation of the 1998 recruitment $\left(N_{1998,0}\right)$.
2. Use this information to generate values of the abundance indices $I_{97}^{\text {cpue }}$ and $I_{98}^{\text {sur }}$.
3. From these abundance indices, calculate $s_{1998}$ as per Equation 2 and the associated text, and then use Equation 1 to compute $T A C_{1999}$.
4. Project the numbers-at-age $N_{1998, a}$ forward under a catch $T A C_{1998}$ to determine $N_{1998, a}$ (in this process it is assumed that the post-Independence, i.e. post1990, commercial age-specific selectivity pattern estimated in the assessment in Step 1 remains unchanged).
5. Repeat steps $1-4$ for as long a period as desired, and at the end of that period assess the performance of the candidate IMP under review by considering statistics such as the total catch taken and the final biomass of the resource.

## Accounting for basic uncertainties

The process used to provide the resource assessment is an age-structured production model (ASPM), the data inputs to which were specified and agreed during the October 1997 International Workshop, and which is described in detail in the Appendix. As indicated above, the key uncertainty in the results that followed from this process is related to assumptions made about the past Dr Fridtjof Nansen surveys. If these are assumed to provide abundance estimates in absolute terms (i.e. the catchability parameter, $q$, is fixed equal to 1 - see subsection: "Abundance data" in the Appendix), the current resource biomass is estimated to be low ( $B_{1998}^{2+}=525000$ tons; see Table I). However, if the Dr Fridtjof Nansen results are assumed to be reliable only as relative indices, so that the constant of proportionality $q$ is estimated by Equation App.16, the consequent estimates are $q=0.223$ and $B_{1998}^{2+}=3386000$ tons.

To reflect this range of uncertainty, the performance of candidate IMPs was tested over a range of ASPM assessment results corresponding to fixing $q$ at values of $1.0,0.8,0.6,0.4$ and 0.2 . Ideally, the performance statistics for a candidate should be "robust" across this range, i.e. show acceptable results whatever value of $q$ best reflects the present real situation of the hake resource.
There are further uncertainties that also had to be taken into account in assessing candidate IMP performance:

- indices of abundance forthcoming in future years will not be exactly proportional to true abundance, but will also be subject to observation error ( $\varepsilon_{y}^{i}$ in Appendix Equation App.12);
- future recruitments will not be exactly specified by the deterministic stock-recruitment relationship of Appendix Equation App.4, but will be subject to fluctuation
about the levels indicated by that relationship;
- the starting numbers-at-age and stock-recruit parameters ( $N_{1998, a} ; \alpha, \beta$ ) estimated by the assessment process, even given a fixed value for $q$, are not known exactly, but are subject to estimation error.
These concerns are handled as follows.
- Log-normal observation error is added to the expected value of the abundance index ( $I$ ) evaluated from forward-projected numbers-at-age ( $N_{y, a}$ ) for the resource, i.e.

$$
\begin{equation*}
I_{y}^{i}=q^{i} \boldsymbol{B}_{y}^{i} \mathrm{e}^{\varepsilon_{y}^{i}} \varepsilon_{y}^{i} \text { from } N\left(0,\left(\sigma^{i}\right)^{2}\right) \tag{3}
\end{equation*}
$$

where $B_{y}^{i}$ is the biomass determined from Appendix Equations App. 10-11 as appropriate, $i$ reflects commercial срие ("срие") or the Dr Fridtjof Nansen surveys ("sur"), $q^{\text {sur }}$ is set equal to the value assumed (1.0, 0.8, 0.6, 0.4 or 0.2 ) for the associated assessment, and $q^{\text {cpue }}$ is as estimated (Appendix Equation App.15) for that assessment. Calculations conducted assumed:
$\sigma^{s u r}=0.2$ (i.e. a $20 \%$ coefficient of variation, $C V$, for the Dr Fridtjof Nansen survey results, as assumed for the ASPM assessments see Appendix Table App.I), and
$\sigma^{\text {cpue }}=0.3$ (this reflects an average of values of this quantity estimated by Appendix Equation App.14, which ranged from 0.238 to 0.355 as the fixed value of $q$ was changed from 1.0 to 0.2 ).

- Log-normal fluctuations about the stock-recruitment relationship of Appendix Equation App. 4 are assumed, i.e.

$$
\begin{equation*}
R_{y}=\frac{\alpha B_{y}^{s p}}{\beta+B_{y}^{s p}} \mathrm{e}^{\eta_{y}-\sigma_{r}^{2} / 2} \eta_{y} \text { from } N\left(0, \sigma_{r}^{2}\right) \tag{4}
\end{equation*}
$$

The quantity $\sigma_{r}$ is (approximately) the $C V$ of the recruitment variations about their values as predicted by the deterministic component of Equation 4. The calculations that follow assumed that $\sigma_{r}=0.45$. This value was based on the integrated analysis assessment carried out by A. E. Punt during the October 1997 International Workshop (Anon. 1997). That method provided estimates of residuals about the stock recruitment relationship which have some reliability over periods when catch-at-age data are available. The value above corresponds to the period 1964-1988 for the case where the Dr Fridtjof Nansen surveys are treated as providing abundance estimates in absolute terms. The term $-\sigma_{r}^{2} / 2$ in Equation 4 is to correct for bias given the skewness

Table I: Comparison of deterministic results (i.e. all parameter estimates exact; all future abundance indices exact; no future fluctuations about the stock-recruit relationship) for IMPs for different choices for the catch-control law parameter $\lambda$. $C_{97}^{*}$ values in thousand tons

| Input parameters | Average catch ('000 tons) | $B_{1998} / K^{2+}$ | $B_{2003} / K^{2+}$ | $B_{1998}^{2+}$ ('000 tons) | $K^{2+}$ ('000 tons) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & q=1.0, \lambda=1: \\ & C_{97}^{*}=50 \\ & C_{97}^{*}=100 \\ & C_{97}^{*}=150 \\ & C_{97}^{*}=200 \end{aligned}$ | $\begin{array}{r} 45 \\ 84 \\ 117 \\ 143 \end{array}$ | $\begin{aligned} & 0.098 \\ & 0.098 \\ & 0.098 \\ & 0.098 \end{aligned}$ | $\begin{aligned} & 0.118 \\ & 0.078 \\ & 0.046 \\ & 0.019 \end{aligned}$ | 525 | 5347 |
| $\begin{aligned} & q=1.0, \lambda=3: \\ & C_{97}^{*}=50 \\ & C_{97}^{*}=100 \\ & C_{97}^{*}=150 \\ & C_{97}^{*}=200 \end{aligned}$ | $\begin{aligned} & 36 \\ & 63 \\ & 82 \\ & 96 \end{aligned}$ | $\begin{aligned} & 0.098 \\ & 0.098 \\ & 0.098 \\ & 0.098 \end{aligned}$ | $\begin{aligned} & 0.127 \\ & 0.100 \\ & 0.080 \\ & 0.065 \end{aligned}$ |  |  |
| $\begin{aligned} & q=0.8, \lambda=1: \\ & C_{97}^{*}=50 \\ & C_{97}^{*}=100 \\ & C_{97}^{*}=150 \\ & C_{97}^{*}=200 \end{aligned}$ | $\begin{array}{r} 47 \\ 89 \\ 126 \\ 158 \end{array}$ | $\begin{aligned} & 0.128 \\ & 0.128 \\ & 0.128 \\ & 0.128 \end{aligned}$ | $\begin{aligned} & 0.154 \\ & 0.112 \\ & 0.074 \\ & 0.042 \end{aligned}$ | 697 | 5452 |
| $\begin{aligned} & q=0.8, \lambda=3: \\ & C_{97}^{*}=50 \\ & C_{97}^{*}=100 \\ & C_{97}^{*}=150 \\ & C_{97}^{*}=200 \end{aligned}$ | $\begin{array}{r} 41 \\ 73 \\ 97 \\ 114 \end{array}$ | $\begin{aligned} & 0.128 \\ & 0.128 \\ & 0.128 \\ & 0.128 \end{aligned}$ | $\begin{aligned} & 0.160 \\ & 0.128 \\ & 0.103 \\ & 0.084 \end{aligned}$ |  |  |
| $\begin{aligned} & q=0.2, \lambda=1: \\ & C_{97}^{*}=50 \\ & C_{97}^{*}=100 \\ & C_{97}^{*}=150 \\ & C_{97}^{*}=200 \end{aligned}$ | $\begin{array}{r} 56 \\ 112 \\ 166 \\ 219 \end{array}$ | $\begin{aligned} & 0.803 \\ & 0.803 \\ & 0.803 \\ & 0.803 \end{aligned}$ | $\begin{aligned} & 0.900 \\ & 0.867 \\ & 0.834 \\ & 0.802 \end{aligned}$ | 3464 | 4312 |
| $\begin{aligned} & q=0.2, \lambda=3: \\ & C_{97}^{*}=50 \\ & C_{97}^{*}=100 \\ & C_{97}^{*}=150 \\ & C_{97}^{*}=200 \end{aligned}$ | $\begin{array}{r} 70 \\ 137 \\ 199 \\ 257 \end{array}$ | $\begin{aligned} & 0.803 \\ & 0.803 \\ & 0.803 \\ & 0.803 \end{aligned}$ | $\begin{aligned} & 0.889 \\ & 0.847 \\ & 0.808 \\ & 0.772 \end{aligned}$ |  |  |

of the log-normal distribution; it ensures that, on average, recruitments will be as indicated by the deterministic component of the stock-recruitment relationship.

- Imprecision in the $N_{1998, a}, \alpha$ and $\beta$ estimates is taken into account by a Monte Carlo bootstrap process (see the penultimate section of the Appendix) that provides equally likely sets of values of these quantities.

It should be noted that, because observation errors for future abundance indices and also recruitment fluctuations are drawn at random from distributions in projecting the resource biomass forward in time, results will differ from one projection to the next. Therefore, candidate IMP performance in terms of, say, the anticipated cumulative catch over a period is reflected by a distribution rather than a single number. Summary statistics of such distributions are estimated
by carrying out 100 repetitions of the projection process (Steps 1-5 above), with resultant medians and $90 \%$-iles being reported. Monte Carlo bootstrap representation of assessment imprecision is readily incorporated in this process.

## Robustness to further uncertainties

The basic ASPM assessment may not necessarily reflect the actual present resource status and dynamics, so it is important to check that candidate IMP performance is reasonably robust to plausible variations in the hypotheses underlying the assessment results. Two such variations were considered.

1. The stock-recruitment relationship used in the ASPM assessment procedure (Appendix Equation App.4) is deterministic, i.e. past recruitments are set ex-
actly equal to the predictions of that equation without allowance for any fluctuations. To admit more flexibility in the starting number-at-age estimates ( $N_{1998, a}$ ), fluctuations about the relationship were estimated for the period 1990-1997. This is possible because of the availability of catch-at-age information from the Dr Fridtjof Nansen surveys, and it reflects a partial extension of the ASPM assessment towards a fully integrated analysis option explored by A. E. Punt during the October 1997 International Workshop (Anon. 1997);
2. Suggestions had been made that the Namibian ecosystem in the 1990s might be less productive than in the past, reflecting a drop in the carrying capacity $(K)$ for hake; this is investigated by performing an assessment in which the stock-recruit parameters $\alpha$ and $\beta$ each halve in 1990, reflecting a "regime shift" of a $50 \%$ reduction in $K$ at that time, which is taken to continue to apply over the projection period considered.

## MEASURES OF CANDIDATE IMP PERFORMANCE

It was unlikely that an IMP along the lines investigated here would be applied for more than two or at most three years, as modifications would be warranted once more clarity on the effective value of $q$ for the Dr Fridtjof Nansen surveys became available. Nevertheless, the projection period considered here for reporting performance statistics is five years. The reason for this is to clarify longer term risk implications, which may not be evident from shorter projections because they could be confounded by transient effects. Therefore, performance statistics apply to catches anticipated over the period 1998-2002, and the biomass (in terms of ages 2 and above, i.e. $B^{2+}$ ) at the end of this period (the beginning of 2003, i.e. $B_{2003}^{2+}$ ). The performance statistics reported are given below.

- Catch-related:
- Catch at the start of the projection period: $C_{1998}$
- Catch at the end of the projection period: $C_{2003}$
- Average annual catch over that period: $1 / 5 \sum_{y=1998}^{22002} C_{y}$.

Note that the calculations assume that the TAC recommended using Equation 1 is always taken exactly (unless the resource is thereby rendered extinct! although this never occurred for the options considered here).

- Interannual variability in catches:
- The average percentage change in TAC from one year to the next:

$$
\left[1 / 4 \sum_{y=1999}^{2002}\left|C_{y}-C_{y-1}\right| / C_{y}\right] \times 100
$$

- Risk-related (in terms of undesired biomass reduction):
- Depletion at the start of the projection period: $B_{1998} / K^{2+}$
- Depletion at the end of the projection period: $B_{2003} / K^{2+}$
- Change in depletion over this period: $B_{2003} / K^{2+}$ - $B_{1998} / K^{2+}$.

It should be noted that, once stochastic effects (e.g. future observation errors and recruitment fluctuations) are taken into account, the result for each of these quantities on testing a particular candidate IMP for a specific scenario (e.g. specific assumption for $q$ ) is a distribution. Results are reported in the form of medians and $90 \%$ iles of these distributions.

## RESULTS AND DISCUSSION

Table I lists the results of deterministic computations for eight candidate IMPs: all combinations of choices of control parameters $\lambda=1$ or 3 and $C_{97}^{*}=50000$, 100000,150000 or 200000 tons. The word "deterministic" here means that perfect past and future knowledge are assumed, i.e. that the ASPM assessment results are exact, and in future there are no observation errors associated with the abundance indices and no fluctuations about the stock-recruitment relationship. Results are given in Table I for $q=1.0,0.8$ and 0.2 and are also shown graphically in Figures 2a and 2b for the first and last, respectively, of these scenarios for $q$.

Comparative performance for different choices for $\lambda$ is most readily evident from consideration of Figure 2. For the most pessimistic scenario considered ( $q=1$; Fig. 2a), the choice $\lambda=3$ provides faster resource recovery if $C_{97}^{*}=50000$ tons, and much less reduction if $C_{97}^{*}=150000$ tons, than would $\lambda=1$. At the other most optimistic extreme ( $q=0.2$; Fig. 2b), $\lambda=3$ results in a faster increase in the catch without prejudicing the status of the resource. For these reasons, the remaining evaluations of this document are limited to the choice $\lambda=3$.

Table II extends the results of Table I by taking some further uncertainties into account. The listing under "Future errors" shows the consequences of introducing random error in future indices of abundance and fluctuations in future recruitments about the stock-recruitment relationship. Medians of the resultant distributions for the final depletion $B_{2003} / K^{2+}$ are similar to those for the deterministic results. For the higher values of


Fig. 2: Deterministic projections for annual catch and depletion of the $2+$ biomass ( $B / K^{2+}$ ) for four candidate IMPs considered in Table I (all combinations of $C_{97}^{*}=50000$ or 150000 tons, and $\lambda=1$ or 3 ). Results for the scenario with $q=1$ (Dr Fridtjof Nansen survey estimates treated as absolute) are shown in (a) and those for $q=0.2$ (Dr Fridtjof Nansen estimates relative) in (b)
$q$ considered ( 1 and 0.8 ), the $90 \%$-ile for the final depletion spans some $5-10 \%$. This range is much bigger for $q=0.2$, although none of the lower $5 \%$-iles for that case are $<0.5$ so do not give rise to concern in a risk context. Note that depletions can exceed 1 (i.e. $B>K$ ) because of the introduction of fluctuations about the stock-recruitment relationship; $K$ reflects
only the average biomass in the absence of harvesting, and can be exceeded given some better-than-average recruitments.

Table II also lists the results of two of the variations in the assumptions underlying the basic ASPM assessment that are described above. If this assessment allows for recruitment fluctuations over the period 1990-1997,

Table II: The consequences for the 2+ hake biomass in 2003 of introducing random error in future indices of abundance and fluctuations in future recruitments about the stock-recruitment relationship ("Future errors"). Results are also shown for the cases where the age-structured production model allows for such recruitment fluctuations for the years 1990-1997 ("Past recruitment fluctuations"), and for a "Regime shift" reflected by a 50\% drop in K in 1990. All results are for the catch control law parameter $\lambda$ set to 3 ; those for $B / K^{2+}$ in 2003 reflect the distribution median with the associated $90 \%$ probability interval in parenthesis. $C_{97}^{*}$ values in thousand tons

| Input parameters | $B / K^{2+}$ |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Deterministic |  | Future errors |  | Past recruitment fluctuations |  | Regime shift |  |
|  | 1998 | 2003 | 1998 | 2003 | 1998 | 2003 | 1998 | 2003 |
| $\begin{aligned} & q=1.0: \\ & C_{97}^{*}=50 \end{aligned}$ | 0.098 | 0.127 | 0.098 | $\begin{aligned} & 0.120 \\ & (0.087,0.166) \\ & 0.095 \\ & (0.069,0.134) \\ & 0.075 \\ & (0.055,0.133) \\ & 0.062 \\ & (0.043,0.095) \end{aligned}$ | 0.096 | $\begin{aligned} & 0.116 \\ & (0.084,0.161) \end{aligned}$ | 0.200 | $\begin{aligned} & 0.258 \\ & (0.184,0.358) \end{aligned}$ |
| $C_{97}^{*}=100$ | 0.098 | 0.160 | 0.098 |  | 0.096 | $\begin{aligned} & 0.093 \\ & (0.067,0.130) \end{aligned}$ | 0.200 | $\begin{aligned} & 0.208 \\ & (0.146,0.290) \end{aligned}$ |
| $C_{97}^{*}=150$ | 0.098 | 0.080 | 0.098 |  | 0.096 | $\begin{aligned} & 0.074 \\ & (0.054,0.110) \end{aligned}$ | 0.200 | $\begin{aligned} & 0.164 \\ & (0.122,0.248) \end{aligned}$ |
| $C_{97}^{*}=200$ | 0.098 | 0.065 | 0.098 |  | 0.096 | $\begin{aligned} & 0.061 \\ & (0.043,0.093) \end{aligned}$ | 0.200 | $\begin{gathered} 0.138 \\ (0.096,0.212) \end{gathered}$ |
| $\begin{aligned} & q=0.8: \\ & C_{97}^{*}=50 \end{aligned}$ | 0.128 | 0.160 | 0.128 | $\begin{aligned} & 0.153 \\ & (0.111,0.210) \end{aligned}$ | 0.148 | $\begin{aligned} & 0.182 \\ & (0.132,0.252) \end{aligned}$ | 0.300 | $\begin{aligned} & 0.390 \\ & (0.286,0.542) \end{aligned}$ |
| $C_{97}^{*}=100$ | 0.128 | 0.128 | 0.128 | $\begin{aligned} & 0.123 \\ & (0.088,0.170) \end{aligned}$ | 0.148 | $\begin{aligned} & 0.145 \\ & (0.103,0.201) \end{aligned}$ | 0.300 | $\begin{aligned} & 0.314 \\ & (0.220,0.442) \end{aligned}$ |
| $C_{97}^{*}=150$ | 0,128 | 0.103 | 0.128 | $\begin{aligned} & 0.097 \\ & (0.071,0.145) \end{aligned}$ | 0.148 | $\begin{aligned} & 0.112 \\ & (0.082,0.169) \end{aligned}$ | 0.300 | $\begin{aligned} & 0.250 \\ & (0.182,0.376) \end{aligned}$ |
| $C_{97}^{*}=200$ | 0.128 | 0.084 | 0.128 | $\begin{aligned} & 0.079 \\ & (0.055,0.123) \end{aligned}$ | 0,148 | $\begin{aligned} & 0.091 \\ & (0.062,0.141) \end{aligned}$ | 0.300 | $\begin{aligned} & 0.208 \\ & (0.144,0.318) \end{aligned}$ |
| $\begin{aligned} & q=0.2: \\ & C_{97}^{*}=50 \end{aligned}$ | 0.803 | 0.889 | 0.803 | $\begin{aligned} & 0.854 \\ & (0.646,1.127) \end{aligned}$ | 0.805 | $\begin{aligned} & 0.856 \\ & (0.647,1.130) \end{aligned}$ | 0.946 | $\begin{aligned} & 1.166 \\ & (0.834,1.634) \end{aligned}$ |
| $C_{97}^{*}=100$ | 0.803 | 0.847 | 0.803 | $\begin{aligned} & 0.805 \\ & (0.609,1.079) \end{aligned}$ | 0.805 | $\begin{aligned} & 0.808 \\ & (0.612,1.084) \end{aligned}$ | 0.946 | $\begin{aligned} & 1.126 \\ & (0.808,1.596) \end{aligned}$ |
| $C_{97}^{*}=150$ | 0.803 | 0.808 | 0.803 | $\begin{aligned} & 0.766 \\ & (0.575,1.034) \end{aligned}$ | 0.805 | $0.770$ | 0.946 | $\begin{aligned} & 1.092 \\ & (0.784,1.558) \end{aligned}$ |
| $C_{97}^{*}=200$ | 0.803 | 0.772 | 0.803 | $\begin{aligned} & 0.737 \\ & (0.543,0.995) \end{aligned}$ | 0.805 | $\begin{gathered} 0.741 \\ (0.547,1.001) \end{gathered}$ | 0.946 | $\begin{aligned} & 1.064 \\ & (0.762,1.522) \end{aligned}$ |

the projected final depletion statistics generally differ little from those when such fluctuations are ignored. In the interests of computational efficiency, therefore, the option of continuing to estimate these past fluctuations was not pursued in subsequent computations. The "regime shift" results ( $K$ halving in 1990) generally reflect both a healthier resource (less depleted relative to the changed $K$ ), and a greater extent of resource recovery over the 5-year period of implementation of the IMP candidates. Therefore, the robustness of these candidates to this possibility does not seem to be an issue.
The "Future errors" results of Table II are extended further in Table III by there taking account also of the imprecision of the ASPM assessment (via the bootstrap procedure, as described above). In this case, focus on the final depletion $B_{2003} / K^{2+}$ distribution
statistics alone can be misleading because the initial depletion $B_{1998} / K^{2+}$ is also not known exactly. The "change" statistic for depletion provides an indication of the biomass trend to be expected under the candidate IMP, and the degree of uncertainty associated with this. Thus, for the two higher values of $q$ considered (corresponding to the more pessimistic appraisals of resource status), median biomass changes range from $-5 \%$ to $+5 \%$ (of $K^{2+}$ ) for the various $C_{97}^{*}$ values investigated, but with associated $90 \%$ uncertainty ranges of 7-13\%.

Some of the results in Table III are also shown graphically in Figures 3 and 4. Comparison of the projections in Figure 3 with their deterministic counterparts in Figure 1 shows the consequences for predictions of assessment imprecision and future noise in recruitment and abundance indices.


Fig. 3: Repeat of the projections shown in Figure 2, but in this case for the fully stochastic simulation trials reported in Table III, and only for the case $\lambda=3$. The plots show medians and $90 \%$ probability intervals. The plots in (a) are for the $q=1$ scenario and those in (b) for $q=0.2$

Figure 4 provides probably the best basis to commence comparison of the performance of the different candidate IMPs. Figure 4a compares across different choices for the value of the IMP control parameter $C_{97}^{*}$ for the scenario $q=0.8$. Ideally, the candidate to be chosen should provide reasonable average catch without too much of the distribution for the change in depletion (the centre plot) lying below the zero line, i.e. reflecting biomass reduction. However, this tradeoff has to be evaluated across all plausible scenarios for $q$, as illustrated in Figure 4b for the candidate

IMP with $C_{97}^{*}=150000$ tons. The lower the value of $q$, the healthier the resource, and hence the higher the cumulative catch to be expected, which is as achieved except for the extreme case $q=0.2$ (a consequence of the resource being currently close to $K$ in this instance, a feature which the design of these simple candidate IMPs precludes them being able to handle well). The widening of the change in depletion distribution for lower $q$ values may appear problematic, but this is not actually the case because the final depletions in these scenarios are above 0.5 (see Table III).





Fig. 4: Performance statistics (average catch, change in depletion and average annual change in catch, projected over the five-year period commencing in 1998) compared for different choices for the $C_{97}$ control parameter, whereas in (b) this parameter is set to 15000 tons and

Table III: Performance statistics for the catch control law parameter $\lambda$ set to 3 for the case where both "future errors" are taken into account (i.e. random errors in future abundance indices and future fluctuations about the stock-recruitment relationship), and allowance is made (through bootstrapping) for imprecision in the estimates of the 1998 stock status and values of the parameters of the stock-recruitment relationship. Distribution medians, together with $90 \%$ probability intervals in parenthesis, are shown. $C_{97}^{\star}$ values in thousand tons

| Input parameters | Catch |  |  |  | $B / K^{2+}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1998 | 2002 | Average | Average \% change | 1998 | 2003 | Change |
| $\begin{aligned} & q=1.0: \\ & C_{97}^{*}=50 \\ & C_{97}^{*}=100 \\ & C_{97}^{*}=150 \\ & C_{97}^{*}=200 \end{aligned}$ | 36 $(26,46)$ 72 $(52,91)$ 108 $(78,137)$ 143 $(104,182)$ | $\begin{gathered} 52 \\ (19,104) \\ 74 \\ (25,161) \\ 75 \\ (23,166) \\ 66 \\ (7,146) \end{gathered}$ | $\begin{aligned} & 41 \\ & (22,69) \\ & 71 \\ & (40,115) \\ & 92 \\ & (52,146) \\ & 105 \\ & (64,166) \end{aligned}$ | 17 $(5,32)$ 15 $(6,29)$ 14 $(5,30)$ 19 $(6,42)$ | 0.104 $(0.076,0.141)$ 0.104 $(0.076,0.141)$ 0.104 $(0.076,0.141)$ 0.104 $(0.076,0.141)$ | 0.136 $(0.074,0.210)$ 0.104 $(0.058,0.167)$ 0.081 $(0.050,0.120)$ 0.067 $(0.037,0.098)$ | $\begin{gathered} 0.029 \\ (-0.011,0.095) \\ -0.004 \\ (-0.036,0.039) \\ -0.025 \\ (-0.058,0.017) \\ -0.040 \\ (-0.078,0.001) \end{gathered}$ |
| $\begin{aligned} & q=0.8: \\ & C_{97}^{*}=50 \\ & C_{97}^{*}=100 \\ & C_{97}^{*}=150 \\ & C_{97}^{*}=200 \end{aligned}$ | $\begin{aligned} & 36 \\ & (25,45) \\ & 72 \\ & (50,90) \\ & 108 \\ & (75,135) \\ & 144 \\ & (100,180) \end{aligned}$ | $\begin{aligned} & 50 \\ & (22,81) \\ & 75 \\ & (32,127) \\ & 81 \\ & (33,147) \\ & 75 \\ & (16,141) \end{aligned}$ | $\begin{aligned} & 41 \\ & (25,61) \\ & 74 \\ & (45,104) \\ & 99 \\ & (62,136) \\ & 117 \\ & (77,160) \end{aligned}$ | $\begin{aligned} & 16 \\ & (7,28) \\ & 14 \\ & (5,27) \\ & 13 \\ & (5,28) \\ & 16 \\ & (7,36) \end{aligned}$ | 0.125 $(0.111,0.147)$ 0.125 $(0.111,0.147)$ 0.125 $(0.111,0.147)$ 0.125 $(0.111,0.147)$ | 0.175 $(0.115,0.254)$ 0.119 $(0.083,0.173)$ 0.099 $(0.069,0.152)$ 0.082 $(0.052,0.126)$ | $\begin{aligned} & 0.047 \\ & (0.000,0.127) \\ & -0.005 \\ & (-0.043,0.041) \\ & -0.030 \\ & (-0.061,0.015) \\ & -0.047 \\ & (-0.080,-0.001) \end{aligned}$ |
| $\begin{aligned} & q=0.2: \\ & C_{97}^{*}=50 \\ & C_{97}^{*}=100 \\ & C_{97}^{*}=150 \\ & C_{97}^{*}=200 \end{aligned}$ | $\begin{aligned} & 42 \\ & (32,52) \\ & 84 \\ & (65,105) \\ & 126 \\ & (97,157) \\ & 169 \\ & (130,209) \end{aligned}$ | $\begin{aligned} & 90 \\ & (51,145) \\ & 171 \\ & (95,260) \\ & 241 \\ & (137,371) \\ & 301 \\ & (170,469) \end{aligned}$ | 65 $(42,97)$ 126 $(82,186)$ 184 $(122,269)$ 239 $(159,344)$ | 23 $(12,37)$ 22 $(11,35)$ 21 $(11,33)$ 20 $(10,32)$ | 0.803 $(0.765,0.831)$ 0.803 $(0.765,0.831)$ 0.803 $(0.765,0.831)$ 0.803 $(0.765,0.831)$ | 0.845 $(0.651,1.111)$ 0.805 $(0.623,1.063)$ 0.772 $(0.576,1.018)$ 0.739 $(0.557,0.977)$ | 0.055 $(-0.173,0.312)$ 0.012 $(-0.208,0.259)$ -0.031 $(-0.237,0.214)$ -0.063 $(-0.270,0.162)$ |

## WHERE TO NEXT?

When the results above were presented to stakeholders in the management of the Namibian hake resource, it was made clear that further evaluations were possible including, for example, that

- different $\left(\lambda, C_{97}^{*}\right)$ combinations could be investigated;
- candidate IMPs could be subjected to a wider range of robustness tests to plausible variations in the assumptions of the population models used (e.g. evidence for correlation between observation errors for the Dr Fridtjof Nansen surveys and commercial срие, instead of the independence assumed above, should be sought, and if found, incorporated in future evaluations);
- constraints could be introduced in the IMPs, such as a limitation on the percentage change in TAC allowed from one year to the next.

However, before such further evaluations might be pursued, the authors recommended that stakeholders review the results presented above, so as to carefully define the areas where further results would be of most interest, to best facilitate rapid progress. In particular, it was stressed that such a review should seek to narrow the range of $\lambda$ and $C_{97}^{*}$ values upon which it was considered appropriate to focus investigations.

## AFTERWORD

The results of this paper were presented and thoroughly reviewed at a meeting of scientists, industry and Ministry officials held in Walvis Bay, Namibia, on 25 February 1998. After extensive discussion, consensus was reached on use of the class of IMPs investigated as the basis for recommending TACs for Namibian hake for the next 2-3 years, and furthermore on the
choice of the $\lambda=3, C_{97}^{*}=150000$ tons option.
The question is to what extent was this IMP applied in the years that followed?

- 1998: With results available for the 1997 срие and the 1998 Dr Fridtjof Nansen summer survey, the IMP indicated a TAC of 140000 tons. The industry, however, petitioned for a minimum TAC of 150000 tons on socio-economic grounds. The TAC set was 150000 tons and the final catch for the year was 150800 tons.
- 1999: With higher survey abundance estimates persisting, and an enhanced cpue, the IMP formula suggested a TAC of 210000 tons. Namibian scientists expressed concerns that either the value of $\lambda$ should be reduced, or a limit placed on the extent to which the TAC could change from one year to the next. The TAC for that year was set at 210000 tons, but the final catch was lower, at 160700 tons. The reason for the shortfall was, first, that initial quota allocations reduced the figure to 180000 tons, keeping the balance in reserve; subsequently, some new entrants to the fishery and some existing quota-holders were unable (as a result of poor internal management allied to attempts to maximize the value of their fish on the international market) to catch all of their allocations before the end of the year (B. W. Oelofsen, Ministry of Fisheries and Marine Resources, pers. comm.).
- 2000: Originally, it had been foreseen that the IMP would have been fully re-examined and revised by this time. As this had not been achieved by this date, the existing IMP continued to be applied, but with an ad hoc reduction in the value of $\lambda$ to 1 . The resultant TAC recommendation implemented was for 194000 tons.

In summary, a slight increase in cpue and a more substantial increase in Dr Fridtjof Nansen survey results over the three-year period led to increases in the trends in these indices, and hence to a rise in the TAC suggested by the IMP. Although the IMP formula was not followed precisely, it still served as the basis for management decisions and facilitated consensus on increasing the TAC to some 200000 tons by the year 2000, a considerably improved situation from the divergent views held in 1997 about appropriate levels of $T A C$. In retrospect, however, it seems that it would have
been advisable to have incorporated a constraint on the extent of interannual TAC variability in the IMP formulation.

By 2000, it was recognized that the IMP had played its required role and was due for replacement. Consequently, research is currently in progress towards the development and full simulation testing of a Revised Management Procedure, in circumstances of a somewhat reduced range of uncertainty about stock status than prevailed in 1997, as a result of further cpue and survey estimates forthcoming since that time.

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## APPENDIX

## An age-structured production model assessment of the Namibian hake resource

## Data

Annual total catch-by-mass $\left(C_{y}\right)$, and relative and absolute abundance indices $\left(I_{y}^{i}\right)$ are given in Table App.I. The relative abundance indices, as agreed for use by the October 1997 International Workshop (Anon. 1997), consist of five series:

- ICSEAF сриe for Divisions 1.3+1.4 from 1964 to 1988
- ICSEAF cpue for Divisions 1.5 from 1964 to 1988
- GLM standardized сриe from 1991 to 1996
- Spanish summer survey from 1984 to 1990
- Spanish winter survey from 1983 to 1989

The Dr Fridtjof Nansen summer and winter surveys are treated as absolute abundance series with an (assumed) associated coefficient of variation of 0.2 .

Commercial catches-at-age $\left(C_{\mathrm{y}, a}^{c o m}\right)$ for pre- and post-1990 are given in Table App.II, and summer and winter survey catch-at-age ( $C_{\text {var }}^{\text {sur }}$ ) are given in Tables App.III and App.IV respectively.

## Age-structured production model

The resource dynamics are modelled by the equations

$$
\begin{equation*}
N_{y+1,0}=R_{y+1} \tag{App.1}
\end{equation*}
$$

Table App.I: Total annual catch, cpue and survey abundance data for hake off Namibia (ICSEAF Divisions 1.3, 1.4 and 1.5) for the period 1964-1997

| Year | Catch (tons) | CpueICSEAFDivs 1.3+1.4(tons h | CpueICSEAFDiv. 1.5(relative index) | Cpие <br> GLM <br> ( $\mathrm{kg} \mathrm{h}^{-1}$ ) | Spanish survey ('000 tons) |  | Dr Fridtjof ('000 ton | sen survey $V=0.2)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  | Summer | Winter | Summer | Winter |
| 1964 | 47852 |  |  |  |  |  |  |  |
| 1965 | 193200 | 1.78 | 2.24 |  |  |  |  |  |
| 1966 | 334627 | 1.31 | 2.62 |  |  |  |  |  |
| 1967 | 394445 | 0.91 | 1.47 |  |  |  |  |  |
| 1968 | 630392 | 0.96 | 1.38 |  |  |  |  |  |
| 1969 | 526657 | 0.88 | 1.15 |  |  |  |  |  |
| 1970 | 627198 | 0.90 | 1.10 |  |  |  |  |  |
| 1971 | 595215 | 0.87 | 1.44 |  |  |  |  |  |
| 1972 | 820110 | 0.72 | 1.00 |  |  |  |  |  |
| 1973 | 667965 | 0.57 | 1.00 |  |  |  |  |  |
| 1974 | 514558 | 0.45 | 0.70 |  |  |  |  |  |
| 1975 | 488208 | 0.42 | 0.82 |  |  |  |  |  |
| 1976 | 601045 | 0.42 | 0.58 |  |  |  |  |  |
| 1977 | 431483 | 0.49 | 0.69 |  |  |  |  |  |
| 1978 | 379390 | 0.43 | 0.56 |  |  |  |  |  |
| 1979 | 310175 | 0.40 | 0.74 |  |  |  |  |  |
| 1980 | 171848 | 0.45 | 0.71 |  |  |  |  |  |
| 1981 | 211534 | 0.55 | 0.85 |  |  |  |  |  |
| 1982 | 307078 | 0.53 | 0.84 |  |  |  |  |  |
| 1983 | 339590 | 0.58 | 0.90 |  |  | 708.50 |  |  |
| 1984 | 364993 | 0.64 | 0.93 |  | 2187.60 | 2128.26 |  |  |
| 1985 | 386184 | 0.66 | 1.03 |  |  | 1215.84 |  |  |
| 1986 | 381189 | 0.65 | 0.93 |  | 1018.61 | 938.29 |  |  |
| 1987 | 300249 | 0.61 | 0.88 |  |  | 721.02 |  |  |
| 1988 | 336000 | 0.63 | 0.84 |  | 532.55 | $562.59$ |  |  |
| 1989 | 309329 |  |  |  | $1737.84$ | $485.68$ |  |  |
| 1990 | 132379 |  |  |  | 1957.13 |  | 525.939 | 467.013 |
| 1991 | 56135 |  |  | 677.162 |  |  | 500.979 |  |
| 1992 | 87497 |  |  | 808.500 |  |  | 730.578 |  |
| 1993 | 108000 |  |  | 935.692 |  |  | 924.813 | 695.078 |
| 1994 | 112206 |  |  | 628.609 |  |  | 733.521 | 761.512 |
| 1995 | 130362 |  |  | 460.480 |  |  | 491.558 | 480.618 |
| $1996$ | 129102 |  |  | 410.092 |  |  | $743.621$ | 534.350 |
| 1997 | 120170 |  |  |  |  |  | 524.894 |  |

Table App.II: Commercial catches-at-age and masses-at-age for the hake fishery off Namibia. Catches-at-age for the period 1968-1988 are for ICSEAF Divisions 1.3, 1.4 and 1.5, and those for 1996 are from unpublished Ministry records for the fishery off Namibia

| Year | Numbers caught at age ( $\times 10^{-6}$ ) |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8+ |
| 1968 |  | 2.72 | 124.01 | 725.39 | 322.57 | 106.88 | 31.77 | 7.18 | 3.45 |
| 1969 |  | 5.33 | 112.33 | 328.48 | 308.58 | 87.76 | 30.15 | 13.44 | 6.51 |
| 1970 |  | 0.13 | 171.65 | 445.64 | 298.72 | 140.44 | 34.85 | 12.46 | 4.63 |
| 1971 |  | 1.11 | 58.33 | 261.82 | 372.38 | 113.09 | 37.58 | 16.63 | 6.52 |
| 1972 |  | 6.30 | 155.47 | 722.94 | 436.25 | 146.74 | 52.08 | 20.86 | 4.33 |
| 1973 |  | 31.80 | 142.30 | 669.32 | 465.63 | 78.57 | 29.07 | 11.76 | 9.78 |
| 1974 |  | 69.27 | 284.20 | 283.86 | 150.56 | 129.81 | 75.04 | 24.79 | 4.89 |
| 1975 |  | 27.50 | 142.84 | 401.43 | 181.67 | 99.90 | 42.95 | 18.65 | 8.75 |
| 1976 |  | 84.38 | 438.85 | 651.93 | 300.64 | 68.13 | 17.97 | 4.68 | 2.32 |
| 1977 |  | 112.05 | 120.06 | 379.78 | 279.99 | 86.09 | 11.96 | 8.32 | 4.86 |
| 1978 |  | 75.87 | 511.81 | 437.32 | 143.10 | 70.31 | 29.74 | 10.06 | 3.19 |
| 1979 |  | 23.64 | 176.77 | 239.90 | 145.86 | 87.34 | 33.32 | 14.69 | 6.07 |
| 1980 |  | 47.02 | 51.57 | 87.79 | 71.34 | 36.90 | 21.25 | 8.12 | 4.21 |
| 1981 |  | 51.53 | 133.77 | 139.17 | 102.08 | 63.01 | 32.89 | 9.93 | 4.15 |
| 1982 |  | 144.85 | 345.79 | 230.79 | 124.13 | 59.84 | 39.70 | 21.29 | 9.94 |
| 1983 |  | 1037.94 | 871.53 | 182.28 | 65.47 | 19.86 | 10.53 | 3.67 | 1.64 |
| 1984 |  | 87.95 | 807.64 | 446.81 | 117.01 | 38.38 | 13.85 | 4.65 | 1.84 |
| 1985 |  | 102.03 | 254.42 | 406.52 | 205.98 | 52.52 | 12.81 | 3.17 | 1.24 |
| 1986 |  | 53.10 | 434.90 | 279.90 | 187.80 | 104.50 | 35.30 | 14.20 | 3.40 |
| 1987 |  | 30.40 | 202.00 | 337.70 | 185.50 | 74.00 | 28.60 | 7.40 | 1.70 |
| 1988 |  | 28.40 | 332.30 | 560.60 | 250.40 | 51.30 | 13.50 | 4.20 | 1.40 |
| 1989 |  |  |  |  |  |  |  |  |  |
| 1990 |  |  |  |  |  |  |  |  |  |
| 1991 |  |  |  |  |  |  |  |  |  |
| 1992 |  |  |  |  |  |  |  |  |  |
| 1993 |  |  |  |  |  |  |  |  |  |
| 1994 |  |  |  |  |  |  |  |  |  |
| 1995 |  |  |  |  |  |  |  |  |  |
| 1996 |  | 0.89 | 26.19 | 76.66 | 103.80 | 56.81 | 18.88 | 7.23 | 6.83 |
| $w_{a}(\mathrm{~g})$ | 0 | 17 | 85 | 226 | 448 | 752 | 1130 | 1539 | 2071 |
| $w_{a+1 / 2}(\mathrm{~g})$ | 4 | 43 | 145 | 327 | 590 | 932 | 1344 | 1816 | 2336 |

$$
\begin{array}{r}
N_{y+1, a+1}=N_{y, a} \mathrm{e}^{-\left(M_{a}+S_{y, a}^{\text {com }} F_{y}\right)}=N_{y, a} \mathrm{e}^{-Z_{y, a}} \\
\text { for } 0 \leq a<m-2 \text { (App.2) }
\end{array}
$$

$$
\begin{align*}
N_{y+1, m} & =N_{y, m-1} \mathrm{e}^{-\left(M_{m-1}+S_{y, m-1}^{c o m} F_{y}\right)} \\
& +N_{y, m} \mathrm{e}^{-\left(M_{m}+S_{y, m}^{c o m} F_{y}\right)}, \tag{App.3}
\end{align*}
$$

where $N_{y, a}$ is the number of fish of age $a$ at the start of year $y, M_{a}$ denotes the natural mortality rate on fish of age $a, S_{v, a}^{c o m}$ the age-specific commercial selectivity for year $y, F_{y}$ the fully selected fishing mortality in year $y$, and $m$ the maximum age considered (taken to be a plus-group).
The number of recruits at the start of year $y$ is related deterministically to the spawner stock size by the Beverton-Holt stock-recruit relationship

$$
\begin{equation*}
R_{y}=\frac{\alpha B_{y}^{s p}}{\beta+B_{y}^{s p}}, \tag{App.4}
\end{equation*}
$$

where $\alpha$ and $\beta$ are spawner biomass-recruitment parameters and $B_{v}^{s p}$ is the spawner biomass at the start of year $y$, given by

$$
\begin{equation*}
B_{y}^{s p}=\sum_{a=1}^{m} f_{a} w_{a} N_{y, a} \tag{App.5}
\end{equation*}
$$

where $w_{a}$ is the begin-year mass of a fish of age $a$ and $f_{a}$ is the proportion of fish of age $a$ that are mature.

In order to work with estimable parameters that are more meaningful biologically, the stock-recruit relationship is re-parameterized in terms of the pre-exploitation equilibrium spawning biomass, $K^{s p}$, and the "steepness" of the stock-recruit relationship (recruitment at $B^{s p}=0.2 K^{s p}$ as a fraction of recruitment at $\left.B^{s p}=K^{s p}\right)$ :

$$
\alpha=\frac{4 h R_{1}}{5 h-1}
$$

(App.6)
and

$$
\beta=\frac{K^{s p}(1-h)}{5 h-1},
$$

(App.7)

Table App.III: Summer survey catches-at-age for the Namibian hake fishery from the Dr Fridtjof Nansen surveys

| Year | Numbers caught at age $(\times 10-6)$ |  |  |  |  |  |  |  |  |
| :---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | :---: | :---: |
|  | 2 |  |  |  |  |  |  |  |  |
| 1990 | 2404.04 | 1046.50 | 257.27 | 35.76 | 16.37 | 7.12 | 4.49 |  |  |
| 1991 | 419.77 | 782.85 | 255.91 | 162.34 | 82.71 | 26.24 | 13.38 |  |  |
| 1992 | 1593.29 | 682.35 | 325.88 | 173.89 | 100.06 | 61.47 | 45.23 |  |  |
| 1993 | 3431.17 | 938.26 | 228.36 | 165.45 | 87.13 | 65.63 | 72.14 |  |  |
| 1994 | 2339.75 | 703.50 | 218.69 | 136.35 | 113.33 | 44.42 | 50.40 |  |  |
| 1995 | 2023.26 | 486.36 | 182.56 | 118.11 | 48.91 | 24.60 | 25.40 |  |  |
| 1996 | 1721.64 | 1147.28 | 270.35 | 185.22 | 99.46 | 40.65 | 20.92 |  |  |
| 1997 | 544.35 | 917.95 | 283.97 | 137.86 | 69.50 | 24.14 | 14.32 |  |  |

where

$$
R_{1}=K^{s p} /\left[\sum_{a=1}^{m-1} f_{a} w_{a} \mathrm{e}^{-\sum_{a=0}^{a-1} M_{a^{\prime}}}+f_{m} w_{m} \frac{\mathrm{e}^{-\sum_{a=0}^{m-1} M_{a^{\prime}}}}{1-\mathrm{e}^{-M_{m}}}\right] .
$$

(App.8)
The total catch by mass in year $y$ is given by

$$
C_{y}=\sum_{a=0}^{m} w_{a+1 / 2} N_{y, a} \frac{S_{y, a}^{c o m} F_{y}}{Z_{y, a}}\left(1-\mathrm{e}^{-Z_{y, a}}\right), \text { (App.9) }
$$

where $w_{a+\frac{1}{2}}$ denotes the midyear mass of a fish at age $a$.

The model estimate of biomass corresponding to an abundance index is given by

$$
\begin{equation*}
B_{y}^{i}=\sum_{a=0}^{m} w_{a} S_{y, a}^{i} N_{y, a} \tag{App.10}
\end{equation*}
$$

for begin-year biomass and

$$
\begin{equation*}
B_{y}^{i}=\sum_{a=0}^{m} w_{a+\frac{1}{2}} S_{y, a}^{i} N_{y, a} \mathrm{e}^{-Z_{y, a} / 2} \tag{App.11}
\end{equation*}
$$

for midyear biomass, where $B_{y}^{i}$ is the model estimate of abundance for year $y$ and series $i$, and $S_{y, a}^{i}$ is the age-specific selectivity for year $y$ corresponding to the $i$ th abundance series ( $S_{y, a}^{c o m}$ or $S_{y, a}^{\text {sur }}$ in this case).

It is assumed that the resource is at the deterministic equilibrium that corresponds to an absence of harvesting at the start of the initial year of harvesting $\left(B_{1964}^{s p}=K^{s p}\right)$.

## The likelihood function

The model is fitted to cpue, survey biomass and commercial and survey catch-at-age data to estimate model parameters (see Data section above). Contributions by each of these to the negative of the log-likelihood $(-\ell n L)$ are as follows.

## ABUNDANCE DATA

The likelihood is calculated assuming that the observed abundance index is log-normally distributed about its expected value:

$$
I_{y}^{i}=q^{i} B_{y} \mathrm{e}^{\varepsilon_{y}^{i}} \text { or } \varepsilon_{y}^{i}=\ln \left(I_{y}^{i}\right)-\ln \left(q^{i} B_{y}^{i}\right),(\text { App.12 })
$$

where $I_{y}^{i}$ is the abundance index for year $y$ and series $i$, $B_{y}^{i}$ the model estimate of biomass given by Equation App. 10 or App.11, $q^{i}$ the constant of proportionality for abundance series $i$, and $\varepsilon_{y}^{i}$ is generated from $N\left(0,\left(\sigma_{y}^{i}\right)^{2}\right)$.

The contribution of the abundance data to the negative of the log-likelihood function (after removal of constants) is given by

Table App.IV: Winter survey catches-at-age for the Namibian hake fishery from the Dr Fridtjof Nansen surveys

| Year | Numbers caught at age $(\times 10-6)$ |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 2 | 3 | 4 | 5 | 6 | 7 | $8+$ |  |
| 1990 | 1212.26 | 617.22 | 145.49 | 59.12 | 16.25 | 4.94 | 4.20 |  |
| 1991 |  |  |  |  |  |  |  |  |
| 1992 |  |  |  |  |  |  |  |  |
| 1993 | 1408.69 | 387.97 | 157.64 | 101.41 | 56.31 | 34.37 | 58.29 |  |
| 1994 | 958.50 | 525.20 | 260.27 | 137.56 | 76.03 | 24.30 | 43.94 |  |
| 1995 | 1936 | 332.76 | 171.24 | 99.83 | 59.13 | 18.06 | 14.94 |  |
| 1996 | 1917.37 | 456.48 | 183.35 | 69.41 | 29.37 | 7.96 | 5.46 |  |

$$
-\ln L=\sum_{i}\left[\sum_{y}\left(\varepsilon_{y}^{i}\right)^{2} / 2\left(\sigma_{y}^{i}\right)^{2}+\ln \sigma_{y}^{i}\right]
$$

Relative abundance - In the case of relative abundance series, it is assumed that $\sigma_{y}^{i}=\sigma^{i}$ the standard deviation of the residuals for the logarithms of abundance series $i$, estimated in the fitting procedure by its maximum likelihood value

$$
\begin{equation*}
\hat{\sigma}^{i}=\sqrt{1 / n_{i} \sum_{y}\left(\ln I_{y}^{i}-\ln q^{i} B_{y}^{i}\right)^{2}} \tag{App.14}
\end{equation*}
$$

where $n_{i}$ is the number of data points for abundance series $i$, and $q^{i}$ is the catchability coefficient for abundance series $i$, estimated by its maximum likelihood value

$$
\begin{equation*}
\ln \hat{q}^{i}=1 / n \sum_{y}\left(\ln I_{y}^{i}-\ln B_{y}^{i}\right) . \tag{App.15}
\end{equation*}
$$

Absolute abundance - In the case of the survey abundance series, $\sigma_{v}^{i}$ is the estimate of the coefficient of variation ( CV ) of the resource biomass estimate for year $y$, which is input. The constant of proportionality (effectively the multiplicative bias) for this abundance series is either input ( $q^{i}=1$ for absolute abundance series $i$ ), or estimated by its maximum likelihood value which, for the case of a log-normal error distribution, is given by

$$
\begin{equation*}
\ln \hat{q}^{i}=\frac{\sum_{y} 1 /\left(\sigma_{y}^{i}\right)^{2}\left(\ln I_{y}^{i}-\ln B_{y}^{i}\right)}{\sum_{y} 1 /\left(\sigma_{y}^{i}\right)^{2}} \tag{App.16}
\end{equation*}
$$

For the scenarios considered for testing the candidate IMPs, the value of $q$ for the Dr Fridtjof Nansen surveys is set at various fixed values: $1.0,0.8,0.6,0.4$, 0.2 . Values of $q<1$ correspond to assuming a fixed negative bias in these survey results.

## COMMERCIAL CATCHES-AT-AGE

The contribution of the catch-at-age data to the negative of the log-likelihood function when assuming a lognormal error distribution is given by
$-\ell n L=\sum_{y} \sum_{a=2}^{m}\left[\begin{array}{l}\ell n \sigma_{\text {com }}+ \\ \left(\ell n C_{y, a}^{c o m}-\ell n \hat{C}_{y, a}^{c o m}\right)^{2} /\left(2 \sigma_{c o m}^{2}\right)\end{array}\right]_{\text {(App }}$,
(App.17)
where $C_{y, a}^{c o m}$ is the observed numbers-at-age caught by the commercial fleet in year $y, \hat{C}_{y, a}^{\text {com }}$ the model predicted numbers-at-age caught in year $y$, given by

$$
\begin{equation*}
\hat{C}_{y, a}^{c o m}=N_{y, a} \frac{S_{y, a} F_{y}}{Z_{y, a}}\left(1-\mathrm{e}^{-Z_{y, a}}\right), \tag{App.18}
\end{equation*}
$$

and $\sigma_{\text {com }}$ is the standard deviation associated with these catch-at-age data, estimated in the fitting procedure by

$$
\hat{\sigma}_{c o m}=\sqrt{1 / n_{c o m} \sum_{y} \sum_{a=2}^{m}\left(\ell n C_{y, a}^{c o m}-\ell n \hat{C}_{y, a}^{c o m}\right)^{2}},(\text { App.19) }
$$

where $n_{\text {com }}$ is the number of data points in the summation.

Ages 0 and 1 are omitted from Equation App. 17 because of the small sizes of the associated catches-at-age.

A log-normal distribution is assumed for the commercial catches-at-age, rather than, say, a multinomial because these data are affected by interannual variability in selectivity-at-age as a result of changes in the distribution of fishing effort as well as by sampling variability.

## SURVEY CATCHES-AT-AGE

The contribution of the summer and winter survey catches-at-age to the negative of the log-likelihood when assuming a log-normal error distribution is given by

$$
-\ln L=\sum_{y} \sum_{a=2}^{m}\left[\begin{array}{l}
\ln \sigma_{s u r}+  \tag{App.20}\\
\left(\ln C_{y, a}^{s u r}-\ln \hat{C}_{y, a}^{s u r}\right)^{2} /\left(2 \sigma_{\text {sur }}^{2}\right)
\end{array}\right],
$$

where $C_{y, a}^{s u r}$ is the number of fish from catches in a survey in year $y$ found to have age $a, \hat{C}_{v, a}^{s u r}$ is the model-predicted number of fish of age $a$ in year $y$ in the survey, given by

$$
\begin{equation*}
\hat{C}_{y, a}^{s u r}=\hat{\rho}_{y, a} \sum_{a^{\prime}=2}^{m} C_{y, a^{\prime}}^{s u r}, \tag{App.21}
\end{equation*}
$$

where $\hat{\rho}_{y, a}$ is the expected proportion of fish of age $a$ in year $y$ in the survey, given by

$$
\begin{equation*}
\hat{\rho}_{y, a}=\frac{S_{y, a}^{s u r} N_{y, a}}{\sum_{a^{\prime}=2}^{m} S_{y, a^{\prime}}^{s u r} N_{y, a^{\prime}}}, \tag{App.22}
\end{equation*}
$$

and $\sigma_{\text {sur }}$ is the standard deviation associated with these catch-at-age data, estimated in the fitting procedure by

$$
\begin{equation*}
\hat{\sigma}_{s u r}=\sqrt{1 / n_{s u r} \sum_{y} \sum_{a=2}^{m}\left(\ell n C_{y, a}^{s u r}-\ell n \hat{C}_{y, a}^{s u r}\right)^{2}}, \mathrm{~A}_{1} \tag{Ảpp.23}
\end{equation*}
$$

where $n_{\text {sur }}$ is the number of data points in the summation.

Ages 0 and 1 are omitted from the likelihood calculations owing to apparent undersampling of these age groups in the surveys.

The assumption of a multinomial rather than a lognormal distribution might be argued for the survey catches-at-age, because they should not be subject to
interannual variations in the selectivity-at-age, as is the case for the commercial catches. There are difficulties, however, in estimating the appropriate effective number of fish aged for each survey, which is required for such an approach (see Geromont and Butterworth 1997). Use of the log-normal in these circumstances should not bias estimates, but would mean that the estimator is marginally non-optimal in terms of precision.

## Bootstrap replicates

## ABUNDANCE DATA

The calculation of standard errors is effected by a parametric bootstrap procedure. Bootstrap samples are generated from the predicted abundance series obtained by fitting the model to the data. Error is then added to the predicted abundance indices according to the formula
$I_{y}^{i, U}=\hat{q}^{i} \hat{B}_{y}^{i} \mathrm{e}^{\varepsilon_{y}^{i, U}}$ where $\varepsilon_{y}^{i, U} \sim N\left(0,\left(\hat{\sigma}_{y}^{i}\right)^{2}\right),($ App.24)
where $I_{y}^{i, U}$ is the abundance index for year $y$ and series $i$ in bootstrap dataset $U, \widehat{B}_{y}^{i}$ the estimate of the abundance index for year $y$ and series $i$ obtained by fitting the model to the actual data, and $\hat{\sigma}_{y}^{i}$ is the estimate of the standard error for log relative abundance (see Equation App.14) or input value for absolute abundance series $i$.

## COMMERCIAL CATCHES-AT-AGE

For commercial catch-at-age data, the replicate process assumes a log-normal error distribution, and that these errors occur only in the assignment of the ages of the fish, rather than real fluctuations in selectivity at age from year to year. Bootstrap samples are generated from the model-fitted catches-at-age as follows:
$C_{y, a}^{U}=\hat{C}_{y, a}^{c o m} \mathrm{e}^{\varepsilon_{y, a}^{U}}$ where $\varepsilon_{y, a}^{U} \sim N\left(0, \hat{\sigma}_{c o m}^{2}\right)$, (App.25) where $C_{y, a}^{U}$ is the number of fish caught of age $a$ in year $y$ in bootstrap dataset $U, \hat{C}_{y}^{\text {com }}$, the corresponding model estimate given by Equation App.18, and $\hat{\sigma}_{\text {com }}$ is the associated estimate of standard deviation given by Equation App.19.

The pseudo catches-at-age are then scaled so that $\sum_{a} C_{y, a}^{U}=\sum_{a} \hat{C}_{y, a}^{c o m}$ for each year $y$.

## SURVEY CATCHES-AT-AGE

For the survey catch-at-age data, the bootstrap samples are generated in exactly the same manner as the commercial catches-at-age when assuming a log-normal
error distribution.
The population model is fitted to each bootstrap dataset in turn. Summary statistics, including standard error estimates, are then derived from the resultant set of estimates for quantities of interest.

## Model parameters

## NATURAL MORTALITY

Age-dependent natural mortality, $M_{a}$, is estimated as follows:

$$
M_{a}= \begin{cases}M_{a+1} & \text { for } a \leq 1  \tag{App.26}\\ \alpha^{M}+\frac{\beta^{M}}{a+1} & \text { for } a \geq 2\end{cases}
$$

$M_{0}$ and $M_{1}$ are set equal to $M_{2}$ as there is virtually no information in the data taken into account in the likelihood which would allow independent estimation of $M_{0}$ and $M_{1}$.

## COMMERCIAL SELECTIVITY-AT-AGE

A simple time-varying logistic curve is assumed for the commercial selectivity

$$
\begin{equation*}
S_{y, a}^{i}=S_{y, a}^{c o m}=\frac{1}{1+\mathrm{e}^{-\left(a-a_{y}^{c}\right) / \delta_{y}}} \tag{App.27}
\end{equation*}
$$

where $a_{v}^{c}$ years is the age-at- $50 \%$ selectivity, and $\delta_{y}$ year ${ }^{-1}$ defines the steepness of the selectivity curve for year $y$.

To account for a likely change in the selectivity pattern at young ages over time in the commercial catches-at-age as a result of changed patterns of fishing and changed regulations since Namibian Independence (forbidding discarding, exclusion of vessels from fishing in shallow water), the selectivity parameters $a_{y}^{c}$ and $\delta_{y}$ are estimated for two separate periods of catches, the first from 1964 to 1989 and the second from 1990 to the present.

## SURVEY SELECTIVITY-AT-AGE

$$
\begin{equation*}
S_{y, a}^{i}=S_{a}^{s u r}=1 \text { for } a \geq 2 . \tag{App.28}
\end{equation*}
$$

Because of apparent undersampling of ages 0 and 1 in the survey, data for these ages are not considered in the associated likelihood (see Equation App.19), so the survey selectivity for those two ages need not be specified. Uniform selectivity is assumed for ages 2 and above, corresponding to assumed random sampling by the survey. As the survey abundance estimates include some 0 - and 1 -year old fish, this formulation makes the assumption that the biomass of those fish

Table App.V: Parameters used for the von Bertalanffy growth curve and mass-at-age computations

| Parameter | Value |
| :--- | :---: |
| $\alpha(w$ in g$)$ | 0.0055 |
| $\beta$ | 3.1 |
| $L_{\infty}(\mathrm{cm})$ | 120 |
| $\kappa\left(\right.$ year $\left.^{-1}\right)$ | 0.0893 |
| $t_{0}($ years $)$ | -0.3177 |

relative to the total biomass measured by the survey is small and has low variability in absolute terms, so that the survey abundance estimates can be taken to be proportional to the biomass of $2+$ hake.

## AGE-AT-MATURITY

The proportion of fish of age $a$ that are mature is ap-
proximated by $f_{a}=1$ for $a>3$ years (i.e. $f_{a}$ for $a=0$, $1,2,3)$.

## MAXIMUM AGE

This is input as $m=8$, and is taken as a plus-group.
MASS-AT-AGE
The mass ( $w$ ) of a fish at age (a) is assumed to be related to the von Bertalanffy growth curve:

$$
w_{a}=\alpha\left[L_{\infty}\left(1-\mathrm{e}^{-\kappa\left(a-t_{0}\right)}\right)\right]^{\beta}
$$

where the values assumed for the growth parameters are given in Table App.V. The resulting begin- and midyear mass-at-age vectors are shown at the bottom of Table App.II.


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