BAJOPAS Volume 15 Number 1, June, 2022 **http://dx.doi.org/10.4314/bajopas.v15i1.14**

Bayero Journal of Pure and Applied Sciences, 15(1): 105 - 112
Received: May, 2022
Accepted: June, 2022
ISSN 2006 – 6996

EFFICIENCY OF MODIFIED GENERALIZED IMPUTATION SCHEME FOR ESTIMATING POPULATION MEAN WITH KNOWN AUXILIARY INFORMATION

Adejumobi, A.,[1*] Audu, A.,[2] Yunusa, M. A.[2] and Singh, R. V. K.[1]
[1]Department of Mathematics, Kebbi State University of Science and Technology, Aliero, Nigeria
[2]Department of Statistics, Usmanu Danfodiyo University, Sokoto, Nigeria
[*]Corresponding Author Email: awwaladejumobi@gmail.com

ABSTRACT
Different authors for estimating population mean have proposed several Imputation schemes. Recently, some authors have suggested generalized imputation schemes that their estimators are functions of unknown parameters of the study variable. These unknown parameters need to be estimated for the estimators to be applicable and this may require additional resources. This paper considered a class of imputation scheme that is independent of unknown parameter and the point estimator of the suggested scheme for estimating population mean was derived. The properties (bias and MSE) of an efficient estimators presented were derived up to first order approximation and also conditions for which the estimators of the proposed scheme is more efficient than other estimators of the existing schemes considered in the study were also examined. The result of the empirical study revealed that the suggested estimators are more efficient than the existing ones considered in the study.
Keywords: Imputation, Missing Information, Population mean, auxiliary variable, Efficiency.

INTRODUCTION

In sample surveys, missing information on sampled units is a relevant and crucial observation. The demographic surveys, social-economic survey, clinical and agricultural experiments are the fundamental examples of this. It has been proved by some survey researchers in their findings that the inferences of unknown population parameters can be spoiled due to missing information. Therefore, the suitable methodology of estimating population parameters may be used to handle the statistical datasets in case of missing or incomplete information. The common technique used to handle situations where data is missing is Imputation. Missing values can be completed with specific substitutes and data can be analyzed using standard methods. Information about unit of characteristic of interest observed and auxiliary variable help improve the accuracy of demographic parameter estimates (Pandey et al., 2021).

Hansen and Hurwitz (1946) was the first researcher who considered the problem of non-response. Many researchers also worked on imputation methods to deal with non-response and missing values among them include; Lee et al. (1994), Singh and Horn (2000), Singh and Deo (2003), Toutenburg et al. (2008), Singh (2009), Wang and Wang (2006), Kadilar and Cingi (2008), Diana and Perri (2010), Al-Omari et al. (2013), Singh et al. (2014), Gira (2015), Singh et al. (2016), Bhushan and Pandey (2016), Prasad (2016), Audu et al. (2020a,b,c), Audu et al. (2021a,b,c,d), Audu and Singh (2021), Yusuf et al. (2022). However, the estimators of the scheme proposed by Audu et al. (2020a) and Pandey et al. (2021) are functions of the unknown parameters of the study variable which makes the schemes and estimators impracticable in real life application unless if the unknown parameters are estimated using large sample which may required additional resources in the conduct of survey.

Audu et al., (2020) suggested On the Class of Exponential-Type Imputation Estimators of Population Mean with Known Population Mean of Auxiliary Variable.

$$y_{.i} = \begin{cases} \theta_1 \dfrac{n}{r} y_i & i \in R \\ \dfrac{n}{n-r} \bar{y}_r \left(\theta_2 \left(\dfrac{\bar{X}}{\bar{x}_r} \right)^{\kappa_1} + \theta_3 \left(\dfrac{\kappa_2 \left(\bar{X} - \bar{x}_r \right)}{\bar{X} + \bar{x}_r} \right) \right) & i \in R^c \end{cases} \quad (1)$$

where $\kappa_1, \kappa_2 \in (1, -1)$

The point estimators of population mean from the proposed schemes in (1) are obtained as

$$t_p = \bar{y}_r \left(\theta_1 + \theta_2 \left(\frac{\bar{X}}{\bar{x}_r} \right)^{\kappa_1} + \theta_3 \exp\left(\frac{\kappa_2 \left(\bar{X} - \bar{x}_r \right)}{\bar{X} + \bar{x}_r} \right) \right) \quad (2)$$

$$Bias\left(t_p\right) = \bar{Y} \lambda_{r,N} \left(\left(\theta_2 \frac{\kappa_1 (\kappa_1 + 1)}{2} + \theta_3 \frac{\kappa_2 (\kappa_2 + 2)}{8} \right) C_X^2 - \left(\theta_2 \kappa_1 + \theta_3 \frac{\kappa_2}{2} \right) \rho_{XY} C_X C_Y \right) \quad (3)$$

$$MSE\left(t_p\right) = \bar{Y}^2 \lambda_{r,N} \left(C_Y^2 + \psi^2 C_X^2 - 2\psi \rho_{XY} C_X C_Y \right) \quad (4)$$

where $\psi = \rho_{XY} C_Y / C_X$

The expressions for $\theta_i, i = 1,2,3,$ are as follows

$$\left. \begin{aligned} \theta_3 &= 4\left(2^{-1}(\kappa_1 + 1) C_X - \rho_{XY} C_Y\right) \rho_{XY} C_Y / \kappa_2 \left(\kappa_1 - \kappa_2/2\right) C_X^2 \\ \theta_2 &= -\left(2^{-1}(\kappa_2 + 2) C_X - 2\rho_{XY} C_Y\right) \rho_{XY} C_Y / \kappa_1 \left(\kappa_1 - \kappa_2/2\right) C_X^2 \\ \theta_1 &= 1 + \begin{pmatrix} 2\left(4^{-1} \kappa_2 (\kappa_2 + 2) - \kappa_1 (\kappa_1 + 1)\right) C_X \\ -(\kappa_2 - 2\kappa_1) \rho_{XY} C_Y \end{pmatrix} \rho_{XY} C_Y / \kappa_1 \kappa_2 \left(\kappa_1 - \kappa_2/2\right) C_X^2 \end{aligned} \right\} \quad (5)$$

Audu and Singh (2021) proposed Exponential-type regression compromised imputation class of estimators, the generalized class of imputation scheme given as

$$y_{.i} = \begin{cases} y_i & i \in \Phi \\ \dfrac{\hat{\mu}_0 + \hat{\beta}_{rg} \left(\bar{X} - \bar{x}_r \right)}{\pi_1 \bar{x}_r + \pi_2} \left(\pi_1 \bar{X} + \pi_2 \right) \exp\left(\dfrac{\varpi_1 \left(\bar{X} - \bar{x}_r \right)}{\varpi_1 \left(\bar{X} + \bar{x}_r \right) + 2\varpi_2} \right) & i \in \Phi^c \end{cases} \quad (6)$$

where π_1 and π_2 are known functions of auxiliary variables like coefficient of skewness $\beta_{1(x)}$, kurtosis $\beta_{2(x)}$, variation C_x, standard deviation S_x etc.

Note that $\pi_1 \neq \pi_2$ and $\pi_1 \neq 0$

The estimator, bias and MSE of the Imputation scheme in (6) are given as in (7), (8) and (9) respectively.

$$\mu_i^{(*)} = \frac{r}{n} \hat{\mu}_0 + \left(1 - \frac{r}{n}\right) \frac{\hat{\mu}_0 + \hat{\beta}_{rg} \left(\bar{X} - \bar{x}_r \right)}{\pi_1 \bar{x}_r + \pi_2} \left(\pi_1 \bar{X} + \pi_2 \right) \exp\left(\frac{\varpi_1 \left(\bar{X} - \bar{x}_r \right)}{\varpi_1 \left(\bar{X} + \bar{x}_r \right) + 2\varpi_2} \right) \quad (7)$$

$$Bias\left(\mu_i^{(*)}\right) = \psi_{r,N} \left(1 - \frac{r}{n}\right) \begin{pmatrix} \left(\beta_{rg} \bar{X} (\eta_1 + \eta_2) + \bar{Y} \left(\eta_1^2 + \eta_1 \eta_2 - 1.5\eta_2^2 \right) \right) S_x^2 \\ -\bar{Y} (\eta_1 + \eta_2) C_{YX} \end{pmatrix} \quad (8)$$

$$MSE\left(\mu_i^{(*)}\right) = \psi_{r,N} \left(S_Y^2 + \gamma^2 S_X^2 - 2\gamma S_{YX} \right) \quad (9)$$

where $\eta_1 = \dfrac{\pi_1 \bar{X}}{\pi_1 \bar{X} + \pi_2}$, $\eta_2 = \dfrac{\varpi_1 \bar{X}}{2(\varpi_1 \bar{X} + \varpi_2)}$ and $\gamma = \left(1 - \dfrac{r}{n}\right)\left(R(\eta_1 + \eta_2) + \beta_{rg}\right)$,

$\beta_{rg} = S_{YX} / S_X^2$

Pandey et al., (2021) proposed improved estimators for mean estimation in presence of missing information.

$$y_{.i} = \begin{cases} y_i & \text{i} \in A \\ \left\{ n\left\{\alpha_j \bar{y}_r + \lambda_j(\bar{x}_n - \bar{x}_r)\right\} \exp\left\{\dfrac{F_j(1 - \bar{x}_r/\bar{X})}{1 + \left((a\bar{x}_r + b)/(a\bar{X} + b)\right)}\right\} - r\bar{y}_r \right\} \dfrac{x_i}{\sum_{i \in A^c} x_i} & \text{if } \text{i} \in A^c \end{cases} \tag{10}$$

where

$F_j = \left((a\bar{X})/(a\bar{X} + b)\right);\ j = 1, 2, ..., 10$ for different choices of $a(\neq b)$ and b.

Under the Pandey et al. (2021) suggested imputation methods, the corresponding point estimators of population mean $\bar{Y}$ are derived as

$$d_{pj} = \left[\alpha_j \bar{y}_r + \lambda_j(\bar{x}_n - \bar{x}_r)\right] \exp\left\{\frac{F_j(1 - \bar{x}_r/\bar{X})}{1 + \left((a\bar{x}_r + b)/(a\bar{X} + b)\right)}\right\} \ (j = 1, 2, ..., 10) \tag{11}$$

The bias and MSE(d_{pj}) are as follows

$$Bias(d_{pj}) = \bar{Y}\left[\alpha_j\left(1 + \frac{1}{2}\theta F_j C_x\left(\frac{3}{4}F_j C_x - \rho_{yx} C_y\right)\right) + \frac{1}{2R_1}\lambda_j F_j(\theta - \theta_1)C_x^2 - 1\right] \tag{12}$$

$$MSE(d_{pj}) = \left(1 - 2\alpha_j + (1 + C_y^2\theta)\alpha_j^2\right)\bar{Y}^2 + \frac{1}{4}C_x^2\begin{pmatrix} -4\theta_1\lambda_j\bar{X}\left(\lambda_j\bar{X} + (-1 + 2\alpha_j)\bar{Y}F_j\right) \\ +\theta\begin{pmatrix} 4\lambda_j^2\bar{X}^2 + 4(-1 + 2\alpha_j)\lambda_j\bar{X}\bar{Y}F_j \\ +\alpha_j(-3 + 4\alpha_j)\bar{Y}^2F_j^2 \end{pmatrix}\end{pmatrix} \tag{13}$$

$$+ C_y C_x \alpha_j \bar{Y}\left(2\theta_1\lambda_j\bar{X} + f\left(-2\lambda_j\bar{X} + \bar{Y}F_j - 2\alpha_j\bar{Y}F_j\right)\right)\rho$$

The optimal values are

$$\alpha_{j(opt.)} = -\frac{-8 + C_x^2(\theta - 4\theta_1)F_j^2 + 4\theta_1 F_j \rho C_y C_x}{8\left(1 + C_x^2\theta_1 F_j^2 - 2\rho C_y C_x \theta_1 F_j + C_y^2(\theta - \theta\rho^2 + \theta_1\rho^2)\right)} \tag{14}$$

and

$$\lambda_{j(opt.)} = \frac{-8 + C_x^2(\theta - 4\theta_1)F_j^2 + 4\theta_1 F_j \rho C_y C_x}{8\left(1 + C_x^2\theta_1 F_j^2 - 2\rho C_y C_x \theta_1 F_j + C_y^2(\theta - \theta\rho^2 + \theta_1\rho^2)\right)} \tag{15}$$

$$MSE(d_{pj})_{opt.} = \left(1 - 2\alpha_j^* + (1 + C_y^2\theta)\alpha_j^{*2}\right)\bar{Y}^2 + \frac{1}{4}C_x^2\begin{pmatrix} -4\theta_1\lambda_j^*\bar{X}\left(\lambda_j^*\bar{X} + (-1 + 2\alpha_j^*)\bar{Y}F_j\right) \\ +\theta\begin{pmatrix} 4\lambda_j^{*2}\bar{X}^2 + 4(-1 + 2\alpha_j^*)\lambda_j^*\bar{X}\bar{Y}F_j \\ +\alpha_j^*(-3 + 4\alpha_j^*)\bar{Y}^2F_j^2 \end{pmatrix}\end{pmatrix} \tag{16}$$

$$+ C_y C_x \alpha_j^* \bar{Y}\left(2\theta_1\lambda_j^*\bar{X} + f\left(-2\lambda_j^*\bar{X} + \bar{Y}F_j - 2\alpha_j^*\bar{Y}F_j\right)\right)\rho$$

The aim of this study is to modify the imputation scheme proposed by Pandey et al. (2021) and test for the efficiency of the proposed estimator and some existing related estimators considered in the study theoretically using real life data.

MATERIALS AND METHODS

The Proposed Estimator under imputation

Having studied the imputation schemes suggested by Pandey *et al.* (2021) for estimation of $\bar{Y}$ using the information on auxiliary variable, we proposed the following improved and efficient exponential type imputation methods.

$$y_{.i}=\begin{cases} y_i & i\in\tau \\ \left\{ n\left(\frac{\bar{y}_r}{2}\left(\frac{\bar{x}_n}{\bar{x}_r}+\frac{\bar{x}_r}{\bar{x}_n}\right)+w_1\left(\bar{x}_n-\bar{x}_r\right)+w_2\bar{y}_r\right)\exp\left(\frac{F_s\left(1-\frac{\bar{x}_r}{\bar{X}}\right)}{1+\left(\left(a\bar{x}_r+b\right)/\left(a\bar{X}+b\right)\right)}\right)-r\bar{y}_r\right\}\frac{x_i}{\sum_{i\in\tau^c}x_i} & \text{if } i\in\tau^c \end{cases} \quad (17)$$

where $F_s=\left(\left(a\bar{X}\right)/\left(a\bar{X}+b\right)\right);\ s=1,2,.......,10$ for different choices of $a(\neq 0)$ and b.

The point estimators of finite population mean under the proposed scheme is obtained as:

$$T_{fs}=\frac{1}{n}\left(\sum_{i\in\tau}y_i+\sum_{i\in\tau^c}\left[\frac{\bar{y}_r}{2}\left(\frac{\bar{x}_n}{\bar{x}_r}+\frac{\bar{x}_r}{\bar{x}_n}\right)+w_1\left(\bar{x}_n-\bar{x}_r\right)+w_2\bar{y}_r\right]\exp\left[\frac{F_s\left(1-\frac{\bar{x}_r}{\bar{X}}\right)}{1+\left(a\bar{x}_r+b\right)/\left(a\bar{X}+b\right)}\right]\right) \quad (18)$$

$$T_{fs}=\left[\frac{\bar{y}_r}{2}\left(\frac{\bar{x}_n}{\bar{x}_r}+\frac{\bar{x}_r}{\bar{x}_n}\right)+w_1\left(\bar{x}_n-\bar{x}_r\right)+w_2\bar{y}_r\right]\exp\left[\frac{F_s\left(1-\frac{\bar{x}_r}{\bar{X}}\right)}{1+\left(a\bar{x}_r+b\right)/\left(a\bar{X}+b\right)}\right] \quad (19)$$

Remark 2.2: The proposed class of imputation estimators is independent of unknown parameter, hence it is practically applicable.

Table 1: Some member of $T_{fs}\left(s=1,2,.....,10\right)$ for different values of a and b

s	Proposed members of the class	a	b
1.	$T_{f1}=\left[\frac{\bar{y}_r}{2}\left(\frac{\bar{x}_n}{\bar{x}_r}+\frac{\bar{x}_r}{\bar{x}_n}\right)+w_1\left(\bar{x}_n-\bar{x}_r\right)+w_2\bar{y}_r\right]\exp\left[\frac{F_1\left(1-\left(\bar{x}_r/\bar{X}\right)\right)}{1+\left(\bar{x}_r/\bar{X}\right)}\right]$	1	0
2.	$T_{f2}=\left[\frac{\bar{y}_r}{2}\left(\frac{\bar{x}_n}{\bar{x}_r}+\frac{\bar{x}_r}{\bar{x}_n}\right)+w_1\left(\bar{x}_n-\bar{x}_r\right)+w_2\bar{y}_r\right]\exp\left[\frac{F_2\left(1-\left(\bar{x}_r/\bar{X}\right)\right)}{1+\left(\left(\bar{x}_r+1\right)/\left(\bar{X}+1\right)\right)}\right]$	1	1
3.	$T_{f3}=\left[\frac{\bar{y}_r}{2}\left(\frac{\bar{x}_n}{\bar{x}_r}+\frac{\bar{x}_r}{\bar{x}_n}\right)+w_1\left(\bar{x}_n-\bar{x}_r\right)+w_2\bar{y}_r\right]\exp\left[\frac{F_3\left(1-\left(\bar{x}_r/\bar{X}\right)\right)}{1+\left(\left(\bar{x}_r+\beta_{2(x)}\right)/\left(\bar{X}+\beta_{2(x)}\right)\right)}\right]$	1	$\beta_{2(x)}$
4.	$T_{f4}=\left[\frac{\bar{y}_r}{2}\left(\frac{\bar{x}_n}{\bar{x}_r}+\frac{\bar{x}_r}{\bar{x}_n}\right)+w_1\left(\bar{x}_n-\bar{x}_r\right)+w_2\bar{y}_r\right]\exp\left[\frac{F_4\left(1-\left(\bar{x}_r/\bar{X}\right)\right)}{1+\left(\left(\bar{x}_r+\beta_{1(x)}\right)/\left(\bar{X}+\beta_{1(x)}\right)\right)}\right]$	1	$\beta_{1(x)}$
5.	$T_{f5}=\left[\frac{\bar{y}_r}{2}\left(\frac{\bar{x}_n}{\bar{x}_r}+\frac{\bar{x}_r}{\bar{x}_n}\right)+w_1\left(\bar{x}_n-\bar{x}_r\right)+w_2\bar{y}_r\right]\exp\left[\frac{F_5\left(1-\left(\bar{x}_r/\bar{X}\right)\right)}{1+\left(\left(\bar{x}_r+\rho\right)/\left(\bar{X}+\rho\right)\right)}\right]$	1	ρ
6.	$T_{f6}=\left[\frac{\bar{y}_r}{2}\left(\frac{\bar{x}_n}{\bar{x}_r}+\frac{\bar{x}_r}{\bar{x}_n}\right)+w_1\left(\bar{x}_n-\bar{x}_r\right)+w_2\bar{y}_r\right]\exp\left[\frac{F_6\left(1-\left(\bar{x}_r/\bar{X}\right)\right)}{1+\left(\left(S_x\bar{x}_r+1\right)/\left(S_x\bar{X}+1\right)\right)}\right]$	S_x	1
7.	$T_{f7}=\left[\frac{\bar{y}_r}{2}\left(\frac{\bar{x}_n}{\bar{x}_r}+\frac{\bar{x}_r}{\bar{x}_n}\right)+w_1\left(\bar{x}_n-\bar{x}_r\right)+w_2\bar{y}_r\right]\exp\left[\frac{F_7\left(1-\left(\bar{x}_r/\bar{X}\right)\right)}{1+\left(\left(S_x\bar{x}_r+\beta_{1(x)}\right)/\left(S_x\bar{X}+\beta_{1(x)}\right)\right)}\right]$	S_x	$\beta_{1(x)}$
8.	$T_{f8}=\left[\frac{\bar{y}_r}{2}\left(\frac{\bar{x}_n}{\bar{x}_r}+\frac{\bar{x}_r}{\bar{x}_n}\right)+w_1\left(\bar{x}_n-\bar{x}_r\right)+w_2\bar{y}_r\right]\exp\left[\frac{F_8\left(1-\left(\bar{x}_r/\bar{X}\right)\right)}{1+\left(\left(S_x\bar{x}_r+\rho\right)/\left(S_x\bar{X}+\rho\right)\right)}\right]$	S_x	ρ

9.	$T_{f9}=\left[\frac{\bar{y}_r}{2}\left(\frac{\bar{x}_n}{\bar{x}_r}+\frac{\bar{x}_r}{\bar{x}_n}\right)+w_1\left(\bar{x}_n-\bar{x}_r\right)+w_2\bar{y}_r\right]\exp\left[\frac{F_9\left(1-\left(\bar{x}_r/\bar{X}\right)\right)}{1+\left(\left(\beta_{1(x)}\bar{x}_r+\beta_{2(x)}\right)/\left(\beta_{1(x)}\bar{X}+\beta_{2(x)}\right)\right)}\right]$	$\beta_{1(x)}$	$\beta_{2(x)}$
10.	$T_{f10}=\left[\frac{\bar{y}_r}{2}\left(\frac{\bar{x}_n}{\bar{x}_r}+\frac{\bar{x}_r}{\bar{x}_n}\right)+w_1\left(\bar{x}_n-\bar{x}_r\right)+w_2\bar{y}_r\right]\exp\left[\frac{F_{10}\left(1-\left(\bar{x}_r/\bar{X}\right)\right)}{1+\left(\left(C_x\bar{x}_r+\beta_{2(x)}\right)/\left(C_x\bar{X}+\beta_{2(x)}\right)\right)}\right]$	C_x	$\beta_{2(x)}$

Properties of the Suggested Estimator

In this section, the bias and MSE of the suggested estimators in this paper are derived and discussed properly.

$$\left.\begin{array}{l}\frac{\bar{y}_r}{\bar{Y}}=\left(1+e_0\right),\ \frac{\bar{x}_r}{\bar{X}}=\left(1+e_1\right),\ \frac{\bar{x}_n}{\bar{X}}=\left(1+e_2\right)\\ E\left(e_0\right)=E\left(e_1\right)=E\left(e_2\right)=0,\ E\left(e_0^2\right)=\theta C_y^2 E\left(e_1^2\right)=\theta C_x^2,\\ E\left(e_2^2\right)=E\left(e_1e_2\right)=\theta_1C_x^2,\ E\left(e_0e_1\right)=\theta\rho C_yC_x,\ E\left(e_0e_2\right)=\theta_1\rho C_yC_x\\ \theta=\left(\frac{1}{r}-\frac{1}{N}\right),\ \theta_1=\left(\frac{1}{n}-\frac{1}{N}\right),\ \frac{\bar{X}}{\bar{Y}}=\gamma,\ \theta_s=\frac{F_s}{2},\ \gamma=\frac{\bar{X}}{\bar{Y}}\end{array}\right\} \quad (20)$$

Theorem 1: To $O(n^{-1})$, the bias of the suggested estimator T_{fs} is:

$$Bias\left(T_{fs}\right)=\bar{Y}\left[\begin{array}{l}\left(\left(1+3\theta_s^2\right)\frac{\lambda C_x^2}{2}-\frac{\lambda_1C_x^2}{2}-\theta_s\lambda\rho C_yC_x\right)+w_1\gamma\theta_sC_x^2\left(\lambda-\lambda_1\right)\\ +w_2\left(1+\frac{3\theta_s^2\lambda C_x^2}{2}-\theta_s\lambda\rho C_yC_x\right)\end{array}\right] \quad (21)$$

Proof: Express (19) in terms of e_i^s, we have

$$T_{fs}=\left[\begin{array}{l}\frac{\bar{Y}\left(1+e_0\right)}{2}\left[\left(1+e_2\right)\left(1+e_1\right)^{-1}+\left(1+e_1\right)\left(1+e_2\right)^{-1}\right]\\ +w_1\bar{X}\left(e_2-e_1\right)+w_2\bar{Y}\left(1+e_0\right)\end{array}\right]\exp\left[\frac{F_s\left(1-\left(1-e_1\right)\right)}{1+\left(a\bar{X}+a\bar{X}e_1+b\right)/\left(a\bar{X}+b\right)}\right] \quad (22)$$

$$=\left[\frac{\bar{Y}\left(1+e_0\right)}{2}\left[2+e_1^2+e_2^2-2e_1e_2\right]+w_1\bar{X}\left(e_2-e_1\right)+w_2\bar{Y}\left(1+e_0\right)\right]\exp\left[\frac{-F_se_1}{2+F_se_1}\right] \quad (23)$$

where $F_s=\frac{a\bar{X}}{a\bar{X}+b}\left(s=1,2,....,10\right)$ for different suitable choices of a and b, and $\left|F_se_1\right|<1$ so that the term $\left(1+F_se_1\right)^{-1}$ is convergent.

where

$$\begin{array}{l}F_1=1,\ F_2=\bar{X}\left(\bar{X}+1\right)^{-1},\ F_3=\bar{X}\left(\bar{X}+\beta_{2(x)}\right)^{-1}\\ F_4=\bar{X}\left(\bar{X}+\beta_{1(x)}\right)^{-1},\ F_5=\frac{\bar{X}}{\bar{X}+\rho_{yx}},\ F_6=\frac{S_x\bar{X}}{S_x\bar{X}+1},\\ F_7=\frac{S_x\bar{X}}{S_x\bar{X}+\beta_{1(x)}},\ F_8=\frac{S_x\bar{X}}{S_x\bar{X}+\rho_{yx}},\ F_9=\frac{\beta_{1(x)}\bar{X}}{\beta_{1(x)}\bar{X}+\beta_{2(x)}},\\ F_{10}=\frac{C_x\bar{X}}{C_x\bar{X}+\beta_{2(x)}}\end{array} \quad (24)$$

Simplify (23) up to $O(n^{-1})$, we have

$$T_{fs}-\bar{Y}=\bar{Y}\left(e_0-\theta_s e_1+\frac{e_1^2}{2}+\frac{3\theta_s^2 e_1^2}{2}+\frac{e_2^2}{2}-e_1e_2-\theta_s e_0 e_1\right)+w_1\frac{\bar{X}}{\bar{Y}}\left(e_2-\theta_s e_1 e_2-e_1+\theta_s e_1^2\right)$$
$$+w_2\bar{Y}\left(1-\theta_s e_1+\frac{3\theta_s^2 e_1^2}{2}+e_0-\theta_s e_0 e_1\right) \qquad (25)$$

Subtract $\bar{Y}$ from both sides of (25), take expectation and apply the results of (20), Theorem (20) is proved.

Theorem 2: To $O(n^{-1})$, the MSE of the suggested estimator T_{fs} is:

$$MSE\left(T_{fs}\right)=\bar{Y}^2\left[A+w_1^2B+w_2^2C+2w_1D+2w_2E+2w_1w_2F\right] \qquad (26)$$

where $A=\lambda\left(C_y^2-2\theta_s\rho C_yC_x\right)$, $B=\gamma^2C_x^2\left(\lambda-\lambda_1\right)$,

$C=1+\lambda\left(C_y^2+3\theta_s^2C_x^2-4\theta_s\rho C_yC_x\right)+\theta_s^2\lambda_1C_x^2$, $D=\gamma\left(\lambda_1\left(\rho C_yC_x-\theta_sC_x^2\right)-\lambda\left(\rho C_yC_x+\theta_sC_x^2\right)\right)$,

$E=\lambda\left(C_y^2-3\theta_s\rho C_yC_x+\left(2.5\theta_s^2+0.5\right)C_x^2\right)-0.5\lambda_1C_x^2$, $F=\gamma\left(2\theta_sC_x^2\left(\lambda-\lambda_1\right)+\rho C_yC_x\left(\lambda_1-\lambda\right)\right)$

Differentiating (27) partially with respect w_1 and w_2 equate to zero and solve for w_1 and w_2 simultaneously, we obtained $w_1=\frac{EF-CD}{BC-F^2}$ and $w_2=\frac{DF-BE}{BC-F^2}$. Substituting the results in (27), we obtained the minimum $MSE\left(T_{fs}\right)$.

$$MSE\left(T_{fs}\right)=\bar{Y}^2\left[A+\frac{CD^2+BE^2+2DEF}{\left(BC-F^2\right)}\right] \qquad (27)$$

RESULTS AND DISCUSSION

In this section, In order to elucidate the performance of suggested estimators to deal missing data with respect to some existing related estimators by using two data sets below.

Yadav and Zaman (2021)

Population 1: Y **=** The production (Yield) of peppermint oil in kilogram and X = The area of the field in Bigha (2529.3 Square Meter).

$N=150,\ n=40,\ \gamma=0.018333,\ \bar{Y}=79.58,\ \bar{X}=6.5833,\ \rho=0.9363$

$C_y=0.781333,\ C_x=0.661726,\ S_y^2=3866.165,\ S_x^2=18.97791,\ \beta_1=1.4984$

$\beta_2=5.408,$

Murthy (1967)

Population 2: Y = Output for 80 factories in a region and X = Number of workers

$N=80,\ \ n=20,\ \ \bar{Y}=51.8264,\ \ \bar{X}=11.2646,\ \ \rho=0.9413$

$C_y=0.3542,\ \ C_x=0.7505,\ \ \beta_1=1.0500,\ \ \beta_2=-0.0634,$

Table 2: MSE of the Proposed and some related existing estimators using population 1

ESTIMATORS	MSE	ESTIMATORS	MSE
Sample mean	167.5338	$\left(\pi_1=\beta_2(x),\pi_2=C_x\right)$	13967.32
Lee et al. (1994)	83.57332	$\left(\pi_1=\beta_2(x),\pi_2=\beta_1(x)\right)$	13938.67
Singh and Horn (2000)	82.80116	$\left(\pi_1=\beta_2(x),\pi_2=S_x\right)$	13850.05
Singh and Deo (2003)	82.80123	$\left(\pi_1=S_x,\pi_2=C_x\right)$	13961.75
Singh (2009)	84.34548	$\left(\pi_1=S_x,\pi_2=\beta_1(x)\right)$	13926.69
Gira (2015)	82.80123	$\left(\pi_1=S_x,\pi_2=\beta_2(x)\right)$	13786.15
Singh et al. (2016)	59.37374	Audu et al. (2020)	
	166.8763	$\left(\kappa_1=1,\kappa_2=1\right)$	167.5818
Kadilar and Cingi (2008)	94.86998	$\left(\kappa_1=1,\kappa_2=-1\right)$	142.7133
	56.87213	$\left(\kappa_1=-1,\kappa_2=1\right)$	321.5926
Al-Omari et al. (2013)	98.97785	$\left(\kappa_1=-1,\kappa_2=-1\right)$	518.1431
	86.471	Pandey et al. (2021)	6293.872
Audu & Singh (2021) $\left(\pi_1=1,\pi_2=0\right)$	13875.25	**Proposed estimators** $\left(T_{fs}\right)$	
$\left(\pi_1=1,\pi_2=C_x\right)$	13872.86	T_{f1}	**8.409117**
$\left(\pi_1=1,\pi_2=\beta_{1(x)}\right)$	13751.76	T_{f2}	**20.0644**
$\left(\pi_1=1,\pi_2=\beta_{2(x)}\right)$	13412.72	T_{f3}	**45.62799**
$\left(\pi_1=1,\pi_2=S_x\right)$	13479.75	T_{f4}	**24.61472**
$\left(\pi_1=C_x,\pi_2=\beta_{1(x)}\right)$	13661.32	T_{f5}	**19.43115**
$\left(\pi_1=C_x,\pi_2=\beta_{2(x)}\right)$	13282.55	T_{f6}	**11.44761**
$\left(\pi_1=C_x,\pi_2=S_x\right)$	13350.62	T_{f7}	**12.87232**
$\left(\pi_1=\beta_1(x),\pi_2=C_x\right)$	13909.62	T_{f8}	**11.26137**
$\left(\pi_1=\beta_1(x),\pi_2=\beta_2(x)\right)$	13535.92	T_{f9}	**38.2639**
$\left(\pi_1=\beta_1(x),\pi_2=S_x\right)$	13596.86	T_{f10}	**53.06286**

Table 3: MSE of the Proposed and some related existing estimators using population 2

ESTIMATORS	MSE	ESTIMATORS	
Sample mean	134.2434	$(\pi_1=\beta_2(x),\pi_2=C_x)$	46694.13
Lee et al. (1994)	92.22131	$(\pi_1=\beta_2(x),\pi_2=\beta_1(x))$	46670.56
Singh and Horn (2000)	87.16081	$(\pi_1=\beta_2(x),\pi_2=S_x)$	46579.75
Singh and Deo (2003)	87.16081	$(\pi_1=S_x,\pi_2=C_x)$	46694.67
Singh (2009)	97.28181	$(\pi_1=S_x,\pi_2=\beta_1(x))$	46672.12
Gira (2015)	106.8948	$(\pi_1=S_x,\pi_2=\beta_2(x))$	46589.92
Singh et al. (2016)	55.89374	Audu et al. (2020)	
	133.5788	$(\kappa_1=1,\kappa_2=1)$	140.2786
Kadilar and Cingi (2008)	121.043	$(\kappa_1=1,\kappa_2=-1)$	118.6738
	33.42507	$(\kappa_1=-1,\kappa_2=1)$	170.8189
Al-Omari et al. (2013)	122.1959	$(\kappa_1=-1,\kappa_2=-1)$	246.6052
	100.6903	Pandey et al. (2021)	3234.955
Audu & Singh (2021) $(\pi_1=1,\pi_2=0)$	46672.08	**Proposed estimators** (T_{fs})	
$(\pi_1=1,\pi_2=C_x)$	46642.72	T_{f1}	**24.12438**
$(\pi_1=1,\pi_2=\beta_{1(x)})$	46531.59	T_{f2}	**28.6813**
$(\pi_1=1,\pi_2=\beta_{2(x)})$	46224.38	T_{f3}	**41.61604**
$(\pi_1=1,\pi_2=S_x)$	46209.44	T_{f4}	**30.65192**
$(\pi_1=C_x,\pi_2=\beta_{1(x)})$	46395.04	T_{f5}	**28.43718**
$(\pi_1=C_x,\pi_2=\beta_{2(x)})$	45981.32	T_{f6}	**24.99556**
$(\pi_1=C_x,\pi_2=S_x)$	45964.47	T_{f7}	**25.4184**
$(\pi_1=\beta_1(x),\pi_2=C_x)$	46663.26	T_{f8}	**24.94527**
$(\pi_1=\beta_1(x),\pi_2=\beta_2(x))$	46345.55	T_{f9}	**37.3705**
$(\pi_1=\beta_1(x),\pi_2=S_x)$	46332.96	T_{f10}	**49.81802**

Table 2 and 3 shows the MSEs of the suggested and other existing estimators considered in the study by using information of two different populations. The results revealed that the suggested estimator have the minimum MSE compared to the conventional estimators from each population. This means that the proposed methods shown a high level of efficiency on others considered in the study, and can produce a better estimate of the average population in the presence of a missing observation on average.

CONCLUSION

From the empirical study, the results showed that the suggested estimators were more efficient than the existing estimators considered in the study. So, therefore its use is recommended to estimate the population average when certain values of the variables of the study are missing in the study.

REFERENCES

Al-Omari, A.I., Bouza, C.N., and Herrera, C. (2013). Imputation methods of missing data for estimating the population mean using simple random sampling with known correlation coefficient. *Quality and Quantity*, 47, 353-365.

Audu, A., and Singh, R.V.K. (2020). Exponential-Type Regression Compromised Imputation class of estimators, *Journal of Statistics and Management Systems*, DOI:10.1080/09720510.2020.1814501.

Audu, A., Ishaq O.O., Isah, U., Muhammed, S., Akintola, A.K., Rashida, A., and Abubakar, A. (2020). On the class of Exponential-Type Imputation estimators of population mean with known population mean of auxiliary variable, *Journal of Science and Technology Research*, 2(4), pp. 1-11.

Audu, A., Singh, R. and Khare, S. (2021a): New Regression-Type Compromised Imputation Class of Estimators with known Parameters of Auxiliary Variable. *Communication in Statistics-Simulation and computation,* 1-13. DOI:10.1080/03610918.2021.1970182

Audu, A., Danbaba, A., Ahmad, S. K., Musa, N., Shehu, A., Ndatsu, A. M. and Joseph, A. O. (2021b). On the Efficient of Almos Unbiased Mean Imputation When the Population Mean of Auxiliary Variable is Unknown. *Asian Journal of Probability and Statistics,* 15(4): 235-250. DOI:10.9734/AJPAS/2021/v15i430377

Audu, A., Yunusa, M. A., Zoramawa, A. B., Buda, S. and Singh, R. V. K. (2021c). Exponential Ratio-Type Imputation Class of Estimators using Nonconventional Robust Measures of Dispersions*. Asian Journal of Probability and Statistics,* 15(2): 59-74. DOI:10.9734/AJPAS/2021/v15i23035

Audu, A., Ishaq, O. O., Abubakar, A., Akintola, K. A., Isah, U., Rashida, A. and Muhammad, S. (2021d): Regression- type Imputation Class of Estimators using Auxiliary Attribute. *Asian Research Journal of Mathematics,* 17(5): 1-13. DOI: 10.9734/ARJOM/2021/v17i530296

Audu, A., Ishaq O. O., Muili, J., Zakari, Y., Ndatsu, A. M. and S. Muhammed (2020b): On the Efficiency of Imputation Estimators using Auxiliary Attribute. Continental J. Applied Sciences, 15(1), 1-13. DOI:10.5281/zenodo.3721046

Audu, A., Ishaq, O. O., Zakari, Y., Wisdom, D. D., Muili, J. O. and Ndatsu, A. M. (2020c): Regression-cum-exponential ratio imputation class of estimators of population mean in the presence of non- response. *Science Forum Journal of Pure and Applied Science,* 20, 58-63. DOI: http://dx.doi.org/10.5455/sf.71109

Bhushan, S., and Pandey, A.P. (2016). Optimal Imputation of missing data for estimation of population mean. *Journal of Statistics and Management Systems*, 19(6), 755-769.

Diana, G., and Perri, P.F. (2010). Improved estimators of population mean for missing data, *Communication in Statistics-Theory and Methods*, 39, 3245-3251.

Gira, A.A. (2015). Estimation population mean with a new imputation methods. *Applied Mathematical Sciences*, 9(34), 1663-1672.

Hansen, M.N., and Hurwitz, W.N. (1946). The problem of non-response in sample surveys. *Journal of the American Statistical Association*, 41:517-529.

Kadilar, C., and Cingi, H. (2008). Estimators for the population mean in the case of missing data, *Communications in Statistics-Theory and Methods*, 37, 2226-2236.

Lee, H., Rancourt, E., and Saerndal, C.E. (1994). Experiments with variance estimation from survey data with imputed values. *J. Official Statistics-Stockholm*, 10, 231.

Pandey, A.K., Singh, G.N., and Neveen S.A. (2021). Improved estimators for mean estimation in presence of missing information, *Alexandria Engineering Journal*, 60, 5977-5990.

Prasad, S. (2016). A study on new methods of ratio exponential type imputation in sample surveys. H*acettepe Journal of Mathematics and Statistics*, DOI: 10.15672/HJMS.2016.392.

Singh, A.K., Singh, P., and Singh, V.K. (2014). Exponential-Type Compromised Imputation in Survey Sampling, *J. Stat. Appl*. 3(2), 211-217.

Singh, S. (2009). A new method of imputation in survey sampling. *Statistical Papers*, 44, 555-579.

Singh, S., and Deo, B. (2003). Imputation by power transformation. *Statistical Papers*, 44, 555-579.

Singh, S., and Horn, S. (2000). Compromised Imputation in Survey Sampling. *Metrika,* 51, 267-276.

Singh, G.N., Maurya, S., Khetan, M., and Kadilar, C. (2016). Some imputation methods for missing data in sample surveys. *Hacettepe Journal of Mathematics and Statistics*, 45(6), 1865-1880.

Toutenburg, H., Srivastava, V.K., and Shalabh, A. (2008). Imputation versus Imputation of missing values through ratio method in sample surveys. *Statistical Papers*, 49, 237-247.

Wang, L., and Wang, Q. (2006) Empirical likelihood for parametric model under imputation for missing data. *Journal of Statistics and Management Systems*, 9(1), 1-13.

Yadav, S.K., and Zaman, T. (2021). Use of some conventional and non-conventional parameters for improving the efficiency of ratio-type estimators. *Journal of Statistics and Management Systems*, DOI:10.1080/09720510.2020.1864939.

Yusuf H. Aliyu, Amos A. Adewara, Ahmed Audu, Omotayo A. Abidoye, Issa Sulaiman, Muhammed B. Aliyu (2022). Modified Compromized Type Method of Imputation for Estimating Population Mean. *Journal of Scientific Research,* 66(1), 404-410. DOI:10.37398/JSR.2022.66014.