Reliability Analysis of Reinforced Concrete Beams Produced with Metakaolin

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Abstract

The purpose of this paper is to evaluate the reliability of reinforced concrete beams produced with metakaolin as a supplementary cementitious material under provision of BS 8110 (1985: 1997) and BS 1881: Part 118: 1983,experimental design was adopted where representative statistics and appropriate probability distribution of basic resistance and load variables are obtained. The data obtained for compressive strength test, tensile strength of reinforcement, self-weight of beam, imposed load on beams, length, breadth, and height for reinforced concrete beams produced with 0%, 10%, 20% and 30% replacements of cement with metakaolin were fitted to five statistical distributions models. The distribution fitting was achieved with Kolmogorov-Smirnov test method using statistical computer package Easy-fit. The mean values of the variables, distribution models, and standard deviations obtained from this research were acceptable when compared with the values in the existing literature and found suitable for a running reliability analysis. FORTRAN programing language was used to develop subroutine and performance function equation for shear, bending and deflection of the beams. Prediction of structural performance based on target safety index of 3.7 and 4.7 according to the Joint Committee of Structural Safety JCSS (2001) was achieved.

Keywords: *Reliability, Safety index, Bending, Shear, Deflection,*

Introduction

The study of reliability analysis has become interesting, with the tendency of the reliability analysis providing factors of safety for structures or structural elements whenever it is required. According to Thoft-Cristense and Baker (2012), structural reliability is the probability that a system does not reach a defined limit state under a given reference period.

Research works conducted on reliability of reinforced concrete structures by Renjian,Luo & Conte, (1994); Amana, (2009); Mohammed, Aliyu & Mohammed, (2014) ;Mohammed, Lawan, Aliyu&Suleiman, (2014); Onwuka and Sule, (2014); Kigha, (2014); Mohammadi and Keshtegar (2015); Abdulbasit, (2015) ; Faruq (2016) ; Saul (2016); Usmam (2016); Mahdi and Kioumarsi (2017) shown several problems in reinforced concrete structures and element.

While some of the problems were resolved. In the studies the problems were using of First Order Reliability Methods (FORM), Second Order Reliability Methods (SORM) Monte Carlo Simulation. Recently admixtures are used in the production of concrete which may influence the concrete properties. The safety of any structure depends on its strength and resistance properties of that particular structure, in reliability analysis random variable can be described using their distribution models and parameters. Therefore, investigating the effect of mineral admixture in random variables for reliability analysis is important.

Methods

In this study, reliability analysis of reinforced concrete beams produced with metakaolin was conducted using a structural reliability program written in *Fortran* Programming language. Representative statistics and appropriate probability distribution of basic resistance and load variables were obtained from an experimental work. The data obtained for compressive strength test, tensile strength of reinforcement, self-weight of beam, imposed load on beams, length, breadth, and height for reinforced concrete beams produced with 0%, 10%, 20% and 30% replacements of cement with metakaolin were fitted to five statistical distributions models.

The distribution fitting was achieved using Kolmogorov-Smirnov test method via a statistical computer package; Easy-fit (2010). The mean values of the variables, distribution models, and standard deviations obtained in this research were favorably compared with the values in the existing literature and found suitable for running reliability analysis.

Basis for Reliability Analysis and Transformations

$$P_f = \{g(X) < 0\} = \int_{g(x)<0} f_x(X)dx \qquad (1.0)$$

$$R - 1 = P_f = \{g(X) < 0\} = \int_{g(x)<0} f_x(X)dx$$

The effort of this method is to alleviate the computational difficulties by simplifying the integrand f_x(**X**) and approximating the performance function g(**X**) using the concept of first order Taylor expansion (linearization) (DU 2005).

$$g(X) = 0 \qquad (1.1)$$

$$F_{xi}(x_i) = \Phi(u_i) \qquad (1.2)$$

$$\Phi(.)$$

$$U_i = \Phi^{-1}[F_{xi}(X_i)] \qquad (1.3)$$

$$U = \Phi^{-1}[F_X(X)] = \Phi^{-1}\Phi\left[\left(\frac{X-\mu}{\sigma}\right)\right] = \frac{X-\mu}{\sigma} \qquad (1.4)$$

$$X = \mu + \sigma U \qquad (1.5)$$

$$Y = g(U) \qquad (1.6)$$

$$P_f = P\{g(U) < 0\} = \int_{g(u)<0} \phi(U)du \qquad (1.7)$$

$$\phi(U) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}u_i^2\right) \qquad (1.8)$$

$$P_f = \int_{g(u_1,u_2,\ldots u_n)} \ldots\ldots \int \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}u_i^2\right) du_1, du_2 \ldots\ldots du_n \qquad (1.9)$$

FORM uses a liner approximation (i.e. the first order Taylor expansion) to approximate the integration boundary.

$$g(\mathbf{U}) \approx L(\mathbf{U}) = g(u^*) + \nabla g(u^*)(U-u^*)^T \qquad (2.0)$$

Where L(U) is the linearized performance function of $u = (u_1, u_2, \ldots u_n)$ is the expansion point, T stands for the transpose, and $\nabla g(u^*)$ is the gradient of g(**U**) at u* and is given by in 2.2 to 2.7

$$\nabla g(u^*) = \left(\frac{\partial g(U)}{\partial g U_1}, \frac{\partial g(U)}{\partial g U_2}, \ldots\ldots\ldots\ldots \frac{\partial g(U)}{\partial g U_n} \right)_{U^*} \qquad (2.1)$$

$$\begin{cases} \max u \\ \textit{subject to } g(u) = 0 \end{cases} \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2} u^2{}_i \right) \qquad (2.2)$$

$$\prod_{i=1}^{n} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2} u^2{}_i \right) = \frac{1}{\sqrt{2\pi}} \exp \sum_{i=1}^{n} u^2{}_1 \qquad (2.3)$$

Maximizing $\prod_{i=1}^{n} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2} u^2{}_i \right)$ is equivalent to minimizing $\sum_{i=1}^{n} u^2{}_1$ the model for the most probable point (MPP) search can be written as;

$$\begin{cases} \min_{\mathbf{u}} \|\mathbf{u}\| \\ \text{subject to} \quad g(\mathbf{u}) = 0 \end{cases} \qquad (2.4)$$

||. || The length of vector which is defined as

$$\|\mathrm{u}\| = u_i^2 \sqrt{u_i^2} + u_2^2 + u_n^2 + = \sum_{i=1}^{n} u_1^2 \qquad (2.5)$$

The solution to the equation is the MMP and is given by

$$(u_1^*, u_1^*, \ldots\ldots u_n^*) \qquad (2.6)$$

The MMP is the shortest point from limits state origin g(u) =0 to the origin **O** in U space and the minimum distance $\beta = \|u^*\|$ is called reliability index

Performance Functions

The performance functions for various failure modes known as limit state functions or objective functions are presented in equations 3.0-3.8

Bending

Limit state function for the compression section are expressed in 3.0 to 3.8

$$G(x_{1a}) = M_u - M_A \qquad (3.0)$$

$$G(x_{1a}) = 0.156\, fcubd^2 - 0.125\omega l^2 x10^6 \qquad (3.1)$$

Limit state function for the tensile section are expressed in 3.0 to 3.3 BS8110 (1985)

$$G(x_{1b}) = M_u - M_A \qquad (3.2)$$

$$G(x_{1b}) = 0.87\, fyzA_S - 0.125\omega l^2 x10^6 \qquad (3.3)$$

Shear

Limit state function for shear are expressed in 3.24 to 3.26

$$G(x)_2 = V_c - V_A \qquad (3.4)$$

$$R_{wl}\, \frac{0.79K_1K_2}{r_m}\left(\frac{100AS}{bd}\right)\left(\frac{400}{d}\right)^{1/4} - \frac{(\gamma_g G_k + \gamma_q Q_k)}{2}xl \qquad (3.5)$$

$$\frac{wl}{2} = \frac{(\gamma_g G_k + \gamma_q Q_k)}{2}xl \qquad (3.6)$$

Deflection

Limit state function for deflection are expressed in equation 3.4 to 3.5

$$G(x)_2 = \delta l - \delta_A \qquad (3.7)$$

$$G(x)_2 = 0.004l - \frac{0.1775\omega l^3}{IE} \qquad (3.8)$$

Basic Variables

Distribution Fitting

The Kolmogorov-Smirnov test performs Normality test to check the deviation of the probability density function (PDF) of the random variable from Normal distribution. The normal (PDF) plot has a perfect bell shape. Other distributions are measured by their skewness either to the left or right of the normal plot as well as their measured peak point flatness to the peak point of the normal plot (i.e kurtosis).

Summary of Results for Distribution Models

To test the normality of concrete compressive strength five statistical distributions models, namely Normal distribution, Log-normal distribution, Gumbel Max, Frecher and Weibull distributions were considered.

Table 1.0: Ranking of distribution for concrete compressive strength at 0%, 10%, 20% and 30% using Kolmogorov-Smirnov distribution model

S/N	Percentage of metakaolin (%)	Distribution ranking				
		Normal	Lognormal	Gumbel Max	Frechet	Weibull
1	0%	2	1	3	5	4
2	10%	1	3	5	2	4
3	20%	1	3	2	4	5
4	30%	3	2	1	4	5
Mean distance with least ranking		1	2	3	4	5

The interpretation of the results is presented in Table 1.0, the sequence of the fitness is defined by ranking, the optimum distribution model was assigned the highest rank of 1 while the least distribution was assigned least ranking of 5. Table 1.0 presents the summary of distribution ranking. It is observed that most appropriate distribution model for cube compressive strength is normal distribution with rank 1. According to JCSS (2001) log-normal distribution can be used for materials properties.

Table 2.0: Ranking live load (Q_k) distribution at 0%, 10%, 20% and 30% using Kolmogorov-Smirnov distribution model

S/N	Percentage of metakaolin (%)	Distribution ranking				
		Normal	Lognormal	Gumbel Max	Exponential	Gamma
1	0%	1	3	4	5	2
2	10%	1	3	4	5	2
3	20%	1	2	4	5	3
4	30%	3	4	1	5	2
Mean distance with least ranking		1	3	4	5	2

To test the normality of live load of beam at 0%, 10%, 20% and 30% five statistical distributions models, namely Normal distribution, Log-normal distribution, Gumbel Max, Gamma and Exponential were considered. The interpretation of the results is presented in Table 1.0, the sequence of the fitness is defined by ranking, the optimum distribution model was assigned the highest rank of 1 while the least distribution was assigned least ranking of 5. Table 2.0 presents the summary of the distribution ranking. It is observed that the most appropriate distribution model for live load is the normal distribution with rank 1. According to JCSS (2001) permanent load should be represented by normal distribution while the probability distribution of the largest extreme could be approximated by one of the asymptotic extreme-value distributions Gumbel Max or Frechet. In this research the live load was considered as the ultimate or extreme, so Gumbel Max distribution was selected.

Table 3.0: Ranking of distribution for dead load (G_k) at 0%, 10%, 20% and 30% using Kolmogorov-Smirnov distribution model

S/N	Percentage of metakaolin (%)	Distribution ranking				
		Normal	Lognormal	Gumbel Max	Frechet	Weibull
1	0%	3	4	1	5	2
2	10%	3	2	1	5	4
3	20%	3	5	1	2	4
4	30%	2	3	1	4	5
Mean Distance with least ranking		2	3	1	5	4

To test the normality of the dead load on beam, five statistical distributions models, namely Normal distribution, Log-normal distribution, Gumbel Max, Frechet distribution, Gumbel Max, Frechet and Weibull distributions were considered. The interpretation of the results is presented in Table below 3.0 The sequence of the fitness is define by ranking, with the optimum distribution model having the highest rank of 1 and the least distribution having the least rank of 5. Table 3.0 present the summary of distribution ranking. From the Table it is observed that most appropriate distribution model for dead load is Gumbel Max. The result obtained is not in line with the recommendation of JCSS (2001) which mention that the permanent load should be represented by the normal distribution.

Table 4.0: Ranking of distribution for effective length, width and depth of the beam and tensile strength of reinforcement (Y10) using Kolmogorov-Smirnov distribution model

S/N		Distribution ranking				
		Normal	Lognormal	Gumbel Max	Frechet	Weibull
1	Effective length	2	1	4	5	3
2	Width	1	2	4	5	3
3	Depth	1	2	4	5	3
4	Tensile strength of reinforcement	1	2	3	5	4

To test the normality for the length, breadth, depth and tensile strength of reinforcement, five statistical distributions models, namely Normal distribution, Log-normal distribution, Gumbel maximum, Frechet distribution, and Weibull distribution were considered. The interpretation of the results is presented in Table 4.0. The sequence of the fitness is defined by ranking, with the optimum distribution model having the highest rank of 1 and the least distribution having the least ranking of 5. Table 4.0 presents the summary of distribution ranking for effective length, width, depth and tensile strength of reinforcement. From the Table it is observed that most appropriate distribution model for effective length, width; depth and tensile strength of reinforcement (f_y) are Log-normal, Normal, Normal and Normal respectively. The results obtained agree with the recommendations of the JCSS (2001) which mention that the dimensional variables can be adequately represented by Normal or Log-normal distribution.

Table 5.0: Parameters identified for reliability analysis

Parameter	Variable	Distribution	Mean	STD	Cov.	JSCC (2000)
X1	Fcu_{15}	Normal	15.59-9.94 N/mm^2	1.819	0.12	Lognormal or Normal
X2	$Q_{k\ 15}$	Gumbel	166.62-141.95 kN/m	8.9354	0.05	Gumbel or Frechet
X3	G_{k15}	Log-normal	0.65156-0.57393 kN/m	0.021194	0.04	Lognormal or Normal
X4	*b*	Normal	153.46mm	4.22	0.027	Lognormal or Normal
X5	d_{eff}	Normal	124.93mm	5.13	0.040	Lognormal or Normal
X6	l_{eff}	Log-normal	453.7mm	2.983	0.00657482	Lognormal or Normal
X7	fy	Log-normal	727.24N/mm^2	13.14	0.0189	Lognormal or Normal

Table 5.0 shows the parameters identified for reliability analysis. It can be deduced from the table that concrete compressive strength and the steel yield strength has Normal distribution. These findings are not in line with the findings of (Yusuf, 2002; Farsani and Keshtegar, 2015; Kioumarsi, *et al*., 2017). It is also observed from the Table 5.0 that, the self -weight of the beam has Normal distribution; this finding agrees with that of (Kioumarsi *et al*., 2017) which identified that the self-weight of a beam has Normal distribution. Furthermore, distribution of section width and height are obtained as normal and, and these findings are also in agreement with findings of (Farsani and Keshtegar, 2015; Kioumarsi *et al.,* 2017.), and disagrees with Yusuf (2002) who found the section width as Log-normal. The live load was obtained to have Normal distributions which disagree with Mohammed *et al*., (2014) and disagrees with Kioumarsi *et al*., (2017) and Yusuf (2002) that identified live load as Gumbel distribution. This may be due to the assignment of distribution models to a random variable.

Relationship between safety index (β) and imposed load (Q_k) at the top of reinforced concrete beam produced with metakaolin under bending (grade 15)

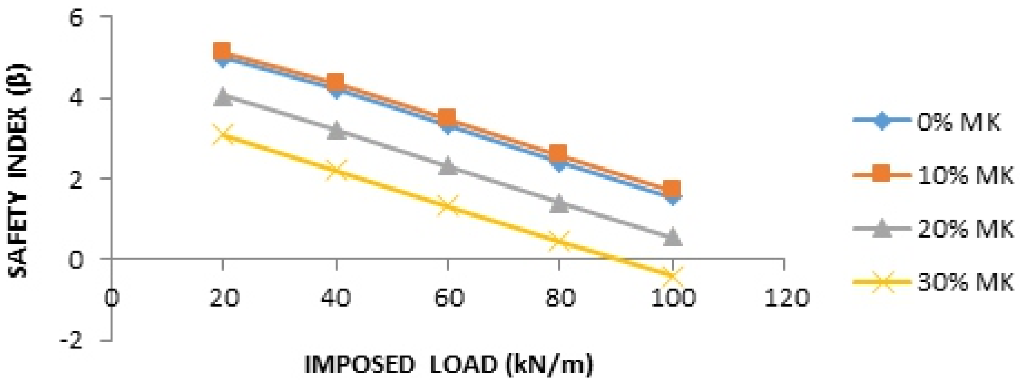

Figure 1: Relationship between safety index (β) and imposed load (Q_k) on reinforced concrete beam produced with metakaolin under bending, grade 15

Figure 1, 2 and 3 show the relationship between the safety index and imposed load for grade 15 concrete. Imposed load was varied while other parameters such as compressive strength, effective length of the beam, depth of the beam and length of the beam were kept constant. It is noticed from the plot that the bending failure on the beam prism was sensitive to the imposed load.

Discussion

The safety indices for the 0%, 10%, 20% and 30% replacements of metakaolin decreased from 4.979 to 1.536, 5.136 to 1.705, 4.046 to 0.553, and 3.097 to -0.409 respectively, while the probability of failure increased from 0.32×10^{-6} to 0.623×10^{-1}, 0.141×10^{-6} to 0.441×10^{-1}, 0.261×10^{-4} to 0.290 and 0.978×10^{-3} to 0.659 respectively. The probability of failure obtained were favorably compared with the probability of failure specified by the Joint Committee of Structural Safety (2001) and found that they are within the acceptable range of 10^{-4} for 3.7 safety index to 10^{-6} for 4.7 safety index. Also, the safety index of the control beams obtained was compared with the safety index obtained by Usman (2016) under the same failure criteria and found that the safety index obtained at an imposed load of 15 kN/m for grade 15 concrete under bending failure criteria was noted to be approximately 5.0 and which when compared to the result obtained in this research at imposed load of 15kN/m the safety index is 3.734. The variation of safety indices could be due to the variation of the material properties.

From the figure 1, it is also observed that, at an imposed load of 100 kN/m, the safety indices of beam prism of 750mm × 150mm ×150mm were 1.536, 1.705, 0.553, - 0.409 for the 0%, 10%, 20% and at 30% metakaolin replacement respectively. This indicates that 30% metakaolin replacement falls within the failure region with negative safety indices. However to guard against negative safety indices for beams with 30% metakaolin replacement the live load should not exceed 80kN/m. The safety indices also decrease as the percentages of metakaolin exceed 10% replacement.

It is also observed that the curves follow similar trends for specimen with 0%, 10%, 20% and 30% replacement of cement with metakaolin. Hence the beam's reliability is decreases with increas ing imposed load. This could be due to the similar behavior 0% metakaolin replacemet beams with 10%, 20% and 30% replacement of metakaolin beams, the ultimate moment of resistance will also be affected, and could be the reason for having lower values of safety indices at higher dosages of metakaolin beyond 10% metakaolin replacement.

Furthermore, increases in imposed load could lead to an increase in the overall stresses in beam section thereby weakening the bond between the binders and aggregate in concrete section, and the beam section will fail when its moment of resistance is exceeded.

Relationship between safety index (β) and imposed load (Q_k) of reinforced concrete beam produced with metakaolin under shear (grade 15)

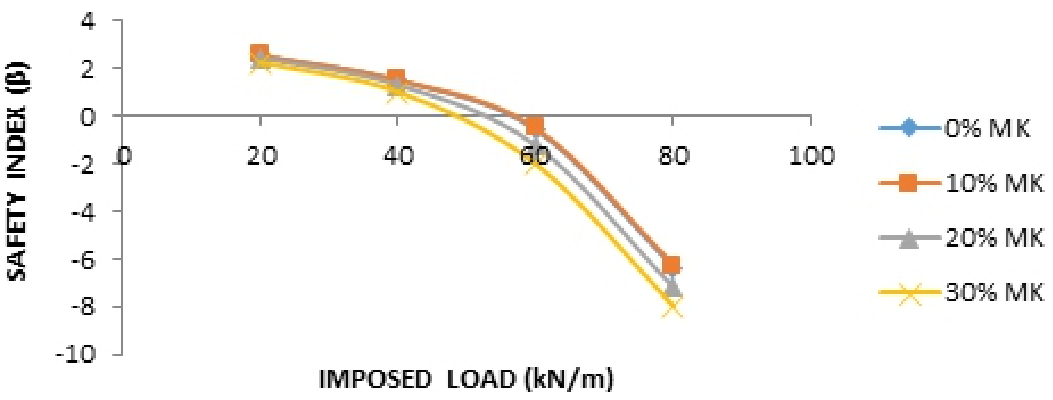

Figure 2: Relationship between safety index (β) and imposed load (Q_k) of reinforced concrete beam produced with metakaolin under shear (grade 15)

It can be observed from the figure 2 that the shear failure of the beam prism is sensitive to imposed load. This can be because of an increase in imposed load resulted an overall increase in stresses in concrete section thereby weakening the bond between the binders and the aggregate particles in concrete section, which could also lead to the shearing of the concrete section as the applied shear force exceeded shear capacity of the beam section.

The safety indices for 0%, 10%, 20% and 30% replacements of cement with metakaolin beams decreased from 2.574 to -6.379, 2.593 to -6.270, 2.417 to -7.194 and 2.229 to -8.001 respectively, while the probability of failure increased from 0.502×10^{-2} to 1.00, 0.475×10^{-2} to 1.00, 0.782×10^{-2} to 1.00 and 0.129×10^{-1} to 1.00 respectively.

The probability of failure obtained were compared with the value recommended for the ultimate limit state which is 10^{-4} and 10^{-6} for 3.7 and 4.7 and deduced that the values obtained did not fall within this range of values recommended for ultimate limits state, but when those values where compared with the range of values recommended for serviceability limit state that has the probability of failure of 10^{-2} and safety index of 2.3., it was observed that the values fall within the recommended values for serviceability limit state.

The results also compared with the findings of Usman (2016) who's obtained the safety index of grade 15 concrete at an imposed load of 15 kN/m under shear failure criterion to be 1.75, while the safety index obtained in this research under the same failure criterion, at an imposed load of 20 KN/m was obtained as 2.574.

The variation of safety indices could be due to variation in material properties used, which could lead to the variation in safety indices. It is observed that shear failure is the most critical mode of failure as compared with the bending mode of failure.

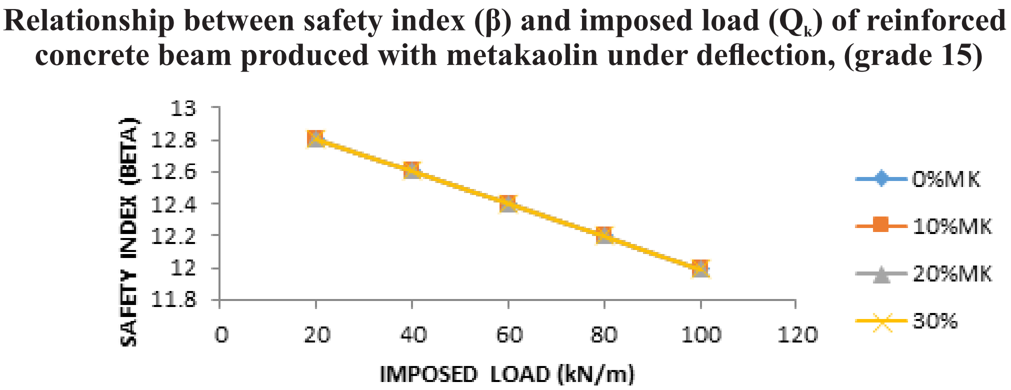

Figure 3: Relationship between safety index (β) and imposed load (Q_k) of reinforced concrete beam produced with metakaolin under deflection, (grade 15)

Figure 3 shows the relationship between the safety indices and imposed load in reinforced concrete beam produced with metakaolin, while concrete compressive strength, breadth, depth and length of the beam were kept constant. It was observed from the plot that the failure due to deflection of the beam is sensitive to the imposed load.

The safety indices for the 0%, 10%, 20% and 30%, decrease from 12.805 to 11.991 and the probability of failure increase from 0.794×10^{-37} to 0.204×10^{-32} respectively. The higher safety indices and lower probability of failure obtained as compared with the recommended value of 3.7 and 4.7 given by JCSS (2001) for ultimate and serviceability limits state indicates that the recommended safety index and probability of failure could not be archived. It is further observed that the higher safety indices obtained under higher imposed loads indicates that beam of 450mm length will not fail by deflection, but it can fail by bending or shear.

Conclusions

i) The distribution models, and standard deviations obtained for this research were favorably compared with the values of existing literature in this area of research and found suitable for executing reliability analysis.

ii) The probability of failure obtained under bending failure criterion were favorably compared with the probability of failure specified by the Joint Committee of Structural Safety (2001) and found that they are within the acceptable range of 10^{-4} for 3.7 safety indices to 10^{-6} for 4.7 safety index. This implies that concrete produced with metakaolin replacement of cement behave in similar way with the concrete without metakaolin replacement.

iii) The probability of failure obtained under shear failure criteria were favorably compared with the recommended value for the ultimate limit state which is 10^{-4} and 10^{-6} for 3.7 and 4.7 and noted that the value did not fall within this range but between the recommended values specified by JCSS (2001) for serviceability limit state that has the probability of failure of 10^{-2} and safety index of 2.3.

iv) The higher safety indices and lower probability of failure obtained as compared with the recommended value of 3.7 and 4.7 given by JCSS

(2001) for ultimate and serviceability limits state indicates that the recommended safety index and probability of failure could not be achieved. It is further observed that the higher safety indices obtained under higher imposed loads indicates that beam of 450mm length may not fail by deflection, but it can fail by bending or shear.

Recommendations

I) The statistical parameters for the basic design variables obtained in this research should be used in the reliability based design of metakaolin/cement reinforced concrete beams.

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