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SERIES SOLUTION FOR THE COMPLETE GOLDEN DYNAMICAL EQUATION OF MOTION FOR THE PHOTON

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ABSTRACTS
In a paper (Howusu, 2004) the complete Golden Dynamical Equation of motion for photon in the gravitational field of a massive body was published. In this paper the series method is used to solve this equation for comparison with the solutions of Einstein Equation for the photon in the same gravitational field. A value of 1.875" was found as the total deflection angle.

Keywords: Gravitational Field, Light, Deflection

INTRODUCTION

In 1916, Einstein, in his Theory of General Relativity (GR) published that light coming from a star and passing near the sun will be deviated by the sun's gravitational field. According to Einstein, the equation for the trajectory of a light ray in a spherically symmetric static gravitational field is given by

$$\frac{d^2u}{d\emptyset^2} + u = \frac{3k_s u^2}{c^2} \qquad (1)$$

$where\ k_s$ = GM

G is the Gravitational constant.

The first exact solution of Einstein field equations was constructed by Karl Schwarzchild using the method of successive approximation.

He derived the full solution of the differential equation as

$$U = \frac{1}{r}sin\emptyset + \frac{2GM}{c^2r^2} - \frac{GM}{c^2r^2}sin^2\emptyset \qquad (2)$$

Where r_0 is the closest approach to the origin (or impact parameter) as shown in Figure

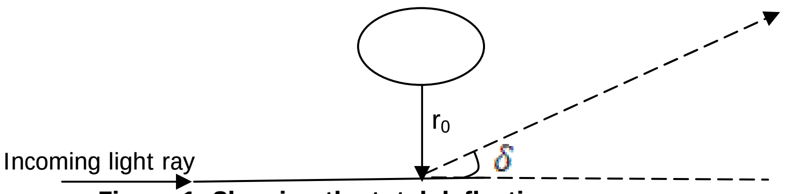

Figure 1: Showing the total deflection

When

$r \to \pm\infty, U \to 0,$

So

$$0 = \frac{1}{r}sin\emptyset + \frac{2GM}{c^2r^2} - \frac{GM}{c^2r^2}sin^2\,\emptyset \qquad (3)$$

at the asymptotes $\emptyset = -\psi_2\ and\ \phi = \psi_1 + \pi$ and taking ϕ < < 1 we get

$$0 = -\frac{1}{r}\psi_1 + \frac{2GM}{c^2r^2} \qquad (4)$$

$$0 = -\frac{1}{r}\psi_2 + \frac{2GM}{c^2r^2} \qquad (5)$$

The total deflection is $\delta = \psi_1 + \psi_2$

or

$$\delta = \frac{4GM}{rc^2} \qquad (6)$$

This works out to be above 1.75". Attempts to observe such a deflection were first carried out by the British solar eclipse expeditions of 1919 and have continued down to the present time.

In a paper "On the Gravitation of Moving Bodies" Physics Essays, volume 4, number 1, 1991. Howusu developed the general equation of motion for a photon in a general gravitational field ϕ_g according to the theories of general mechanics and gravitation as:

$$\frac{d}{dt}u(r,t) - \frac{1}{c^2}\left\{\left[1 + \frac{1}{c^2}\phi_g(r,t)\right]\left|\frac{d}{dt}\phi_g(r,t)\right.\right\}u(r,t) = -\nabla\phi_g(r,t) \quad (7)$$

Also in a paper "The Golden Dynamical Equation of Motion for particles of Nonzero Rest Mass in Gravitational fields" Physics Essays Volume 17, number 3, 2004, Howusu developed the Golden Dynamical Equation of Motion (GDEM) for particles of nonzero rest mass in gravitational fields as:

$$\frac{d}{dt}\left[\left(1+\frac{1}{c^2}\phi_g\right)^{-1}u\right] = -\left(1+\frac{1}{c^2}\phi_g\right)^{-1}\nabla\phi_g \quad (8)$$

Which may equivalently be written as:

$$\frac{d}{dt}u - \frac{1}{c^2}\left(1+\frac{1}{c^2}\right)^{-1}\left(\frac{d}{dt}\phi_g\right)u = -\nabla\phi_g \quad (9)$$

This equation satisfies the principle of equivalence and it completes the corresponding pure Newtonian equation with correction terms and hence effects of all orders of c^{-2}. In the paper it was shown how this equation resolves not only the problem of gravitational deflection of the photon, but also its frequency shift as well. It is however known today that the sun is better described as an oblate spheroidal body (Howusu and Uduh, 2003. Einstein's gravitational field equations exterior and interior to an oblate spheroidal body, Journal of the Nigerian Association of Mathematical Physics 7). In view of this we intend to use the analytical methods to explain the phenomenon of the gravitational deflection of light.

In a paper (Howusu, 2004) the complete dynamic equation of motion for particles of non-zero rest mass in gravitational field was developed as

$$\frac{d^2u}{d\emptyset^2} - \frac{k}{c^2}\left(\frac{du}{d\emptyset}\right)^2 + u = \frac{2k}{c^2}u^2 + \frac{k^2}{c^4}u^3 \quad (10)$$

In a paper (Izam and Makama, 2008), the series method was used to solve Einstein's photon equation. In this paper we again use the series method to solve the complete Dynamical equation of motion for the photon in gravitational fields of a massive body such as the sun.

MATHEMATICAL DEDUCTIONS

We use the analytical method and seek a series solution of the form

$$u(\emptyset) = \sum_{n=0}^{\infty} A_n e^{inw_0\emptyset} \quad (11)$$

Where A_n, W_o are constant coefficients to be determined.
It follows that

$$\frac{du}{d\emptyset} = inw_0 A_n e^{inw_0\emptyset} \quad (12)$$

$$\frac{d^2u}{d\emptyset^2} = -n^2 w^2{}_0 A_n e^{inw_0\emptyset} \quad (13)$$

Substituting equations (11), (12) and (13) into (10) and comparing coefficients of n, we obtain the following results.

$$A_0 = \frac{c^2}{k}(\sqrt{2}-1) or \quad (14)$$

$$A_0 = -\frac{c^2}{k}(\sqrt{2}-1) \quad (15)$$

If

$$A_0 = -\frac{c^2}{k}(\sqrt{2}-1), w_0^2 = (\sqrt{2}-2) \quad (16)$$

If

$$A_0 = \frac{c^2}{k}(\sqrt{2}-1), w_0^2 = (\sqrt{2}+2) \quad (17)$$

Then, for

$$A_o = \frac{c^2}{k}(\sqrt{2} - 1) \quad (18)$$

$$A_2 = 0.2155\frac{k}{c^2}A_1^2 \quad (19)$$

For

$$A_o = -\frac{c^2}{k}(\sqrt{2} - 1) \quad (20)$$

$$A_2 = 0.0774\frac{k}{c^2}A_1^2 \quad (21)$$

A_1 is an arbitrary constant. All the other coefficients of A are written in terms of A_1.

RESULTS

Our equation (11) will then have two solutions as follows

For $A_0 = \frac{c^2}{k}(\sqrt{2}-1), w_0^2 = (\sqrt{2} - 2)$

$$u(\phi) = A_1 e^{iw_0\emptyset} + A_2 e^{2iw_0\emptyset} + \ldots\ldots\ldots\ldots\ldots$$

$$= A_1 e^{iw_0\emptyset} + 0.2155\frac{k}{c^2}A_1{}^2 e^{2iw_0\emptyset} + \ldots\ldots\ldots \quad (22)$$

For, $A_0 = \frac{-c^2}{k}(\sqrt{2} - 1)$

$$u(\emptyset) = A_1 e^{iw_0\emptyset} + 0.0774\frac{k}{c^2}A_1{}^2 e^{2iw_0\emptyset} + \ldots\ldots\ldots \quad (23)$$

These are series solutions that give correction terms to the Einstein's photon equation. A value of 1.875" was found which is better than the value of 1.750" as predicted by Einstein's Relativity theory.

CONCLUSION

We conclude as follows:

i. The complete Golden Dynamical Equation of Motion for the photon in gravitational fields contains some additional correction terms to the Einstein photon equation.

ii. The calculated coefficients slightly defer from the coefficients calculated for the Einstein photon equation.

iii. This effect will certainly introduce additional corrections to the total deflection angle of the photon in gravitational fields.

iv. Our method is versatile enough to generate the solutions for the photon equation for any power of the velocity variable.

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