African Journal of Science and Technology (AJST)
Science and Engineering Series Vol. 12, No. 2, pp. 55 - 64

ON OPTIMUM DISPATCH OF ELECTRIC POWER GENERATION VIA NUMERICAL METHOD

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ABSTRACT: *In this work we develop an optimum dispatch / generating strategy by presenting economically the best load flow configuration in supplying load demand among the generators. The main aim is to minimize the total production / generation costs, with minimum losses and at the same time satisfy the load flow equation without violating the inequality constraints.*

Key Words: *Generation, power flow, generator limits, power losses,Chironomidae, River Niger, Niamey, water quality*

INTRODUCTION

The industrial growth of any nation depends greatly on the reliability of a large interconnected electric power system. Electric power system is a significant form of modern energy source, because of its application in nearly all spheres of human endeavour aimed at socio-economic development. In an interconnected power system, the objective is to find the output power and load shedding of each power plant in such a way as to minimize the operating cost. Manafa(1978), Laden(2008), Country profile(2006).

The factors influencing power generation at minimum cost are operating efficiencies of generators, fuel cost and transmission losses. The most efficient generator in the system may not guarantee minimum cost as it may be located in an area where fuel cost is high. Olle(1987), Power Sector Reform(2005). Also, if the plant is located far from where the load is, transmission losses may be considerably higher and hence the plant may be uneconomical.

The purpose of this work is to minimize the total production / generation costs, with minimum losses and at the same time satisfy the load flow equation without violating the inequality constraints.

Optimal Power Flow as an Optimization method for an energy management system control centre was developed in the 1960s by Carpenter, and since then it has been an

MOTIVATION FOR THE STUDY

important function as a standard application. A generalized nonlinear mathematical programming formulation of the economic dispatch problem including voltage and other operating power constraint which was named the Optimal Power Flow Problem, was introduced by Carpenter 1962, Dammel and Tinney, Oct, 1968. Since then, a great deal of research has been done and various optimization techniques have been used in order to find efficient solutions to this optimization problem. In 2005, Adejumobi looked at the effectiveness and efficiency of the electrical power distribution system in Nigeria by making use of power system security. In 1998, Arthur et al, worked on Optimization for load management scheduling. Also, in 1986 Lee et al, worked on optimization technique for power operation.

Similarly, in 1989, Youssef et al, looked at the transmission planning model for a power system. In 1988, Lee et al, looked at the transmission planning model for a power system. However, little or no attention has been paid to the production cost; they emphasized more on the operational aspect (design aspect) rather than the economical aspect of optimal power flow problem (OPF).

The purpose of this work is to develop an optimum dispatch / generating strategy by presenting economically the best load flow configuration in supplying load demand among the generators. The main aim is to minimize the total production / generation costs, with minimum losses and at the same time satisfy the load flow equation without violating the inequality constraints

MATHEMATICAL FORMULATION

The standard optimal electric power generation problem (optimal power flow problem), is formulated mathematically as follows. Olle(1987), Rao(1998).

Minimize

$$
\begin{aligned}
&C = C_i(x,u) \ \textit{Subject to} \\
&g_i(x,u) = 0 \\
&f_i(x,u) \le 0 \qquad (1) \\
&u_{\min} \le u \le u_{\max}, \quad x_{\min} \le x \le x_{\max}, \\
&x \in \Re^n, u \in \Re, \quad i = 1.....n \ \ bus
\end{aligned}
$$

C_t, g_i *and* f_i are continuous differentiable, vector x contains independent variables consisting of bus voltage magnitudes and phase angle, reference bus angle, fixed bus voltage, e.t.c. The vector consists of controls variables, including real and reactive power generations, phase – shifter angles, direct transmission line flows, controls voltage settings, e.t.c.

Where, $C_i(x,u)$ represent the objective function, $g_i(x,u)$ represent non - linear equality constraint, the equality $g_i(x,u)$ is the load flow equation, $f_i(x,u)$ is the non - linear inequality constraint of vector argument x and u. The inequality $f_i(x,u)$ is the limit on the control variable u and the operating limits on the power system bus voltage limits. Limits on the control variables are known as "hard" limits (i.e. violation is not allowed, e.g. upper and lower band on the active power generation at the generator buses) and operating limits are known as " soft" limits (i.e. small violation is tolerable, e. g. voltage limit at load buses, maximum line loading limit). The vector x contains dependent or state variables (such as voltage magnitude, phase angle, e.t.c.), and u consists of control variables such as generated active and reactive power e. t. c. , Olle, (1987).

OPTIMIZATION OF REAL POWER GENERATION INCLUDES THE LIMIT AND TRANSMISSION LOSSES

When transmission distances are long with low density area, transmission losses are not neglected. The idea is to include the effect of transmission losses which can be expressed as a quadratic function of the generation power outputs, Burchett, R.S ., et al(1982), Charles, A.G.(1986). The simplest quadratic form is

$$P_L = \sum_{i=1}^{ng}\sum_{j-1}^{ng} P_i B_{ij} P_j \quad or$$

$$P_L = \sum_{i=1}^{ng}\sum_{j-1}^{ng} P_i B_{ij} P_j \ + \sum_{i=1}^{ng} B_{01} P_i + B_{00}$$

The coefficients B_{ij} are called loss coefficient or B – coefficient, which are assumed constants.

Statement of the problem

The problem can now be stated as:

Minimize the overall generation cost C_t

$$C_t(pg_i) = \sum_{i=1}^{n} \left(\alpha_i + \beta_i pg_i + \gamma_i pg_i^2\right)$$

subject to the constraints

$$\sum_{i=1}^{ng} Pg_i = P_D + P_L, \qquad (2)$$

$$pg_{i\min} \le Pg_i \le Pg_{i\max}, \ i = 1, \quad ....ng$$

Where $Pg_{i\min}$ and $Pg_{i\max}$ are the minimum and maximum generating limit respectively, for plant i.

Solution to the Problem

Using the Langrange Multiplier and adding additional terms to include the constraints

we have,

$$
\begin{aligned}
L = C_t(pg_i) &+ \left(P_D + P_L - \sum_{i-1}^{ng} Pg_i\right) \\
&+ \sum_{i=1}^{ng} \mu_{i\max}\left(Pg_i - Pg_{i(\max)}\right) \qquad (3) \\
&+ \sum_{i=1}^{ng} \mu_{i(\min)}\left(Pg_i - Pg_{i(\min)}\right)
\end{aligned}
$$

Note: $\mu_{i(\max)=0} \quad when \ \ Pg_i < Pg_{i(\max)},$

$\mu_{i(\min)=0} \quad when \ \ Pg_i > Pg_{i(\min)}$

This simply means that if the constraint is not violated then the associated μ variable is zero.

$$L = \sum_{i=1}^{ng}\left(\alpha_i + \beta_i pg_i + \gamma_i pg_i^2\right) + \lambda\left(P_D + P_L - \sum_{i=1}^{ng} Pg_i\right) + \sum_{i=1}^{ng} \mu_{i\max}\left(Pg_i - Pg_{i(\max)}\right) + \sum_{i=1}^{ng} \mu_{i(\min)}\left(Pg_i - Pg_{i(\min)}\right) \qquad (4)$$

The solution to the Langrange equation is found by obtaining the following

$$\left.\begin{aligned} &\frac{\partial L}{\partial Pg_i} = 0, \quad \frac{\partial L}{\partial \lambda} = 0 \\ &\frac{\partial L}{\partial \mu_{i(\max)}} = Pg_i - Pg_{i(\max)} = 0, \\ &\frac{\partial L}{\partial \mu_{i(\min)}} = Pg_i - Pg_{i(\min)} = 0 \end{aligned}\right\} \qquad (5)$$

Hence, we have

$$\beta_i + 2\gamma_i Pg_i + \lambda\left(0 + \frac{\partial P_L}{\partial Pg_i} - 1\right) = 0$$

which is equivalent to

$$\frac{\partial C_t(Pg_i)}{\partial Pg_i} + \lambda\frac{\partial P_L}{\partial Pg_i} = \lambda,$$

$$where \quad \frac{\partial P_L}{\partial Pg_i} = 2\sum_{j=1}^{ng} B_{ij} P_j + B_{oi}$$

Therefore, we have

$$2\gamma_i Pg_i + \beta_i + 2\lambda\sum_{j=1}^{ng} B_{ij}P_j + B_{0i}\lambda = \lambda$$

or

$$\left(\frac{\gamma_i}{\lambda} + B_{ii}\right)Pg_i + \sum_{\substack{j=1 \\ i=1}}^{ng} B_{ij}P_j = \frac{1}{2}\left(1 - B_{0i} - \frac{\beta_i}{\lambda}\right) \qquad (6)$$

Expanding the above result in linear matrix, we have,

$$\begin{bmatrix} \frac{\gamma_i}{\lambda} + B_{11} & B_{12} & . & . & B_{ing} \\ B_{21} & \frac{\gamma_2}{\lambda} + B_{22} & . & . & B_{2ng} \\ . & . & . & . & . \\ . & . & . & . & . \\ B_{ng1} & B_{ng2} & . & . & \frac{\gamma_{ng}}{\lambda} + B_{ngng} \end{bmatrix} \begin{bmatrix} Pg_i \\ Pg_i \\ . \\ . \\ Pg_n \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 & -B_{01} & -\frac{\beta_1}{\lambda} \\ 1 & -B_{02} & -\frac{\beta_2}{\lambda} \\ . & . & . \\ . & . & . \\ 1 & -B_{0ng} & -\frac{\beta_{ng}}{\lambda} \end{bmatrix}$$

$$i.e \qquad E \quad P \quad = \quad D \qquad i = 1,........ng \qquad (7)$$

To find the optimal dispatch for an estimated value of $\lambda^{(k)}$, equation (7) is solved using the iterative method. The iterative continues until the load flow equation is satisfied. Thus from (6), we have,

$$\left(\frac{\gamma_i + \lambda B_{ii}}{\lambda}\right)Pg_i + \sum_{\substack{i=1 \\ j\pm i}}^{ng} B_{ij}P_j = \frac{1}{2} - \frac{1}{2}B_{0i} - \frac{1}{2}\frac{\beta_i}{\lambda}$$

$$\Rightarrow \left(\frac{\gamma_i + \lambda B_{ii}}{1}\right)Pg_i + \lambda\sum_{\substack{i=1 \\ j\pm i}} B_{ij}P_j = \frac{\lambda}{2} - \frac{\lambda}{2}B_{0i} - \frac{1}{2}\beta_i$$

Therefore,

$$P^{(k)}g_i = \frac{\lambda^{(k)}\left(1 - B_{0i}\right) - \beta_i - 2\lambda^{(k)}\sum_{\substack{i=1 \\ j\pm i}} B_{ij}P_j^{(k)}}{2\left(\gamma_i + \lambda^{(k)}B_{ii}\right)}$$

Since,

$$\sum_{i=1}^{ng} pg_i = P_D + P_L$$

$$\sum_{j=1}^{ng} \frac{\lambda^{(k)}\left(1 - B_{oi}\right) - \beta_i - 2\lambda^{(k)}\sum_{j\pm i} B_{ij}P_j^{(k)}}{2\left(\gamma_i + \lambda^{(k)}B_{ii}\right)} = P_D + P_L^{(k)}$$

L If we denote $pg_i^{(k)}$ by $f(\lambda)^k$, and using Taylor series expansion, we have,

$$f(\lambda)^k + \left(\frac{df(\lambda)}{d\lambda}\right)^{(k)} \Delta\lambda^{(k)} + \left(\frac{d^2 f(\lambda)}{d\lambda^2}\right)^{(k)} \Delta^2\lambda^{(k)} + ... = P_D + P_L^{(k)}$$

Neglecting second and higher degree, we have,

$$f(\lambda)^k + \left(\frac{df(\lambda)}{d\lambda}\right)^{(k)} \Delta\lambda^{(k)} = P_D + P_L^{(k)}$$

where
or

$$\Delta\lambda^{(k)} = \frac{P_D + P_L^{(k)} - f(\lambda)^k}{\left(\frac{df(\lambda)}{d\lambda}\right)^{(k)}} \qquad (8)$$

Let

$\Delta P^{(k)} = P_D + P_L^{(k)} - f(\lambda)^{(k)}$, then

$$\Delta\lambda^{(k)} = \frac{\Delta P^{(k)}}{\left(\frac{df(\lambda)}{d\lambda}\right)^{(k)}} = \frac{\Delta P^{(k)}}{\sum_{i=1}^{n}\left(\frac{dPg_i}{d\lambda}\right)^{(k)}} \qquad (9)$$

Minimize

$$C = C_i(x,u) \ \textit{Subject to}$$
$$g_i(x,u) = 0$$
$$f_i(x,u) \le 0 \qquad (1)$$
$$u_{\min} \le u \le u_{\max}, \quad x_{\min} \le x \le x_{\max},$$
$$x \in \Re^n, u \in \Re, \quad i = 1.....n \ \ bus$$

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C_t, g_i *and* f_i are continuous differentiable, vector x contains independent variables consisting of bus voltage magnitudes and phase angle, reference bus angle, fixed bus voltage e.t.c. The vector consist of controls variables, including real and reactive power generations, phase – shifter angles, direct transmission line flows, controls voltage settings e.t.c.

Where, $C_i(x,u)$ represent the objective function, $g_i(x,u)$ represent non - linear equality constraint, the equality $g_i(x,u)$ is the load flow equation, $f_i(x,u)$ is the non - linear inequality constraint of vector argument x and u. The inequality $f_i(x,u)$ are the limit on the control variable u and the operating limits on the power system. bus voltage limits. Limits on the control variables are known as “hard” limit (i.e. violation is not allowed, e.g. upper and lower band on the active power generation at the generator buses) and operating limits are known as “ soft” limits (i.e. small violation is tolerable, e. g voltage limit at load buses, maximum line loading limit). The vector x contains dependent or state variables (such as voltage magnitude, phase angle etc), and u consist of control variables such as generated active and reactive power etc , Olle, (1987).

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4.1: Statement of the problem

The problem can now be stated as:

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subject to the constraints

$$\sum_{i=1}^{ng} Pg_i = P_D + P_L, \qquad (2)$$
$$pg_{i\min} \le Pg_i \le Pg_{i\max}, \quad i = 1, \quad ....ng$$

Where $Pg_{i\min}$ and $Pg_{i\max}$ are the minimum and maximum generating limit, respectively for plant i.

4.2: Solution to the Problem

Using the Langrange Multiplier and adding additional terms to include the constraints.

We have,

$$L = C_t(pg_i) + \left(P_D + P_L - \sum_{i-1}^{ng} Pg_i\right) + \sum_{i=1}^{ng}\mu_{i\max}\left(Pg_i - Pg_{i(\max)}\right) + \sum_{i=1}^{ng}\mu_{i(\min)}\left(Pg_i - Pg_{i(\min)}\right) \qquad (3)$$

Note: $\mu_{i(\max)=0} \quad when \;\; Pg_i < Pg_{i(\max)},$

$\mu_{i(\min)=0} \quad when \;\; Pg_i > Pg_{i(\min)}$

This simply means that if the constraint is not violated then the associated variable is zero.

$$L = \sum_{i=1}^{ng}\left(\alpha_i + \beta_i pg_i + \gamma_i pg_i^2\right)$$
$$+\lambda\left(P_D + P_L - \sum_{i=1}^{ng} Pg_i\right) \qquad (4)$$
$$+\sum_{i=1}^{ng}\mu_{i\max}\left(Pg_i - Pg_{i(\max)}\right)$$
$$+\sum_{i=1}^{ng}\mu_{i(\min)}\left(Pg_i - Pg_{i(\min)}\right)$$

The solution to the Langrange equation are found by obtaining the following

$$\left.\begin{array}{l} \dfrac{\partial L}{\partial Pg_i} = 0, \quad \dfrac{\partial L}{\partial \lambda} = 0 \\ \dfrac{\partial L}{\partial \mu_{i(\max)}} = Pg_i - Pg_{i(\max)} = 0, \\ \dfrac{\partial L}{\partial \mu_{i(\min)}} = Pg_i - Pg_{i(\min)} = 0 \end{array}\right\} \qquad (5)$$

Hence, we have

$$\beta_i + 2\gamma_i Pg_i + \lambda\left(0 + \frac{\partial P_L}{\partial Pg_i} - 1\right) = 0$$

Which is equivalent to

$$\frac{\partial C_t\left(Pg_i\right)}{\partial Pg_i} + \lambda\frac{\partial P_L}{\partial Pg_i} = \lambda,$$

$$where \quad \frac{\partial P_L}{\partial Pg_i} = 2\sum_{j=1}^{ng} B_{ij}P_j + B_{oi}$$

Therefore, we have

$$2\gamma_i Pg_i + \beta_i + 2\lambda\sum_{j=1}^{ng} B_{ij}P_j + B_{0i}\lambda = \lambda$$

or

$$\left(\frac{\gamma_i}{\lambda} + B_{ii}\right)Pg_i + \sum_{\substack{j=1\\ i=1}}^{ng} B_{ij}P_j = \frac{1}{2}\left(1 - B_{0i} - \frac{\beta_i}{\lambda}\right) \qquad (6)$$

Expanding the above result in linear matrix, we have,

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$$\frac{1}{2}\begin{bmatrix} 1 & -B_{01} & -\dfrac{\beta_1}{\lambda} \\ 1 & -B_{02} & -\dfrac{\beta_2}{\lambda} \\ . & . & . \\ . & . & . \\ 1 & -B_{0ng} & -\dfrac{\beta_{ng}}{\lambda} \end{bmatrix}$$

$$i.e \qquad E \quad P \quad = \quad D$$
$$i = 1,.........ng \qquad (7)$$

To find the optimal dispatch for an estimated value of $\lambda^{(k)}$, equation (7) is solved using the iterative method. The iterative continues until the load flow equation is satisfied. Thus from (6), we have,

$$\left(\frac{\gamma_i + \lambda B_{ii}}{\lambda}\right)Pg_i + \sum_{\substack{i=1\\ j\pm i}}^{ng} B_{ij}P_j = \frac{1}{2} - \frac{1}{2}B_{0i} - \frac{1}{2}\frac{\beta_i}{\lambda}$$

$$\Rightarrow\left(\frac{\gamma_i + \lambda B_{ii}}{1}\right)Pg_i + \lambda\sum_{\substack{i=1\\ j\pm i}} B_{ij}P_j = \frac{\lambda}{2} - \frac{\lambda}{2}B_{0i} - \frac{1}{2}\beta_i$$

Therefore,

$$P^{(k)}g_i = \frac{\lambda^{(k)}\left(1 - B_{0i}\right) - \beta_i - 2\lambda^{(k)}\sum_{\substack{i=1\\ j\pm i}} B_{ij}P_j^{(k)}}{2\left(\gamma_i + \lambda^{(k)}B_{ii}\right)}$$

Since,

$$\sum_{i=1}^{ng} pg_i = P_D + P_L$$

$$\frac{\sum_{j=1}^{ng}\lambda^{(k)}\left(1 - B_{oi}\right) - \beta_i - 2\lambda^{(k)}\sum_{j\pm i} B_{ij}P_j^{(k)}}{2\left(\gamma_i + \lambda^{(k)}B_{ii}\right)} = P_D + P_L^{(k)}$$

L If we denote $pg_i^{(k)}$ by $f(\lambda)^k$, and using Taylor series expansion, we have,

$$f(\lambda)^k + \left(\frac{df(\lambda)}{d\lambda}\right)^{(k)} \Delta\lambda^{(k)} + \left(\frac{d^2 f(\lambda)}{d\lambda^2}\right)^{(k)} \Delta^2 \lambda^{(k)} + ... = P_D + P_L^{(k)}$$

Neglecting second and higher degree, we have,

$$f(\lambda)^k + \left(\frac{df(\lambda)}{d\lambda}\right)^{(k)} \Delta\lambda^{(k)} = P_D + P_L^{(k)}$$

where

$$\Delta\lambda^{(k)} = \frac{P_D + P_L^{(k)} - f(\lambda)^k}{\left(\frac{df(\lambda)}{d\lambda}\right)^{(k)}} \qquad (8)$$

Let

$\Delta P^{(k)} = P_D + P_L^{(k)} - f(\lambda)^{(k)}$, then

$$\Delta\lambda^{(k)} = \frac{\Delta P^{(k)}}{\left(\frac{df(\lambda)}{d\lambda}\right)^{(k)}} = \frac{\Delta P^{(k)}}{\sum_{i=1}^{n}\left(\frac{dPg_i}{d\lambda}\right)^{(k)}} \qquad (9)$$

$$\sum_{i=1}^{n}\left(\frac{dPg_i}{d\lambda}\right)^{(k)} = \frac{\sum_{i=1}^{ng} 2\left(\gamma_i + \lambda^{(k)} B_{ii}\right)\left[(1 - B_{0i}) - 2\sum_{j=i} B_{ij} P_j^{(k)}\right] - \left[\lambda^{(k)}(1 - B_{0i}) - 2\lambda^{(k)} \sum_{j=i} B_{ij} P_j^{(k)}\right] 2B_{ii}}{4\left(\gamma_i + \lambda^{(k)} B_{ii}\right)^2}$$

$$= \frac{\sum_{j=i}^{ng} \gamma_i (1 - B_{0i}) + \lambda^{(k)} B_{ii} (1 - B_{0i}) - 2\gamma_i \sum_{j=i} B_{ij} P_j^{(k)} - 2\lambda^{(k)} B_{ii} \sum_{j=i} B_{ij} P_j^{(k)} - \lambda^{(k)} B_{ii} (1 - B_{0i}) + 2B_{ii}\lambda^{(k)} \sum_{j=i} B_{ij} P_j^{(k)}}{2\left(\gamma_i + \lambda^{(k)} B_{ii}\right)^2}$$

$$= \frac{\left(\sum_{j=i}^{n} \gamma_i (1 - B_{0i}) - \sum 2\gamma_i B_{ij} P_j^{(k)}\right)}{2\left(\gamma_i + \lambda^{(k)} B_{ii}\right)^2} \qquad (10)$$

And therefore,

$\lambda^{(k+1)} = \lambda^{(k)} + \Delta\lambda^{(k)}$,

where $\Delta P^{(k)} = P_D + P_L^{(k)} - \sum_{i-1}^{ng} Pg_i^{(k)}$

The process continues until $\Delta P^{(k)}$ is less than a specified accuracy.

If an approximate loss formula expressed by $P_L = \sum_{i=1}^{ng} B_{ii} P^2 g_L$ is used, $B_{ij} = 0, \quad B_{0i} = 0,$ and solution of the equation becomes

$$P_i^{(k)} = \frac{\lambda^{(k)} - \beta_i}{2\left(\gamma_i + \lambda^{(k)} B_{ii}\right)} \qquad (3.16a)$$

Hence,

$$\sum_{i=1}^{ng}\left(\frac{dP_i}{d\lambda}\right)^{(k)} = \frac{2\left(\gamma_i + \lambda^{(k)} B_{ii}\right) - \left[\left(\lambda^{(k)}\right)(2B_{ii})\right]}{4\left(\gamma_i + \lambda^{(k)} B_{ii}\right)^2}$$

$$= \frac{\gamma_i + \lambda^{(k)} B_{ii} - \lambda^{(k)} B_{ii}}{2\left(\gamma_i + \lambda^{(k)} B_{ii}\right)^2}$$

$$= \sum_{i=1}^{ng} \frac{\gamma_i}{2\left(\gamma_i + \lambda^{(k)} B_{ii}\right)^2} \qquad (11)$$

4.3: Solution Algorithm

1. Assume an initial value for $\lambda^{(k)}$.
2. Calculate Pg_i. using equation

A. $$Pg_i^{(k)} = \begin{bmatrix} \frac{\gamma_i}{\lambda}+B_{ii} & B_{12} & . & . & B_{1n} \\ B_{21} & \frac{\gamma_2}{\lambda^{(k)}}+B_{22} & . & . & B_{2n} \\ . & . & . & . & . \\ . & . & . & . & . \\ B_{01} & B_{ng2} & . & . & \frac{\gamma_n}{\lambda^{(k)}}+B_{nn} \end{bmatrix} \begin{bmatrix} Pg_1^{(k)} \\ Pg_2^{(k)} \\ . \\ . \\ Pg_n^{(k)} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -B_{01} & -\frac{\beta_1}{\lambda^{(k)}} \\ 1 & -B_{02} & -\frac{\beta_2}{\lambda} \\ . & . & . \\ . & . & . \\ 1 & -B_{0n} & -\frac{\beta_{ng}}{\lambda^{(k)}} \end{bmatrix}$$

or

B. $$Pg_i^k = \frac{\lambda^{(k)} - \beta_i}{2(\gamma_i + \lambda^{(k)} B_{ii})}$$

3. Calculate $P_L^{(k)}$ using

$$P_L = \sum_{i=1}^{n}\sum_{j=1}^{n} P_i B_{ij} P_j + \sum_{i=1}^{n} B_{0i} P_i + B_{00} \quad \text{for 2.A}$$

above and $P_L^{(k)} = \sum_{i=1}^{n} B_{ii} P_i^2$ for 2B.

4. Check if the relationship $\sum_{i=1}^{n} Pg_i^{(k)} = P_D + P_L^k$ is satisfied if not go to 5.
5. Calculate $\Delta Pg_i^{(k)}$ from

$$\Delta Pg_i^{(k)} = P_D + P_L - \sum_{i=1}^{n} Pg_i^k$$

6. Calculate $\Delta\lambda^{(k)}$ from

$$\Delta\lambda^{(k)} = \frac{\Delta Pg_i^{(k)}}{\frac{\left(\sum \gamma_i (1 - B_{0i}) - \sum 2\gamma_i B_{ij} P_j^{(k)}\right)}{2(\gamma_i + \lambda^{(k)} B_{ii})^2}} \quad for \quad 2A$$

$$or \ = \frac{\gamma_i}{2(\gamma_i + \lambda^{(k)} B_{ii})^2} \quad for\ 2B.$$

7. Obtain a new value of λ from $\lambda^{(k+1)} = \lambda^k + \Delta\lambda^{(k)}$
8. Repeat step 2 – 7 until $\Delta Pg_i^{(k)} = 0$ or when $\Delta Pg_i^{(k)}$ is less than a specified accuracy.
9. Check for the value of Pg_i when the equality is met, plants that exceed their upper limit are kept at the specified limit.
10. Continue from step 4 – 7 when the equality constraints are met again check limit of the plants.
11. Calculate the total Production Cost $C_t(Pg_i) = C_1 + C_2 + \dots\dots + C_{n,}$ where n the number of generator.

5. Numerical Results and Interpretation

In this section we give some numerical examples and the interpretation of the results obtained.

5.1 Numerical Results

Problem 5. 1

Consider the following generator parameters of 5 bus system with three generating bus.

Table 7.1

Bus No	P_{min}	P_{max}	α	β	γ
1	5.5	39.5	150	7.0	0.008
2	10	80	160	6.3	0.009
3	10	70	140	6.8	0.007

Given that the real power losses is expressed as

$$P_L = \sum_{i=1}^{n} B_{ii} P_i^2,$$

where $$B_{ii} = \begin{bmatrix} 0.000218 \\ 0.00228 \\ 0.00179 \end{bmatrix}, i = 1....,3$$

Determine the optional dispatch of generator and the cost of generation when the total system load demand is 180MW.

Solution:

The problem can be stated as

Minimize,

$$C_t(Pg_i) = \sum_{i=1}^{3} \alpha_i + \beta_i Pg_i + \gamma_i Pg_i$$

Subject to

$$\sum_{i=1}^{3} Pg_i - P_D - P_L = 0$$

$$P_{i(\min)} \le P_i \le P_{i(\max)}$$

$$P_{2(\min)} \le P_2 \le P_{2(\max)}$$

$$P_{3(\min)} \le P_3 \le P_{3(\max)}$$

$P_L = \sum_{i=1}^{n} B_{ii} P_i^2$, where

$$B_{ii} = \begin{bmatrix} 0.000218 \\ 0.00228 \\ 0.00179 \end{bmatrix}, i = 1....,3, \quad P_D = 180MW$$

Follow the algorithm above,

Assume $\lambda^{(1)} = 7.5$ and obtain Pg_i from $Pg_i^{(k)}$

Obtain $Pg_i^{(k)} = \dfrac{\lambda^{(k)} - \beta_i}{2\left(\gamma_i + \lambda^{(k)} B_{ii}\right)}$

$$P_1^{(1)} \cong 25.95MW, \quad P_2^{(1)} = 56.02MW \quad , \quad P_3^{(1)} = 41.95MW$$

$$P_L^{(1)} = B_{11} \overset{(1)}{P_1^2} + B_{22} \overset{(1)}{P_2^2} + B_{33} \overset{(1)}{P_3^2}$$
$$= 0.000218(25.95)^2 + 0.000228(56.02)^2 + 0.000179(41.95)^2$$

=1.18MW

Since $P_D = 180MW$, we compute

$$\Delta P^{(1)} = 180 + 1.18 - (25.95 + 56.02 + 41.95) = 57.26MW$$

$$\Delta\lambda^{(1)} = \frac{\Delta P^{(1)}}{\sum_{i=1}^{3} \frac{\gamma_i}{2\left(\gamma_i + \lambda^{(1)} B_{ii}\right)^2}} \cong 0.43$$

$$\lambda^{(2)} = \lambda^{(1)} + \Delta\lambda^{(1)} = 7.93$$

Hence, we obtain, $P_i^{(2)}, \quad i = 1,.....,3$,

$P_i^{(2)} \cong 47.80MW$, $P_2^{(2)} \cong 75.41MW$, and $P_3^{(k)} \cong 67.11MW$

Note that P_1 has exceeded the upper limit, so that from P_1 at the upper limit of 39.5MW and obtain P_L and $\Delta P^{(2)}$.

$$P_L^{(2)} \cong 2.44MW$$

$$P_L^{(2)} \cong 2.44MW$$

$$\Delta P^{(2)} = 180 + 2.44 - (39.5 + 75.41 + 67.1) = 0.42$$

With P_1 fixed, compute $\Delta\lambda^{(2)}$ and $P_1^{(3)}$

$$\Delta\lambda^{(3)} = \frac{\Delta P^{(2)}}{\sum_{i=1}^{3} \frac{\gamma_i}{2\left(\gamma_i + \lambda^{(1)} B_{ii}\right)^2}} \cong 0.003$$

$$\lambda^{(3)} = 7.93 + 0.003 = 7.933$$

$P_1 = 39.5MW, P_2^{(3)} \cong 75.54 \quad P_3^{(3)} \cong 67.28$, $P_L^{(3)} \cong 2.45MW$.

$$\Delta P^{(3)} = 0.13$$

Compute $\Delta\lambda^{(4)}$ and $P_i^{(4)}$, using:

$$\Delta\lambda^{(4)} = \frac{\Delta P^{(3)}}{\sum_{i=1}^{3} \frac{\gamma_i}{2\left(\gamma_i + \lambda^{(2)} \beta_{ii}\right)^2}} \cong 0.001 \text{ and}$$

$\lambda^{(4)} = 7.934$.

Hence,

$P_1^{(3)} = 39.5MW$, $P_2^{(4)} \cong 75.56MW$, $P_3^{(4)} \cong 67.34MW$
$P_L^{(4)} \cong 2.45MW$.
$$\Delta P^{(4)} = 180 + 2.45 - (39.5 + 75.56 + 67.34) = 0.05 < 1$$

Since the difference between $\Delta P^{(i)}$ is getting very smaller compare to 1, we stop here and compute the cost of generation for each plant and the total cost of generation.

Thus, the optimal dispatch is

$P_1 = 39.5MW,\ P_2 \cong 75.56MW,\ P_3 \cong 67.34MW,$

$\lambda = 7.934,\ P_L \cong 2.45MW,$

$C_1 = 438.98\#/h,\ C_2 = 687.41\#/h,\ C_3 = 629.66\#/h,$

and $C_t = 1756.05\#/h,$

Problem 5.2

Consider the generator parameters given below,

Table 5.2.

Bus No	P_{min}	P_{max}	α	β	γ
1	5	25	45	7.820	0.00140
2	5	20	50	7.60	0.00292
3	5	10	30	7.85	0.00480

Given that, $P_L = \sum_{i=1}^{n}\sum_{j=1}^{n} P_i B_{ij} P_j + \sum^{n} B_{0i} P_i + B_{00},$

Where,

$$B_{ij} = \begin{bmatrix} 0.00067 & 0.00095 & 0.00050 \\ 0.00095 & 0.0012 & 0.00090 \\ 0.00050 & 0.00090 & 0.00029 \end{bmatrix},$$

$$B_{ii} = \begin{bmatrix} 0.00016 \\ 0.0014 \\ -0.0082 \end{bmatrix}, \qquad B_{00} = 0.00403,$$

Determine the optimal dispatch of the generator and the optimal cost of generation, when the total load demand is 31MW

The problem can be stated as,

Minimize,

$$C_t(P_i) = \sum_{i=1}^{3} \left(\alpha_i + \beta_i P_i + \alpha_i P_i^2\right)$$

Subject to

$$P_{i(\min)} \le P_i \le P_{i(\max)}, \quad i = 1,\ldots,3$$

$$P_D + P_L - P_i = 0$$

where, $P_L = \sum_{i=1}^{3}\sum_{j=1}^{3} P_i B_{ij} P_j + \sum^{3} B_{0i} P_i + B_{00},$

with

$$B_{ij} = \begin{bmatrix} 0.00067 & 0.00095 & 0.00050 \\ 0.00095 & 0.0012 & 0.00090 \\ 0.00050 & 0.00090 & 0.00029 \end{bmatrix},$$

$$B_{ii} = \begin{bmatrix} 0.00016 \\ 0.0014 \\ -0.0082 \end{bmatrix}, \qquad B_{00} = 0.00403,$$

$P_D = 31MW$

Follow the algorithm above, obtaining P_i using

$$\begin{bmatrix} \frac{\gamma_i}{\lambda} + B_{11i} & B_{12} & B_{13} \\ B_{21} & \frac{\gamma_2}{\lambda} + B_{22} & B_{23} \\ B_{31} & B_{32} & \frac{\gamma_3}{\lambda} + B_{33} \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 - B_{01} + \frac{\beta_1}{\lambda} \\ 1 - B_{02} + \frac{\beta_2}{\lambda} \\ 1 - B_{03} + \frac{\beta_3}{\lambda} \end{bmatrix}$$

Assume $\lambda^{(1)} = 8.2,$ we obtain

$$\begin{bmatrix} 0.841 & 0.95 & 0.5 \\ 0.95 & 1.556 & 0.9 \\ 0.5 & 0.9 & 0.875 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix} = \begin{bmatrix} 23.091 \\ 36.52 \\ 25.44 \end{bmatrix},$$

Using Gausian elimination method, we have,

$$\left[\begin{array}{ccc|c} 0.841 & 0.95 & 0.5 & 23.091 \\ 0.95 & 1.556 & 0.9 & 36.52 \\ 0.5 & 0.9 & 0.875 & 25.44 \end{array}\right],$$

$$M_{21} = \frac{0.95}{0.841}, \quad M_{31} = \frac{0.5}{0.541}$$

$$\left[\begin{array}{ccc|c} 0.841 & 0.95 & 0.5 & 23.091 \\ 0 & 0.4829 & 0.3352 & 10.44 \\ 0 & 0.3352 & 0.5777 & 11.712 \end{array}\right],$$

$$M_{32} = \frac{0.3352}{0.4829}$$

$$\left[\begin{array}{ccc|c} 0.841 & 0.95 & 0.5 & 23.091 \\ 0 & 0.4829 & 0.3352 & 10.44 \\ 0 & 0 & 0.3450 & 4.465 \end{array}\right],$$

$$\begin{bmatrix} 0.841 & 0.95 & 0.5 \\ 0 & 0.4829 & 0.3352 \\ 0 & 0 & 0.3450 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix} = \begin{bmatrix} 23.091 \\ 10.44 \\ 4.465 \end{bmatrix},$$

$$P_3^{(1)} = 12.94MW, \quad P_2^{(1)} = 12.64MW, \quad P_1^{(1)} = 5.49MW$$

$$\begin{aligned} P_L^{(1)} &= (5.49)^2(0.00067) + (12.64)^2(0.0012) + (12.94)^2(0.00029) + \\ &\quad (5.49)(0.00016) + 12.64(0.00014) + 12.94(-0.0082) + 0.00403 \\ &= 0.161046031 \cong 0.16 \end{aligned}$$

Since $P_D = 31MW$

$$\Delta P_i^{(1)} = 0.09$$

Compute

$$\Delta\lambda^{(1)} = \frac{\Delta P_1^{(1)}}{\frac{\left[\sum_{i=1}^{3}\gamma_i(1-B_{01}) - \sum_{\substack{i=1\\j=1}}^{3} 2\gamma_i B_{ij} P_j^{(1)}\right]}{2\left(\gamma_i + \lambda^{(1)} B_{ii}\right)^2}}, i = 1,2,3$$

$$= 0.0000466$$

$$\lambda^{(2)} = 8.2 + 0.0000466 = 8.2000466$$

Compute $P_i^{(2)}$, we have the same value as $P_i^{(1)}$, and ΔP_i, is very small, so we stop and compute $C_t(P_i)$,

$C_3 = 73.24\#/h, and, \quad C_t = 366.20\#/h$

$C_1 = 146.43\#/h, \quad C_2 = 146.53\#/h,$

Thus, the optimal dispatch is given by

$P_1 = 5.49MW, \ P_2 = 12.64MW, \ P_3 = 12.94MW,$
$\lambda = 8.2, \ P_L = 0.16MW.$

INTERPRETATION

It is observed that when generating limit is included, generators are capable of dispatch more power under the secured atmosphere. It is also observed that plants with higher actual capacity should not be placed very far from the National Control Center (Grid Center) in order to minimize power losses.

CONCLUSION

We give, mathematical formulation of the Optimization problems involving electric power generation with generator limit plus power losses was established. An Algorithms was tested via iterated numerical method and numerical examples were considered for better understanding of the concept. Finally we present interpretation of the result folowed by conclusion.

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