RESETTLEMENT PATTERN MODEL OF INTERNALLY DISPLACED PERSONS DUE TO INSURGENCY

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ABSTRACT
In this paper, the derivation and analysis of a model is considered for resettlement pattern of internally displaced persons due to insurgency. In the derived model, a number of homeless people at some initial time after the insurgency attack are considered. The flow of the population to a temporary state until their final resettlement is analyzed. The analytic solution for the stability analysis is obtained using Routh Harwitz Criterion and the behavior of the model is obtained numerically, using MATLAB R2010a. The result obtained indicates that majority of displaced persons get resettled after a long period of time.

INTRODUCTION
The continuing shortage of accurate and reliable data has resulted in a distorted picture of internally displaced persons (IDPs) assistance needs and an alarming lack of understanding of the country’s displacement dynamics on the part of national authorities and the international community. Those whose homes have been damaged or destroyed by the insurgent attacks have nowhere to go back to (Ogwang, 2014).Inadequate housing and welfare planning by the state to cope with the problems created by activities of insurgency has a negative effect on estimation of the cost of resettlement for victims of insurgency. Nikolopoulos and Tzanetis (2003) considered a model for housing allocation of a homeless population due to disaster. Ogwang (2014) assumed that focus on technical solutions like building infrastructures such as schools and roads at the expense of the underlying political dynamics of the conflict erodes the chance for achieving national reconciliation. Pandya (2013) investigated involuntary displacement being an analysis of the role and contribution of non-government organizations to the Narmada Project affected communities in Western India. Muangi and Kungu (2013) studied the dilemma of integration of internally displaced persons in Gishu, Kenya. Cerneea (1997) studied the risks and reconstruction model for resettling displaced populations. Francis (2017) assumed displacement could occur even when there is no planned process of resettlement. Other studies and reports on resettlement and reintegration of IDPs include Feldman (2018), Terminski (2013), Lango (2005), Oryema (2017), Walter (2008) and Robert (2018) mainly focused on impact of social protection on resettlement of displaced persons, durable solutions and challenges faced in resettlement of IDP. Emeny, Brusset and Yulia (2004) observed that IDP camps have generally been small, and food, water and other essential resources can be found in the vicinity of the camps. See also Shodeinde and Otabor (2018), Tom and Antonella (2005), Gertrud (2017) for studies on human security and the relocation of IDPs. Meanwhile the works of Tajudeen, and Adebayo (2013), Gautam and Dangol (2015), Joireman (2012), Mapiko and Chinyoka (2013), Neil (2016), Nina, and Micheal (2011) and Nicholas (2011) mostly centred toward public-housing allocation problem and provision of homes for extremely low income households. Christopher (2010), Shapps (2012) and Gujar, Zou, and Parkes (2013) worked basically on housing allocations. This study aims at establishing an optimum resettlement pattern for internally displaced persons due to insurgency attack for even distribution of resources and prompt reintegration. Therefore, a model by Nikolopoulos and Tzanetis (2003) is adopted and relate to the resettlement pattern of internally displaced persons due to insurgency by incorporating populations of both injured and hospitalized due to the attack for prompt reintegration and resettlement.
MATERIALS AND METHODS

The model is divided into six (6) compartments namely:

i. Number of displaced persons resettled (R)

ii. Number of persons that become homeless after insurgency attack (W)

iii. Number of displaced persons in camps provided by the state (P)

iv. Number of displaced persons living in self provided temporary tents and unfinished buildings (T)

v. Number of displaced persons accommodated in hospitals due to injuries sustained (H)

vi. Number of displaced persons living with friends and family (F)

Below is the schematic diagram of the proposed model for resettlement of IDPs due to insurgency incorporating injured and hospitalized for reintegration and resettlement.

Figure 1: Schematic Diagram of the Modified Model
Model Description
The variable denoted by $W$ describes the number of persons that become homeless after insurgency attack (having houses destroyed or badly damaged). The parameters $k_i$ and $k_j$ represent the movement of the displaced persons to state camps $P$ and the movement of the displaced persons to self provided temporary tents $T$ respectively. While $k_k$ represents the movement of displaced persons to friends and relatives $F$. Part of the displaced population moved to hospitals $H$ as a result of injuries sustained is represented by $k_l$. The parameter $k_m$ represents the movement of injured persons from hospital $H$ to friends and family $F$. While $k_n$ is the movement of persons loitering in self provided temporary tents $T$ to state camps $P$. The movement of displaced persons from state camps to final resettlement is denoted by $k_o$. The population living in self provided tents $T$ moved to their resettlement is represented by $k_p$. While for those living with friends and family $F$ moved to resettlement $R$ is denoted by $k_q$. Those hospitalized $H$ and eventually moved to their resettlement $R$ is denoted by $k_r$. Finally, the movement of some part of the displaced population moved to their final resettlement is represented by $k_s$.

Model Equations
\[
\begin{align*}
\frac{dW}{dt} &= -k_1 W (F_u - F) - k_2 W (H_u - H) - k_3 W (P_a - P) - k_4 W (T_a - T) - k_{12} W (R_a - R) \quad (2.1) \\
\frac{dH}{dt} &= k_5 W (H_a - H) - k_6 H (P_a - P) - k_7 H (R_a - R) - k_8 H (F_a - F) \quad (2.2) \\
\frac{dT}{dt} &= k_9 W (T_a - T) - k_{10} T (P_a - P) - k_{11} T (R_a - R) \quad (2.3) \\
\frac{dF}{dt} &= k_1 W (F_a - F) + k_6 H (F_a - F) - k_8 F (R_a - R) \quad (2.4) \\
\frac{dP}{dt} &= k_3 W (P_a - P) + k_6 H (P_a - P) - k_{10} P (R_a - R) - k_{10} P (R_a - R) \quad (2.5) \\
\frac{dR}{dt} &= k_8 F (R_a - R) + k_9 H (R_a - R) + k_{10} P (R_a - R) + k_{11} T (R_a - R) + k_{12} W (R_a - R) \quad (2.6) \\
D_0 &= D(0) = W(t) + H(t) + T(t) + F(t) + P(t) + R(t) \quad (2.7)
\end{align*}
\]

3 RESULTS AND DISCUSSION
This work deals with the study which consider a respectable number of displaced persons get settled after a period of time.

3.1 Equilibrium States and Stability Analysis
We scale quantities representing number of persons $W, H, T, F, P, R$ are sealed with the initial number of displaced persons $D_0$ so that $W = X_1 D_0$, $H = X_2 D_0$, $T = X_3 D_0$, $F = X_4 D_0$, $P = X_5 D_0$ and $R = X_6 D_0$.

Therefore, simplifying and expressing equations (2.1) to (2.6) led to the following equations
\[
\begin{align*}
\frac{dx_1}{dt} &= -a_{x_1} (c_4 - x_4) - a_{x_1} (c_2 - x_2) - a_{x_1} (c_5 - x_5) - a_{x_1} (c_5 - x_5) - a_{x_1} (c_5 - x_5) - a_{x_1} (c_5 - x_5) \quad (3.1) \\
\frac{dx_2}{dt} &= a_{x_2} (c_2 - x_2) - a_{x_2} (c_5 - x_5) - a_{x_2} (c_5 - x_5) - a_{x_2} (c_5 - x_5) \quad (3.2) \\
\frac{dx_3}{dt} &= a_{x_3} (c_3 - x_3) - a_{x_3} (c_5 - x_5) - a_{x_3} (c_5 - x_5) - a_{x_3} (c_5 - x_5) \quad (3.3) \\
\frac{dx_4}{dt} &= a_{x_4} (c_4 - x_4) - a_{x_4} (c_4 - x_4) - a_{x_4} (c_4 - x_4) - a_{x_4} (c_4 - x_4) \quad (3.4) \\
\frac{dx_5}{dt} &= a_{x_5} (c_5 - x_5) + a_{x_5} (c_5 - x_5) + a_{x_5} (c_5 - x_5) - a_{x_5} (c_5 - x_5) \quad (3.5)
\end{align*}
\]
\[
\frac{dx}{dt} = a_4 x_4 (c_6 - x_6) + a_5 x_5 (c_6 - x_6) + a_{10} x_5 (c_6 - x_6) + a_1 x_3 (c_6 - x_6) + a_{12} x_1 (c_6 - x_6) \tag{3.6}
\]

For proportion of various compartments, consider the conditions:

\[
W = x_1 D_0 \Rightarrow x_1 = \frac{W}{D_0}; \quad H = x_2 D_0 \Rightarrow x_2 = \frac{H}{D_0}; \quad T = x_3 D_0 \Rightarrow x_3 = \frac{T}{D_0};
\]

\[
F = x_4 D_0 \Rightarrow x_4 = \frac{F}{D_0}; \quad P = x_5 D_0 \Rightarrow x_5 = \frac{P}{D_0}; \quad R = x_6 D_0 \Rightarrow x_6 = \frac{R}{D_0}.
\]

Differentiate \( x_i \) above with respect to \( t \), therefore,

\[
\frac{d x_i}{dt} = \frac{d}{dt} \left( \frac{W}{D_0} \right) = \frac{D_0 \frac{d W}{dt} - W \frac{d D_0}{dt}}{(D_0)^2},
\]

\[
\frac{D_0 \frac{d W}{dt}}{(D_0)^2} = \frac{1}{D_0} \frac{d W}{dt}. \tag{3.7}
\]

Substituting for equation (3.7), equation (2.1) becomes

\[
\frac{dx}{dt} = \frac{1}{D_0} \left( -k_1 W (F_a - F) - k_2 W (H_a - H) - k_3 W (P_a - P) - k_4 W (T_a - T) - k_{12} W (R_a - R) \right)
\]

\[
\left[ -k_1 W (F_a - F) - k_2 W (H_a - H) - k_3 W (P_a - P) - k_4 W (T_a - T) - k_{12} W (R_a - R) \right].
\]

Similarly, equations (2.2), (2.3), (2.4), (2.5) and (2.6) become

\[
\frac{dx}{dt} = a_1 x_1 (c_2 - x_2) - a_6 x_2 (c_5 - x_5) - a_9 x_2 (c_6 - x_6) - a_{11} x_3 (c_4 - x_4),
\]

\[
\frac{dx}{dt} = a_4 x_1 (c_3 - x_3) - a_5 x_3 (c_5 - x_5) - a_1 x_3 (c_6 - x_6),
\]

\[
\frac{dx}{dt} = a_4 x_1 (c_4 - x_4) + a_7 x_2 (c_4 - x_4) - a_6 x_4 (c_6 - x_6),
\]

\[
\frac{dx}{dt} = a_4 x_1 (c_6 - x_6) + a_6 x_2 (c_6 - x_6) + a_{10} x_5 (c_6 - x_6) + a_1 (c_6 - x_6) + a_{12} x_1 (c_6 - x_6)
\]

\[
\frac{dx}{dt} = a_4 x_1 (c_6 - x_6) + a_6 x_2 (c_6 - x_6) + a_{10} x_5 (c_6 - x_6) + a_1 (c_6 - x_6) + a_{12} x_1 (c_6 - x_6).
\]

The solution of the system of algebraic equations coming from equating the right hand side of 3.1 – 3.6 to zero will give the steady states of the system. Therefore from equation (3.1),

\[
x_1 \left[ -a_1 (c_4 - x_4) - a_2 (c_2 - x_2) - a_3 (c_5 - x_5) - a_4 (c_3 - x_3) - a_{12} (c_6 - x_6) \right] = 0
\]

This means that either

\[
x_1 = 0 \text{ or } -a_1 (c_4 - x_4) - a_2 (c_2 - x_2) - a_3 (c_5 - x_5) - a_4 (c_3 - x_3) - a_{12} (c_6 - x_6) = 0.
\]

Similarly, from equations (3.2), (3.3), (3.4), (3.5), (3.6) resulted to:

\[
x_2 = 0 \text{ or } -a_6 (c_5 - x_5) - a_9 (c_6 - x_6) - a_1 (c_4 - x_4) = 0;
\]

\[
x_3 = 0 \text{ or } -a_4 (c_3 - x_3) - a_1 (c_6 - x_6) = 0;
\]

\[
x_4 = 0 \text{ or } -a_8 (c_4 - x_4) = 0;
\]

\[
x_5 = 0 \text{ or } -a_6 (c_6 - x_6) = 0.
\]

Notice that from the \( c_6 = x_6 \), this mean that \( x_6 = c_6 \).

But \( x_6 = R \text{ and } c_6 = R \).

Consequently,
where $D_0 =$ Total number of displaced persons. Hence insurgency free equilibrium $I_0$ is

$$I_0 = \begin{pmatrix} W, H, T, F, P, R \end{pmatrix} = \begin{pmatrix} 0, 0, 0, 0, 0, 1 \end{pmatrix}, \quad (3.8)$$

that of interest in analysis because it expresses the state where by each person is resettled.

To obtain the stability insurgency free equilibrium, the Jacobian matrix of equations (3.1)-(3.6) is

$$J = \begin{pmatrix} A & a_{x_2} & a_{x_3} & a_{x_4} & a_{x_5} & -a_{x_6} \\ a_{c_1} & a_{c_2} & a_{c_3} & a_{c_4} & a_{c_5} & -a_{c_6} \\ a_{e_1} & a_{e_2} & a_{e_3} & a_{e_4} & a_{e_5} & -a_{e_6} \\ a_{d_1} & a_{d_2} & a_{d_3} & a_{d_4} & a_{d_5} & -a_{d_6} \\ a_{r_1} & a_{r_2} & a_{r_3} & a_{r_4} & a_{r_5} & -a_{r_6} \\ a_{s_1} & a_{s_2} & a_{s_3} & a_{s_4} & a_{s_5} & -a_{s_6} \end{pmatrix} \quad (3.9)$$

Substituting equation (3.8) into equation (3.9) we have

$$J_e = \begin{pmatrix} -a_{c_1} - a_{c_2} - a_{c_3} - a_{c_4} - a_{c_5} - a_{c_6} - \lambda \\ a_{c_2} & -a_{c_5} - a_{c_6} - a_{c_1} - \lambda \\ a_{c_4} & 0 & -a_{c_3} - a_{c_6} - \lambda \\ a_{c_5} & 0 & 0 & -a_{c_5} - \lambda \\ a_{c_6} & 0 & 0 & 0 & -a_{c_6} - \lambda \end{pmatrix} \quad (3.10)$$

Given

$$| A - \lambda I | = 0. \quad (3.11)$$

Substituting equation (3.10) into equation (3.11) therefore,

$$\begin{pmatrix} -B - \lambda & 0 & 0 & 0 & 0 \\ 0 & -C - \lambda & 0 & 0 & 0 \\ a_{c_4} & 0 & -D - \lambda & 0 & 0 \\ a_{c_5} & 0 & 0 & -E - \lambda & 0 \\ a_{c_6} & 0 & 0 & 0 & -F - \lambda \end{pmatrix} = 0 \quad (3.12)$$

where $A, B, C, D, E, F$ represent the partial derivatives of equations (3.1)-(3.6) in the matrix. Evaluating equation (3.12);

$$| A - \lambda | = | -B - \lambda | = | C - \lambda | = | -D - \lambda | = | -E - \lambda | = | -F - \lambda | = 0.$$

$$| AB + A\lambda + B\lambda^2 + C\lambda^3 | \left| - D - \lambda \right| \left| - E - \lambda \right| \left| - F - \lambda \right| = 0.$$

$$| ABCD + AB\lambda + AB\lambda^2 + AB\lambda^3 + AB\lambda^4 + \lambda^5 | \left| - E - \lambda \right| = 0.$$
The Routh-Hurwitz criterion states that if the determinants of characteristic polynomials are greater than zero then the system is asymptotically stable. If one of the polynomials is less than zero the system is unstable (Routh, 1905). Therefore, since the determinants of the characteristic polynomials are greater than zero, it means that $\lambda_i < 0$, $i = 1, 2, 3, 4, 5, 6$. This implies that the system is unfairly and
asymptotically stable. By this, it means that the internally displaced persons are fully resettled at their homes.

**Numerical Solution of the Model**

In order to obtain the behavior of the model numerically, MATLAB R2010a used and the following are assumed values:

\[ a_1 = 0.1103; \quad a_2 = 0.0037; \quad a_3 = 0.5133; \quad a_4 = 0.3307; \quad a_5 = 0.0011; \quad a_6 = 0.0026; \quad a_7 = 0.0989; \]
\[ a_8 = 0.0035; \quad a_9 = 0.3656; \quad a_{10} = 0.2918; \quad a_{11} = 0.0126; \quad c_1 = 0.1838; \quad c_2 = 0.3676; \quad c_3 = 0.1618; \]
\[ c_4 = 0.6617; \quad c_5 = 0.8455. \]

The following graphs represent various displaced population based numerical results’ simulation of the modified model generated using MATLAB R2010a.

![Figure 2: Flows of displaced persons, State organized camps and those staying with friends and family](image2.png)

![Figure 3: Flows hospitalized, State organized camps and the displaced population](image3.png)
Figure 4: Flows of displaced population, Tents and state organized camps

Figure 5: The upward movement of displaced persons
We can see from the above graphs that figure 2 shows the flows of displaced persons from their communities, internally displaced persons at state camps and also their pattern of resettlement over a period of time. The downward slope suggest the rapid displacement of people as the result of the attack while the population get settled at internally displaced camps operated by the government and then they begin to get back to their communities over a period of time as a result of less activities of the insurgents. Figure 3 explains the population that was hospitalized has a steady pattern of resettlement. This could be seen in the case where by once the population under attack evacuates their community, there are no new cases or referrals of attacked persons from the insurgents in that
community therefore those already injured and hospitalized get settled over a period of time. Figure 4 explains the flow of displaced persons immediately after the attack which shows the downward slope of the displaced persons, the resettlement behavior is seen to be upward moving alongside the population living with family and friends. Those living with family and friends get to resettle as time goes on. Also figure 5 shows the upward movement of displaced persons which suggest the steady flow of their resettlement as the activities of the insurgents reduces with government intervention. The displaced persons begin to resettle at their communities, villages and towns in order to start over their normal activities. Just as seen in figure 6, there is a downward slope in the graph which implies that there is a rapid displacement of people from their houses and the entire community as the result of the activities of the insurgents.

Communities, Villages and towns are evacuated by its dwellers in order to move to safer places. Finally, figure 7 explains the general interaction of the flows of the displaced persons over a stipulated period of time. For those populations that after displacement move to friends and family, state camps, tents, and hospitals at the initial stage of the attack are on the increase, but with time begin to decrease which suggest that they begin to get settled in their communities, town or villages. Therefore there is a steady rise of resettlement.

3.3 DISCUSSION

For $\lambda_1$, this implies immediately after displacement, there is a steady flow of displaced persons whose houses are badly damaged ($W$), to family and friends ($-a_1c_4$), to hospital emergency wards ($-a_2c_2$), to camps operated by the state ($-a_3c_5$) for those with timely information in that regard while others loiter around in self provided tents, in friends’ houses, street corners ($-a_4c_3$) and finally there is steady flow of persons to their resettlement ($-a_5c_6$). For $\lambda_2$, the displaced persons at the hospital ($H$), there is a steady flow of persons after treatment to state operated camps ($-a_6c_3$) and also for those that have family and friends they eventually move in with them ($-a_7c_4$) finally for those that are financially stable or have houses elsewhere they resettle ($-a_8c_6$). Rapid flow pending on the injury sustained. For $\lambda_3$, the displaced population that loiter around ($T$), there is also a two way steady flow of displaced persons which are; they relocate to state operated camps for better comfort than the street and also food supplied by the government and non-governmental organizations ($-a_9c_6$) and some proportion get settled after a stipulated period of time. For $\lambda_4$, victims already with family and friends have the option of resettlement over time just as shown in the steady flow ($-a_6c_6$). For $\lambda_5$, the state camp being the largest compartment for displaced persons contains IDPs immediately after displacement ($k_1W$) and also those victims discharged from hospitals ($k_6H$) and those loitering on the street ($k_4T$) all converge in the state camp. It has been observed that there is a slow flow of IDPs from state camp to their resettlement. This might be as a result of the continuous activities of the insurgents or as a result of poor financial status. With time the victims might acquire skills to enable them support themselves and also with government intervention thereby leading to eventual closure of the state camp over a period of time. The solution of the system of algebraic equations coming from equating the right-hand side of (3.1), (3.2), (3.3), (3.4), (3.5) to zero will give the steady-states of the system. One steady state is $x_1 = x_2 = x_3 = x_4 = x_5 = 0$, which is of interest in our analysis because it expresses the state when every family is resettled. General interaction of the flows of the displaced persons over a stipulated period of time has been demonstrated in figure (7) above. It is clearly demonstrated that the displaced population has a downward slope due to the persistent attack of the insurgents. Downward slope of the entire population as a result of their displacement has a corresponding upward movement of the population getting resettled. This agrees with the actual situation of the IDPs because with prompt government intervention to combat the insurgents their activities are minimized.
4 CONCLUSION
Having reviewed the flows of displaced persons by insurgency attack, it is important to note that the state operated camps serves as a major accommodation to displaced persons. However, government should put more effort in the welfare of individuals living in state camps. Also, individuals living with friends and family, tents or unfinished houses, hospitals should also be considered when relieve materials are to be distributed. Lastly, we recommend that further study should consider individuals who fled to other countries and also birth and death rate be considered to enable us see how realistic the system will be. There is also need to consider the time that individuals need to settle in a temporary accommodation or to resettle.

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