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ON A NEW WEIBULL LOMAX DISTRIBUTION WITH FIVE PARAMETERS

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ABSTRACT
This study introduces the "New Weibull Lomax Distribution (NWLD)," a five-parameter distribution developed by extending the Weibull-G family and integrating an additional parameter into the existing four-parameter Weibull Lomax distribution. The NWLD aims to provide a robust alternative for modelling that enhances the practical applicability and efficiency of traditional models in statistical analysis. The NWLD's cumulative distribution function and probability density function are shown in this paper. However, the unique distribution's validity was demonstrated. The approach is grounded in the established frameworks of generalized classes of distributions, contributing to the ongoing evolution of probabilistic modelling techniques.
Keywords: Weibull-Lomax distribution, Weibull-G family, Weibull distribution, Lomax distribution, continuous distribution.

INTRODUCTION

Many classical distributions have been extensively utilized over the past few decades to model data across diverse fields, including biology, engineering, actuarial science, environmental and medical sciences, demographics, economics, finance, and insurance. However, there is a clear demand for more expansive variants of these distributions in applications such as insurance, finance, and lifetime analysis. As a result, researchers have vigorously pursued the construction of novel families of distributions (Bourguignon *et al.*, 2014). Numerous innovative approaches have successfully created new probability distribution families that not only extend existing models but also provide significant practical flexibility for data modelling (See Gupta *et al.*, 1998; Eugene *et al.*, 2002; Lee *et al.*, 2013; Cordeiro *et al.*, 2014; Gomes-Silva *et al.*, 2019, etc.). For instance, Gurvich *et al.* (1997) developed a substantial family of univariate distributions by enhancing the traditional Weibull model, effectively deriving new insights from the Weibull distribution. Additionally, Usman *et al.* (2019) introduced a continuous probability distribution known as the Burr X-Topp Leone distribution as an improvement over the Topp-Leone distribution.

Research has flourished concerning generalized classes (G-classes) of distributions, leading to an array of innovative models, including exponentiated-G (Gupta *et al.*, 1998), beta-G (Eugene *et al.*, 2002), Kumaraswamy-G (Cordeiro & De Castro, 2011), McDonald-G (Alexander *et al.*, 2012), odd-gamma-G (Torabi & Montazari, 2012), odd exponentiated generalized-G (Cordeiro *et al.*, 2013), McDonald-G (Afify *et al.*, 2015), truncated exponential-G (Barreto-Souza & Simas, 2013), logistic-G (Torabi & Montazari, 2019), gamma extended Weibull-G (Nascimento *et al.*, 2014), odd Weibull-G (Bourguignon *et al.*, 2014), exponentiated-half-logistic-G (Cordeiro *et al.*, 2014a), Lomax-G (Cordeiro *et al.*, 2014b), modified beta-G (Nadarajah *et al.*, 2014), Weibull-G (Tahir *et al.*, 2016), and exponentiated-Kumaraswamy-G (Gomes-Silva *et al.*, 2019).

The introduction of the transformed-transformer (T-X) family of distributions, encompassing Gamma-X and Weibull-X by Alzaatreh *et al.* (2013), has revolutionized the understanding of generalizing classes of distributions. Notable extensions of this innovative approach include the exponentiated T-X (Alzaghal *et al.*, 2013), Gumbel-X (Al-Aqtash 2013; Al-Aqtash *et al.*, 2014), T-X{Y}-quantile-based method (Aljarrah *et al.*, 2014), T-R{Y} (Alzaatreh *et al.*, 2014), T-Weibull{Y} (Almheidat *et al.*, 2015), T-gamma{Y} (Alzaatreh *et al.*, 2016a), T-Cauchy{Y} (Alzaatreh *et al.*, 2016b), and Logistic-X (Tahir & Cordeiro 2016).

This study confidently adopts the Weibull-G family of distributions (Bourguignon *et al.*, 2014) to transform the three parameters of the Lomax Distribution (LD) (Rajab *et al.*, 2013), resulting in a groundbreaking compound distribution that we are named the "New Weibull Lomax Distribution

(NWLD) with five parameters." By adding an additional parameter to the established Weibull Lomax Distribution (WLD) with four parameters (Tahir *et al.*, 2015), we aim to ensure that this novel distribution outperforms its competitors. It will provide enhanced efficiency for analyzing reliability and lifetime data challenges in engineering and survival analysis, offering significant advantages across relevant disciplines. This development positions the NWLD as a robust alternative to the LD, Weibull, and other distributions with similar characteristics.

MATERIALS AND METHODS

The Weibull-G Family of Distribution

Consider a continuous distribution, $G(x;\xi)$, with density, $g(x;\xi)$, and the Weibull cumulative distribution function (CDF) given as:

$$F(X = x) = 1 - e^{-\alpha x^{\beta}};\ x, \alpha, \beta > 0 \qquad (1)$$

The random variable X in equation (1) was replaced with $\frac{G(x;\,\xi)}{\bar{G}(x;\,\xi)}$, where $\bar{G}(x;\ \xi) = 1 - G(x;\ \xi)$, and defined their class of distributions, say, Weibull–G(α, β, ξ) such that α is the scale parameter and β being the shape parameter (Bourguignon *et al.*, 2014), hence defined as:

$$F(x;\alpha,\beta,\xi) = \alpha\beta \int_0^{\left[\frac{G(x;\,\xi)}{\bar{G}(x;\,\xi)}\right]} x^{\beta-1}\, e^{-\alpha x^{\beta}}\, dx$$

$$= 1 - e^{-\alpha\left[\frac{G(x;\,\xi)}{\bar{G}(x;\,\xi)}\right]^{\beta}};\ x \in \mathbb{R}, \alpha, \beta > 0 \qquad (2)$$

Where $G(x;\xi)$ is the CDF of continuous baseline distribution, which depends on a parameter vector ξ. The Weibull–G(α,β,ξ) family probability density function (PDF) expressed as:

$$f(x;\alpha,\beta,\xi) = \alpha\beta g(x;\xi)\frac{\mathrm{G}(x;\,\xi)^{\beta-1}}{\bar{\mathrm{G}}(x;\,\xi)^{\beta+1}}e^{-\alpha\left[\frac{G(x;\,\xi)}{\bar{G}(x;\,\xi)}\right]^{\beta}} \qquad (3)$$

The Lomax Distribution

The CDF of LD with shape parameter a, and scale parameter λ, is given by:

$$G(y;a,\lambda) = 1 - \left(1 + \frac{y}{\lambda}\right)^{-a};\ y \geq 0, a, \lambda > 0 \qquad (4)$$

The PDF of LD is as follows:

$$g(y;a,\lambda) = \frac{a}{\lambda}\left(1 + \frac{y}{\lambda}\right)^{-(a+1)};\ y \geq 0, a, \lambda > 0 \qquad (5)$$

A three-parameter Lomax distribution had previously presented by Lemonte & Cordeiro (2013) which is an extension of the two-parameter presented in equation (5) using the transformation $y = \mu - x$; hence the equations (4) and (5) become:

Let X be a random variable that follows a three parameter Lomax distribution, i.e. $X \sim Lomax(a, \lambda, \mu)$, the CDF of X is given as thus:

$$G(x;a,\lambda,\mu) = 1 - \left[1 + \left(\frac{x-\mu}{\lambda}\right)\right]^{-a};\ \mu \leq x \leq \infty, a, \lambda, \mu > 0 \qquad (6)$$

Its corresponding PDF becomes:

$$g(x;a,\lambda,\mu) = \frac{a}{\lambda}\left[1 + \left(\frac{x-\mu}{\lambda}\right)\right]^{-(a+1)};\ \mu \leq x \leq \infty, a, \lambda, \mu > 0 \qquad (7)$$

The additional parameter μ is a location parameter which can be interpreted as the guarantee time before failure occurs.

RESULTS AND DISCUSSION

The Proposed New Weibull Lomax Distribution

As established by Bourguignon *et al.* (2014), this study confidently adopts the Weibull-G(α, β, ξ) distribution as the generator of the baseline distribution. The baseline distribution, specifically the LD with three parameters, is seamlessly integrated into equations (2) and (3) by substituting the formulations provided in equations (6) and (7). This process allows us to effectively derive the CDF and PDF of the proposed NWLD, with the specification $\mathrm{G}(x;\ \xi) = G(x;a,\lambda,\mu)$.

$$\frac{\mathrm{G}(x;\ \xi)}{\bar{\mathrm{G}}(x;\ \xi)} = \frac{1 - \left\{\frac{1}{\left[1 + \left(\frac{x-\mu}{\lambda}\right)\right]^{a}}\right\}}{1 - 1 + \left[1 + \left(\frac{x-\mu}{\lambda}\right)\right]^{-a}}$$

$$= \frac{\left\{\left[1+\left(\frac{x-\mu}{\lambda}\right)\right]^{a}-1\right\} \div \left[1+\left(\frac{x-\mu}{\lambda}\right)\right]^{a}}{1 \div \left[1+\left(\frac{x-\mu}{\lambda}\right)\right]^{a}}$$

$$= \frac{\left\{\left[1+\left(\frac{x-\mu}{\lambda}\right)\right]^{a}-1\right\}}{\left[1+\left(\frac{x-\mu}{\lambda}\right)\right]^{a}} \times \frac{\left[1+\left(\frac{x-\mu}{\lambda}\right)\right]^{a}}{1}$$

$$= \left[1+\left(\frac{x-\mu}{\lambda}\right)\right]^{a}-1 \qquad (8)$$

Thus, by substituting equation (8) in equation (2) to obtain the CDF of NWLD as:

$$\therefore F(x;\alpha,\beta,a,\lambda,\mu) = 1 - e^{-\alpha\left\{\left[1+\left(\frac{x-\mu}{\lambda}\right)\right]^{a}-1\right\}^{\beta}}; x \geq \mu, \mu \geq 0, a, \lambda, \alpha, \beta > 0 \qquad (9)$$

Then, by substituting equations (6), (7) and (8) in equation (3) to obtain the PDF of the NWLD, we have:

$$f(x;\alpha,\beta,\xi) = \alpha\beta\frac{a}{\lambda}\left[1+\left(\frac{x-\mu}{\lambda}\right)\right]^{-(a+1)} \frac{\left\{1-\left[1+\left(\frac{x-\mu}{\lambda}\right)\right]^{-a}\right\}^{\beta-1}}{\left\{1-\left\{1-\left[1+\left(\frac{x-\mu}{\lambda}\right)\right]^{-a}\right\}\right\}^{\beta+1}} e^{-\alpha\left\{\left[1+\left(\frac{x-\mu}{\lambda}\right)\right]^{a}-1\right\}^{\beta}}$$

$$f(x;\alpha,\beta,\xi) = \alpha\beta\frac{a}{\lambda}\left[1+\left(\frac{x-\mu}{\lambda}\right)\right]^{-(a+1)} \frac{\left\{1-\left[1+\left(\frac{x-\mu}{\lambda}\right)\right]^{-a}\right\}^{\beta-1}}{\left\{1-\left\{1-\left[1+\left(\frac{x-\mu}{\lambda}\right)\right]^{-a}\right\}\right\}^{\beta+1}} e^{-\alpha\left\{\left[1+\left(\frac{x-\mu}{\lambda}\right)\right]^{a}-1\right\}^{\beta}}$$

$$= \frac{\alpha\beta a}{\lambda}\left[1+\left(\frac{x-\mu}{\lambda}\right)\right]^{a\beta-1}\left\{1-\left[1+\left(\frac{x-\mu}{\lambda}\right)\right]^{-a}\right\}^{\beta-1} e^{-\alpha\left\{\left[1+\left(\frac{x-\mu}{\lambda}\right)\right]^{a}-1\right\}^{\beta}}$$

Let $p = 1 + \left(\frac{x-\mu}{\lambda}\right)$, then, we have:

$$f(p;\alpha,\beta,\xi) = \frac{\alpha\beta a}{\lambda} p^{a\beta-1}(1-p^{-a})^{\beta-1} e^{-\alpha(p^{a}-1)^{\beta}}$$

$$= \frac{\alpha\beta a}{\lambda}\left(\frac{p^{a\beta}}{p}\right)\left(\frac{p^{a}-1}{p^{a}}\right)^{\beta}\left(\frac{p^{a}}{p^{a}-1}\right) e^{-\alpha(p^{a}-1)^{\beta}}$$

$$= \frac{\alpha\beta a}{\lambda}\left(\frac{p^{a\beta}}{p}\right)\frac{(p^{a}-1)^{\beta}}{p^{a\beta}}\left(\frac{p^{a}}{p^{a}-1}\right) e^{-\alpha(p^{a}-1)^{\beta}}$$

$$= \frac{\alpha\beta a}{\lambda} p^{a-1}(p^{a}-1)^{\beta-1} e^{-\alpha(p^{a}-1)^{\beta}}$$

$$\therefore f(x;\alpha,\beta,\xi) = \frac{\alpha\beta a}{\lambda}\left[1+\left(\frac{x-\mu}{\lambda}\right)\right]^{a-1}\left\{\left[1+\left(\frac{x-\mu}{\lambda}\right)\right]^{a}-1\right\}^{\beta-1} e^{-\alpha\left\{\left[1+\left(\frac{x-\mu}{\lambda}\right)\right]^{a}-1\right\}^{\beta}};$$

$$x \geq \mu, \mu \geq 0, a, \lambda, \alpha, \beta > 0 \qquad (10)$$

Where: α and λ are the scale parameters; β and a are shape parameters; and μ is the location parameter.

The Validity Check of the Proposed NWLD

To check the validity of the NWLD, this will be done by integrating the equation (10) over the support: $x \geq \mu$, using the conditions for validating true PDF:

$$f(x) \geq 0; \ -\infty \leq x \leq \infty \qquad (11)$$

For any real values of x chosen arbitrarily from the support of the NWLD, $x \geq \mu$ such that $\mu \geq 0$, then, the PDF $f(x) = f(x;\alpha,\beta,a,\lambda,\mu) \geq 0$.

$$\int_{-\infty}^{\infty} f(x)dx = 1 \qquad (13)$$

This second condition can be shown as what follows from Left Hand Side (LHS) of the condition:

$$\int_{\mu}^{\infty} f(x;\alpha,\beta,a,\lambda,\mu)dx$$

$$= \int_{\mu}^{\infty}\left(\frac{\alpha\beta a}{\lambda}\left[1+\left(\frac{x-\mu}{\lambda}\right)\right]^{a-1}\left\{\left[1+\left(\frac{x-\mu}{\lambda}\right)\right]^{a}-1\right\}^{\beta-1} e^{-\alpha\left\{\left[1+\left(\frac{x-\mu}{\lambda}\right)\right]^{a}-1\right\}^{\beta}}\right)dx$$

$$= \frac{\alpha\beta a}{\lambda}\int_{\mu}^{\infty}\left(\left[1+\left(\frac{x-\mu}{\lambda}\right)\right]^{a-1}\left\{\left[1+\left(\frac{x-\mu}{\lambda}\right)\right]^{a}-1\right\}^{\beta-1} e^{-\alpha\left\{\left[1+\left(\frac{x-\mu}{\lambda}\right)\right]^{a}-1\right\}^{\beta}}\right)dx$$

The above can be transformed by setting $p = \left[1 + \left(\frac{x-\mu}{\lambda}\right)\right]$, then $dx = \lambda dp$; such that its new support becomes: $\underset{x\to\mu}{\ell im}\left[1 + \left(\frac{x-\mu}{\lambda}\right)\right] = 1$ and $\underset{x\to\infty}{\ell im}\left[1 + \left(\frac{x-\mu}{\lambda}\right)\right] = \infty$. Then,

$$\int_{\mu}^{\infty} f(x;\alpha,\beta,a,\lambda,\mu)dx = \frac{\alpha\beta a}{\lambda}\int_{1}^{\infty}\left(p^{a-1}(p^{a}-1)^{\beta-1}e^{-\alpha(p^{a}-1)^{\beta}}\right)\lambda dp$$

By transforming again, we let $Q = p^{a} - 1$, then $dQ = ap^{a-1}dp$, such that the support becoming: $\underset{p\to 1}{\ell im}(p^{a}-1) = 0$ and $\underset{p\to\infty}{\ell im}(p^{a}-1) = \infty$. Then,

$$= \alpha\beta a\int_{0}^{\infty}\left(p^{a-1}Q^{\beta-1}e^{-\alpha Q^{\beta}}\right)\frac{dQ}{ap^{a-1}}$$

$$= \alpha\beta\int_{0}^{\infty}\left(Q^{\beta-1}e^{-\alpha Q^{\beta}}\right)dQ$$

Also, let $R = Q^{\beta}$, then $dR = \beta Q^{\beta-1}dQ$, such that the support becomes: $\underset{Q\to 0}{\ell im}\, Q^{\beta} = 0$ and $\underset{Q\to\infty}{\ell im}\, Q^{\beta} = \infty$. Then,

$$= \alpha\beta\int_{0}^{\infty}\left(Q^{\beta-1}e^{-\alpha R}\right)\frac{dR}{\beta Q^{\beta-1}}$$

$$= \alpha\int_{0}^{\infty} e^{-\alpha R}dR$$

Recall that $\int_{0}^{\infty}\alpha e^{-\alpha R}dR = 1$, that is, an exponential distribution. Hence,

$$= \alpha\left(\frac{1}{\alpha}\right)$$

$$\therefore \int_{\mu}^{\infty} f(x;\alpha,\beta,a,\lambda,\mu)dx = 1$$

CONCLUSION

In this study, we confidently present a continuous probability distribution that enhances the four-parameter Weibull Lomax distribution. We have effectively derived the density and cumulative functions of the new distribution, known as the NWLD. Our rigorous validation process demonstrates that this proposed distribution is a true probability density function (PDF). We are currently conducting further research to decisively establish whether the NWLD surpasses its key competitors, specifically the four-parameter model proposed by Tahir *et al.* (2015) and the five-parameter model developed by Afify *et al.* (2015).

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