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SOLITARY ROSSBY WAVES IN THE LOWER TROPICAL TROPOSPHERE

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ABSTRACT: *Weakly nonlinear approximation is used to study the theoretical comportment of large-scale disturbances around the inter-tropical mid-tropospheric jet. We show here that the Korteweg de Vries (KdV) theory is appropriated to describe the structure of the streamlines around the African easterly jet (AEJ) region. The introduction of the additional velocity C_1 permits to search the stage where the configuration of the wave will correspond in this zone to those of Rossby solitary waves. It was also shown that the configurations of disturbances can be influenced by this parameter so that we can look if the disturbances are in the control or not of their dispersive effects.*

Key words: *Soliton; Rossby waves; KdV equation; African easterly waves.*

INTRODUCTION

The Rossby waves are the most important in large-scale atmospheric flow processes (Holton, 2004). For their analysis, it is usually sufficient to study the horizontal structure of waves. Most theories treating the structure of these waves are based on linear models which only take into account their dispersive behaviour. Nonlinear processes are more interesting because they can help to explain, for example, the hurricane spiral bands observed in the tropical zone (Guinn and Schubert, 1993), and energy exchanges between different modes of the waves (Lenouo et al., 2005). Solitary Rossby waves in a zonal flow appear to have been discovered (analytically) by Long (1964) and have been studies subsequently by Larsen (1965), Benney (1966), Redekopp and Weidman (1978), Miles (1979), Hoskins and Ambrizzi (1993) and Luo (2004). All invoke Rossby's β-plane model, in which the northerly gradient of the vertical component of the earth's rotation is constant.

Many studies have dealt with nonlinear waves and particularly solitary waves in the atmosphere, stating with works by Tepper (1950) and Abdullah (1955). On the theoretical level, nonlinear waves have been examined by Lenouo et al. (2005) in the mid-atmosphere where the African Easterly waves (AEWs) are propagated. In the same region, Dobryshman (1982) showed that the Korteweg de Vries (KdV) theory is appropriate to describe Rossby solitary waves. But the physical interpretation of the results in terms of Rossby solitary waves is not evident and the roles that these waves could influence the structure and energy of these waves have not been examined. Moreover, Huang and Zhang (1988) established that the propagation of Rossby solitary wave has behaviour closer to those of ridges and troughs. They therefore showed that these waves can travel long distances in the northern Hemisphere without a change both in speed or structure, and for any hour.

The first well known studies of Rottman and Einaudi (1993) have helped to identify solitary wave connected to internal gravity waves in the atmosphere. They showed that these waves are described by KdV equations when they move in the upper atmosphere and by the Benjamin-David-Ono (BDO) equation when they appear in the lower level. They therefore correctly analysed the observations of Lin and Goff (1988) by using the KdV model, whereas the observation of Smith and Morton (1984) were better explained by the BDO model. This was the first evidence of these types of wave observed in the atmosphere and their comparison with theoretical models. The solution of the three-dimensional nonlinear Charney–Obukhov equation describing solitary pancake Rossby vortices was found by Kaladze (2000). Its solution was represented in the form of an axially symmetric cylindrical monopole (anticyclonic) vortical structure moving with constant velocity. Whereas the role of westward-travelling planetary (Rossby) waves in the block onset and the deformation of eddies during the interaction between synoptic-scale eddies and an incipient block was examined by Luo (2004). This author has constructed an incipient block that consists of a stationary dipole wave for zonal wavenumber and a westward-travelling monopole wave with constant amplitude for zonal wavenumber.

The role of nonlinear wave was also being studied in oceanography. Hence, time series observations of nonlinear internal waves in the deep basin of the South China Sea are used to evaluate mechanisms for their generation and evolution by Qiang and Farmer (2011). They showed that internal tides are generated by tidal currents over ridges in Luzon Strait and steeper as they travel west, subsequently generating high-frequency nonlinear waves. Although nonlinear internal waves appear repeatedly on the western slopes of the South China Sea, their appearance in the deep basin is intermittent and more closely related to the amplitude of the semidiurnal than the predominant diurnal tidal current in Luzon Strait.

In the present study, we will use the weakly nonlinear theory to examine the behaviour of the large-scale waves around the mid-tropospheric African Easterly Jet (AEJ), where the wave is more intense. Considering that the vertical extent of this jet is smaller than its horizontal extent (Lenouo et al. 2005), we admit in first approximation that the motion of the air in this region is dominated by the effects of the rotation of the earth. Under these assumptions, we will look for the stage where the configurations of these are similar to those having the form of solitary Rossby waves. This study is organised as follows. In Section 2, we will present method used to examine the nonlinear vorticity equation. In Section 3, the linear and nonlinear solutions are discussed whereas conclusion is presented in Section 4.

METHODOLOGY

Basics equations

Rossby solitary waves are sought by using a nonlinear vorticity equation in a barotropic model (Dobryshman, 1982). This equation integrates the horizontal shear in mean zonal wind that the profile permits to characterise the wave instability. We define a coordinate system (x,y,t) where t is the time component and the space components x and y are along the direction of wave propagation in east and north direction respectively. When the flow is no divergent, zonal and meridional components of the velocity can be written as a function of a streamfunction perturbation ψ as

$u = -\partial\psi / \partial y$ and $v = \partial\psi / \partial x$

Otherwise, in a barotropic model, the evolution of streamline is described by the equation (Holton, 2004)

$$\left[\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right]\nabla^2\psi + J(\psi, \nabla^2\psi) + \left[\beta - \frac{d^2U}{dy^2}\right]\frac{\partial\psi}{\partial x} = 0 \qquad (1)$$

with the following boundary conditions:

ψ=0 at y=0 and y=L (2)

In equation (1), J is the Jacobian operator $J(a,b) = \frac{\partial a}{\partial x}\frac{\partial b}{\partial y} - \frac{\partial a}{\partial y}\frac{\partial b}{\partial x}$, $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ the Laplacian operator, U the mean zonal wind component and β the meridional gradient of the coriolis parameter.

The effects of non linearity are introduced through the Jacobian term, which is nonlinear. In the case of weak amplitude waves, the individual oscillations can be represented in the form of linear or nonlinear wave superposition.

Theory

A soliton is localised wave, solution to a nonlinear partial derivatives equation without change of velocity or profile in a weakly dispersive area. By using a multiple scale method, we can write a stream function ψ in the form of a power expansion in a small parameter ϵ so that

$\psi = \epsilon\,\psi_1 + \epsilon^2\,\psi_2 +$ (3)

It is necessary to introduce a convenient space and time variables ζ and τ, adapted to describe a weakly dispersive nonlinear system (Rottman and Einaudi, 1993). In this new Galilean reference frame, the transformations of Rottman and Einaudi (1993) are given by:

$$\zeta = \in^{1/2}(x - C_0 t) \text{ and } \tau = \in^{3/2} t \qquad (4)$$

where C_0 is the phase velocity of the eastward wave. The procedure consists in rewriting equation (1) using ζ and τ, and then seeks a solution in a power series expansion in the amplitude parameter ϵ. Then by collecting terms of order $O(\in^{3/2})$, we obtain the following linear equation in ψ_1:

$$(U - C_0)\frac{\partial}{\partial\zeta}\left(\frac{\partial^2\psi_1}{\partial y^2}\right) + \left[\beta - \frac{d^2U}{dy^2}\right]\frac{\partial\psi_1}{\partial\zeta} = 0 \qquad (5)$$

Also the terms of order $O(\in^{5/2})$ gives:

$$\frac{\partial}{\partial\tau}\left(\frac{\partial^2\psi_1}{\partial y^2}\right) + (U - C_0)\frac{\partial^3\psi_1}{\partial\zeta^3} + \frac{\partial\psi_1}{\partial\zeta}\frac{\partial^3\psi_1}{\partial y^3} - \frac{\partial\psi_1}{\partial y}\frac{\partial}{\partial\zeta}\left(\frac{\partial^2\psi_1}{\partial y^2}\right) = 0 \qquad (6)$$

Solutions of equations (5) and (6) are sought in the nonlinear waves form $A(\zeta,\tau)$, modulate by an amplitude function $\varphi(y)$, as given by the relation:

$$\Psi(\zeta,y,\tau)=A(\zeta,\tau)\,\varphi(y) \quad (7)$$

a)- Determination of φ(y)

By substituting relation (7) into equation (5), we set the following eigenvalue equation for $\varphi(y)$:

$$\varphi''(y)+\frac{\beta-U''}{U-C_0}\varphi(y)=0 \quad (8)$$

where the prime denotes differentiation with respect to y. The boundary conditions are the same as those given by relation (2)

$\varphi=0$ at $y=0$ and $y=L$.

Equation (8) is solved numerically by using GAUSS-SEIDEL's relaxation methods. The shape of horizontal shear is chosen such that the wind is zero at the boundaries. Using centred-difference differentiation, equation (8) is rewritten as:

$$\varphi_i=[\varphi_{i+1}+\varphi_{i-1}]/[2-F_i(\Delta y)^2] \quad (9)$$

where Δy is the grid size; i=1, 2, …, N and $F_i=\left[\frac{\beta-U''}{U-C_0}\right]_i$.

b)- Nonlinear waves

By substituting relation (7) into equation (6), we can obtain a nonlinear equation as KdV-equation in the form

$$\frac{\partial A}{\partial\tau}+a_n\frac{\partial A}{\partial\zeta}+b_n\frac{\partial^3 A}{\partial\zeta^3}=0 \quad (10)$$

where parameters a_n and b_n are determined by eigenfunctions φ and depend on the profile of U(y) (see appendix A for more detail). They are given by the following expression:

$$a_n=\frac{\int_0^L[\varphi\varphi'''-\varphi'\varphi'']dy}{\int_0^L\varphi''\,dy} \quad (11)$$

and

$$b_n=\frac{\int_0^L(U-C_0)\varphi dy}{\int_0^L\varphi''\,dy} \quad (12)$$

We now examine solutions of equation (10) in the form of nonlinear progressive waves (soliton) $A=A(\zeta+C_1\tau)$ where C_1 is the phase velocity of the soliton which is a weak contribution to the principal phase velocity C_0.

Thus, the total velocity of the system is:

$C= C_0+\epsilon C_1$

The solitary waves, solution of equation (10) is given by the following relation (see appendix B for more detail):

$$A(\zeta,\tau) = A_0\mathrm{Sech}^2[\kappa(\zeta+C_1\tau)] \quad (13)$$

Where $A_0=3C_1/a_n$ is the nonlinear wave amplitude and $\kappa=(C_1/b_n)^{1/2}$. Going back to the original variable, we finally have:

$$A(x,t)=A_0Sech^2\left(\frac{x-Ct}{\Delta}\right) \quad (14)$$

where $\Delta=\sqrt{\frac{b_n}{\in C_1}}$ is the soliton characteristic width.

The product of the soliton amplitude and the square of its characteristic width are independent of the soliton phase velocity C_1. It is however proportional to b_n and inversely proportional to a_n or ϵ and is expressed as:

$$\Delta^2A_0=\frac{3b_n}{\in a_n} \quad (15)$$

Since a_n and b_n are given by relation (11) and (12) respectively, the only known remaining is ϵ. Figure 1 shows some profiles of $A(x,t)/A_0$ for different values of additional phase velocity C_1 and $\epsilon=0.01$. For small C_1, the figure shows that A(x,t) is nearly constant in space but takes the form of a pulse when this parameter becomes important. It is to be noticed that this parameter appears explicitly in the expression of Δ. Thus, when C_1 is set to zero, Δ is even larger and the wave becomes evanescent. But as C_1 increases, e.g. $C_1=25m.s^{-1}$, the wave propagate symmetrically below the plan which passed trough the origin where x=Ct. Hence, we can say that the wave amplitude A_0 grows with the additional phase velocity C_1 (Fig.2).This shows that the wave moves faster as its amplitude becomes larger. Fig.2 also shows that the soliton characteristic width Δ decreases with C_1.

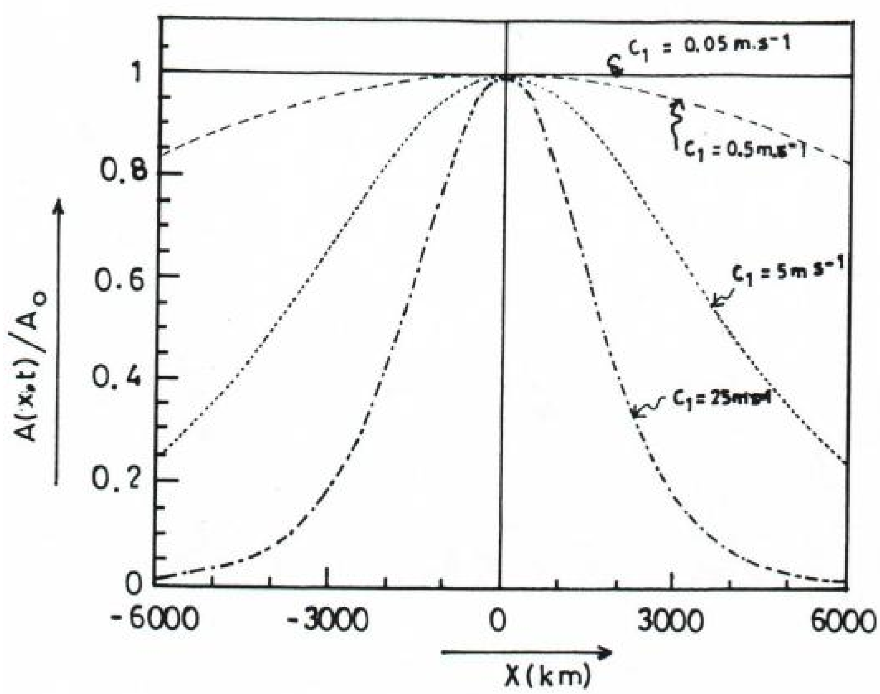

Figure 1: Variations of $A(x,t)/A_0$ in the propagation direction for different values of the additional wind velocity C_1.

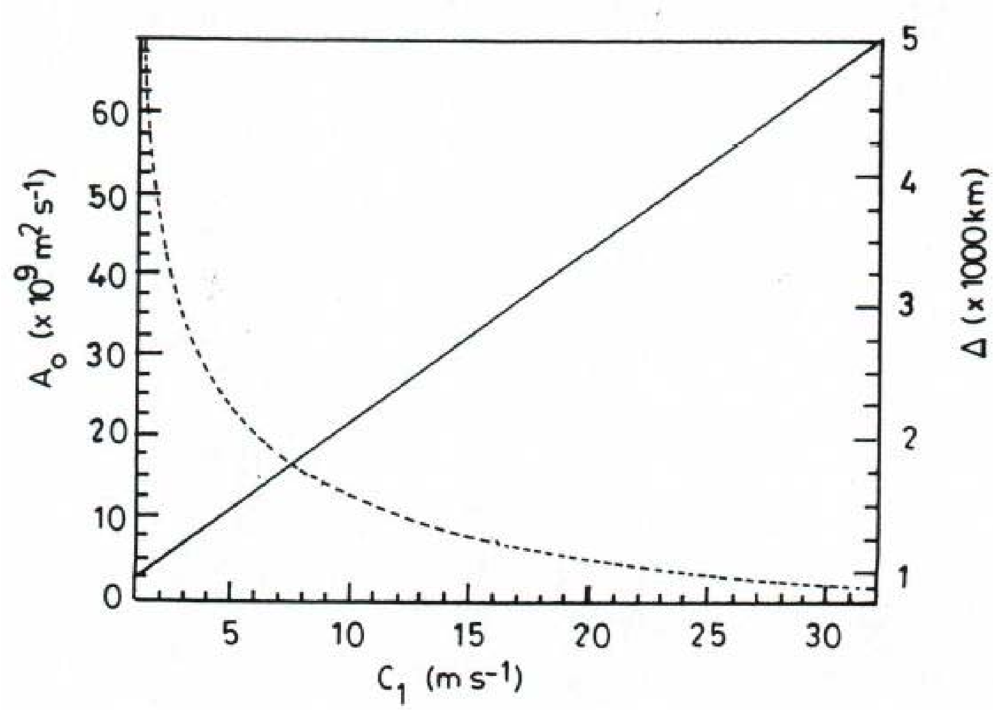

Figure 2: Variations of Amplitude A_0 (in solid line) and the width Δ (in dash line) of the soliton as function of the additional wind velocity C_1.

RESULTS AND DISCUSSIONS

As we know, the solitary wave can be considered only when we define a coordinates system which moves with the wave at the velocity C, for the soliton to appear stationary. At the origin X=0, where X=x-Ct, the amplitude solitary wave is maximal. This is due to the fact that in the translation X=x-Ct of a non fixed distance Ct, the maximum wave amplitude, initially at x=0 stays until we are at X=0.

Explicitly, the streamline depends on the soliton phase velocity C_1 and on the form of the mean zonal wind U(y). We adopt in this work a basic flow with a horizontal shear as proposed by Rennick (1976) to describe the mid-tropospheric jet in the West African Tropical zone. Based on observations the zonal wind can readily be represented in the functional form

$$U(y) = -U_0 \sin^2\left(\frac{\pi}{L} y\right) \qquad (16)$$

where U_0=20m/s is the maximal value of wind at the centre of jet (15°N), L the distance between the Equator (y=0) and 30°N latitude (y=L). This jet corresponds to the one seen in the atmosphere during summer at an altitude of around 3000 m in the Northern African Troposphere. The principal phase velocity is found to be 7.0 m/s (Burpee 1972, Mass 1979, Lenouo and Mkankam 2008).
Before examining the influence of the additional wind velocity C_1 in the present theory, let us first consider the case of linear waves.

Analysis of linear effects

In the linear theory case, the solution of equation (1) without the Jacobian term is sought in the normal mode:

$$\Psi(x,y,t) = Y(y) \exp[ik(x-Ct)] \qquad (17)$$

Where k is the zonal wavenumber, k=2π/λ, λ the zonal wavelength; Y(y) is the amplitude function which depends only on y and solution to the following equation:

$$Y'' + \left[\frac{\beta - U''}{U - C_0} - k^2\right] Y = 0 \qquad (18)$$

This equation differ from equation (8) by the presence of the k^2 term, but it still must verify the boundary conditions Y=0 at y=0 and y=L. The numerical solution of equation (18) is found as earlier by the GAUSS-SEIDEL's relaxation methods.

Figure 3 illustrates the configurations of streamlines in the (X,y) plane obtained from this approximation. We note that the region of instability corresponds to the depression located between two anticyclonics stones, whose centres are along the principal axes of the jet. These streamlines have a quasi-concentric form, on the one hand symmetric respect of the plan passing X=0 and on the other hand to the jet axis.

Nonlinear effects

The streamlines in the case of the weakly nonlinear approximation are presented in figure 4 for different values of the additional wind velocity C_1. We see that this velocity has a predominant role in the configuration of patterns in the domain under consideration.

For C_1=0.05 m.s^{-1} (Fig.4a), the patterns are essentially parallel to the zonal direction. Here, the perturbations are swamped by the mean flow and this explains why for weak value of C_1, one can not observe the track of the wave. The air flow can be assimilated in this case to the displacement of a solid that presents an axis of symmetric.

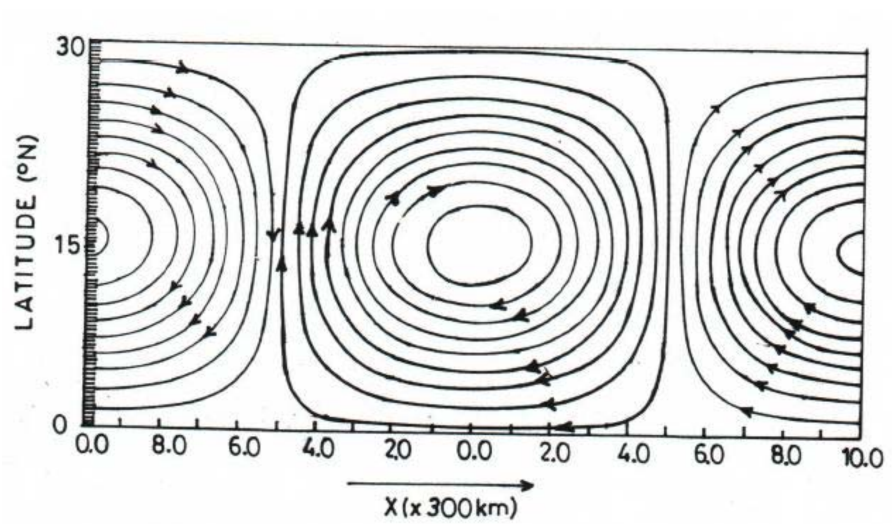

Figure 3: Configuration of streamlines in the case of linear approximation with C_0=7m.s^{-1}

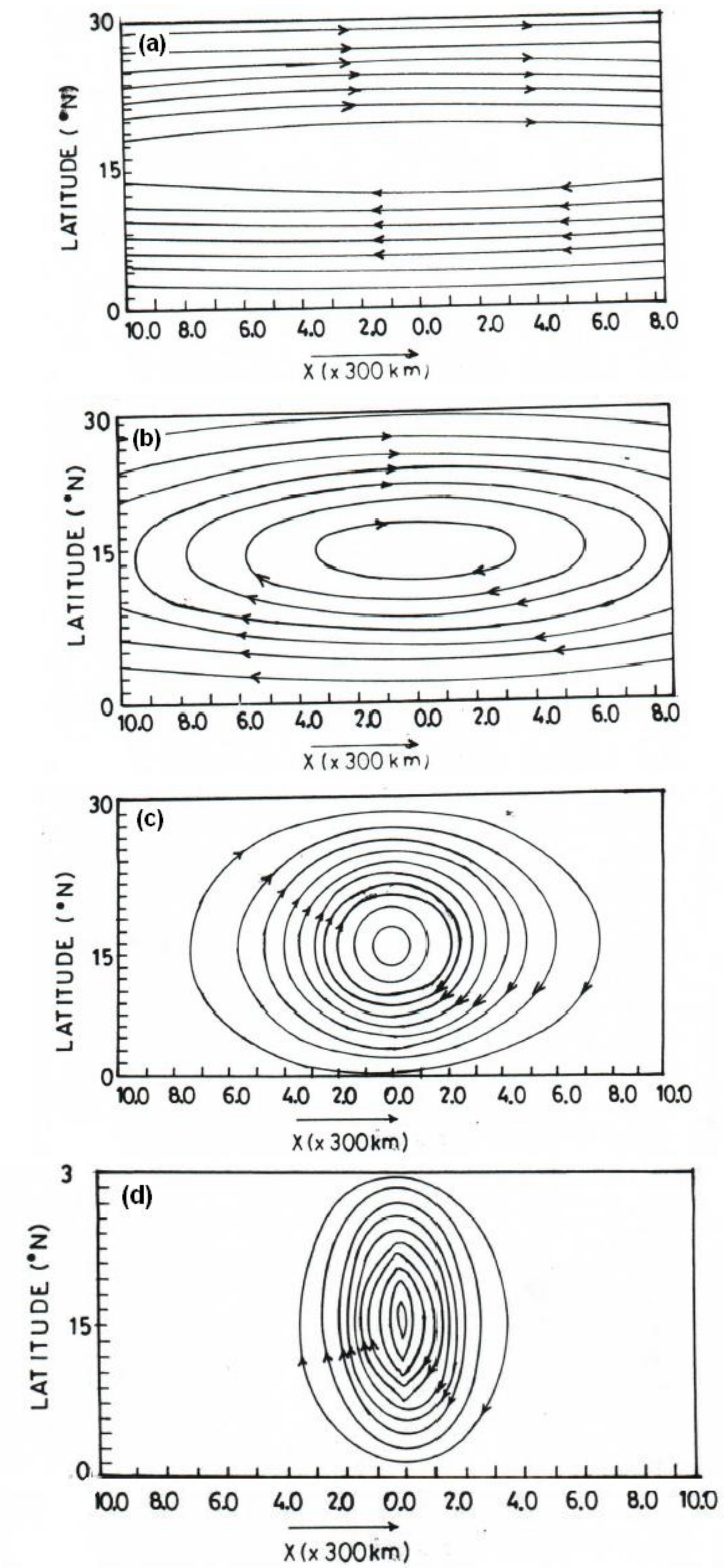

Figure 4: Configuration of streamlines in the case of weakly nonlinear approximation with C_0=7m.s^{-1} and ϵ=0.01 for (a) C_1=0.05m.s^{-1}; (b) C_1=0.5m.s^{-1}; (c) C_1=5m.s^{-1} and (d) C_1=25m.s^{-1}.

When the value of the parameter C_1 is increased, the streamlines have a new configuration as shown in figure 4b. We can deduce that the streamlines tend to unclose and stretch in the zonal direction isolating a depression centred at the region of maximum shear. The presence of a depression characterises the linear effects in the system. The fact that it is presently limited to the maximum disturbance region shows that these effects are in their early stages. This is why the little felt at the boundary of the domain, where only weak deformations of streamlines are observed.

We can continue to increase the value of the phase velocity of the soliton in order to determine a value that for which the weakly nonlinear theory, leads to the same structure of linear waves as given by normal mode theory. Fig. 4c presents the streamlines for $C_1=5 m.s^{-1}$. We note that all the patterns have concentric ovoidal form around the region of maximal instability region where the amplitude of the nonlinear wave is high. The difference with previous configurations is the fact that these patterns are zonally limited at $X=\pm 2100$ km. Contrary to the streamline of Fig.3, the absence of anticyclonic zone here is due to the shape of the solitary wave amplitude A(x,t), obtained in the weak nonlinear approximation. Since our interest is to study the behaviour of wave around the jet, this result is not in contradiction with those obtained in the case of the linear approximation, but matter confirm that the maximum instability of the jet is located in depressionnary area. As also shown in the Fig.4c, this streamline can be superposed to those given by linear approximation (Fig.3). Hence the nonlinear wave is strongly governed by its linear effects. The first manifestation of the nonlinearity effects appear here, as noted by Kadomek (1979), the presence of a weak nonlinearity in the system can produce important effects capable to countering those due to the dispersion. According to the weakly nonlinear approximation, the soliton result from a balance between linear and nonlinear effects. In other term, it is for the value of C_1 equal to $5 m.s^{-1}$ that the Rossby soliton may be observed. Its profile described by relation (14) is represented in Fig.1.

As the parameter C_1 increases, the nonlinear effects grow and the wave patterns are concentrated around the region where their amplitudes grow (Fig.4d). Here, the streamlines tend to stretch along the meridional direction while being confined in a small zonal domain. This shows that the nonlinear waves became strongly localised.

For energy consideration, we admit the principle that the energy of the perturbation is proportional to the square of amplitude of the wave. So, for weak values of C_1, the energy of the wave is dissipated in the space x-Ct. For $C_1=5 m.s^{-1}$, we found that the energy spread in the space x-Ct but in a reasonable interval compared to the purely linear case. However, Fig. 4c shows that nonlinearity, though weak, leads to live for the perturbation.

CONCLUSION

We have presented a nonlinear theory to study the evolution of perturbation due to the shear mean wind in the midtropospheric African jet. Its formulation is necessarily complicated, but we have carefully described all the stages which permit to obtain the final result, so that one can use, in some conditions, these results seem applicable to description of Rossby solitary waves. This requires the choice of additional wind speed C_1. The use of Gardner and Murikawa (1965) transformations help us to introduce the phase velocity of the soliton C_1. Its influence on the structure and the amplitude for streamline is important. Hence, the weakly nonlinear approximation through KdV theory explains how the solitary Rossby waves are propagated over West Africa. The weakly nonlinear theory leads to the same structure of linear waves as given by normal mode theory for $C_1=5 m.s^{-1}$ where the presence of a weak nonlinearity in the system can produce important effects capable to countering those due to the dispersion.

In the tropical zone, the cyclones are some time presented as the soliton. Its interesting to confront this theory with observations since the nonlinear amplitude function A(x,t) which is proportional to pressure, presents a maximum when the eastern wave propagation arrives at X=0.

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APPENDIX

Appendix A: Determination of parameter a_n and b_n

By substituting relation (7) in to equation (6), we obtain

$$\varphi''\frac{\partial A}{\partial \tau}+(U-C_0)\varphi\frac{\partial^3 A}{\partial \zeta^3}+\varphi\frac{\partial A}{\partial \zeta}A\varphi'''-\varphi' A\varphi''\frac{\partial A}{\partial \zeta}=0 \qquad \text{(A-1)}$$

or

$$\varphi''\frac{\partial A}{\partial \tau}+(U-C_0)\varphi\frac{\partial^3 A}{\partial \zeta^3}+(\varphi\varphi'''-\varphi'\varphi'')A\frac{\partial A}{\partial \zeta}=0 \qquad \text{(A-2)}$$

and if we integrate this equation into meridional domain, we have:

$$\int_O^L \varphi''\frac{\partial A}{\partial \tau}dy+\int_0^L (U-C_0)\varphi\frac{\partial^3 A}{\partial \zeta^3}dy+\int_0^L(\varphi\varphi'''-\varphi'\varphi'')A\frac{\partial A}{\partial \zeta}dy=0 \qquad \text{(A-3)}$$

by setting $\int_0^L \varphi'' dy$, (A-3) can be rewritten at last as:

$$\frac{\partial A}{\partial \tau}+a_n\frac{\partial A}{\partial \zeta}+b_n\frac{\partial^3 A}{\partial \zeta^3}=0 \qquad \text{(A-4)}$$

with

$$a_n=\frac{\int_0^L[\varphi\varphi'''-\varphi'\varphi'']dy}{\int_0^L \varphi'' dy} \quad \text{and} \quad b_n=\frac{\int_0^L (U-C_0)\varphi dy}{\int_0^L \varphi'' dy} \qquad \text{(A-5)}$$

Appendix B: solution of KdV-equation

To solve the equation (10), we introduce the following Galilee transformation:

$s=\zeta-C_1\tau$ (B-1)

In this new referential where the wave is propagated with the velocity C_1, the equation (10) becomes:

$$-C_1\frac{\partial A}{\partial s}+a_nA\frac{\partial A}{\partial s}+b_n\frac{\partial^3 A}{\partial s^3}=0 \qquad \text{(B-2)}$$

The integration of (B-2) with respect to s gives

$$-C_1A+a_n\frac{A^2}{2}+b_n\frac{\partial^2 A}{\partial s^2}=0 \qquad \text{(B-3)}$$

By multiplying this last equation by dA/ds and integrating, we have:

$$-C_1\frac{A^2}{2}+a_n\frac{A^3}{6}+\frac{b_n}{2}\left(\frac{dA}{ds}\right)^2=0 \qquad \text{(B-4)}$$

or

$$\frac{dA}{ds}=\left(\frac{C_1}{b_n}\right)^{1/2}A\sqrt{1-\frac{a_n}{3C_1}A} \qquad \text{(B-5)}$$

By setting $\tilde{A}=a_nA/(3C_1)$, we obtain the integral:

$$\int\frac{d\tilde{A}}{\tilde{A}(1-\tilde{A})^{1/2}}=\left(\frac{C_1}{b_n}\right)^{1/2}(s-s_0) \qquad \text{(B-6)}$$

If we assume $s_0=0$, (B-6) can be written as:

$$ArgSech\tilde{A}=\left(\frac{C_1}{b_n}\right)^{1/2}s \qquad \text{(B-7)}$$

And at last:

$$A=\frac{3C_1}{a_n}Sech^2\left[\left(\frac{C_1}{b_n}\right)^{1/2}s\right] \qquad \text{(B-8)}$$