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A NEW GENERALIZED EXPONENTIAL-WEIBULL DISTRIBUTION; ITS PROPERTIES AND APPLICATION

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ABSTRACT
A four-parameter distribution called Generalized Exponential Weibull (GEW) distribution was proposed using a generator introduced. We derived the statistical properties including moment, moment generating function, survival and hazard function, order statistics of the distribution. The distribution parameters were estimated using the method of maximum likelihood and also the asymptotic behavior of the distribution was observed. Applicability of the distribution was also observed using a real data set.
Keywords: Exponential distribution, Exponential Weibull distribution, Reliability analysis, Order statistics and Parameter estimation

INTRODUCTION

Several classical distributions have been widely used over the past decades for modelling data in areas of research such as reliability, engineering, economics, biological studies, environmental, medical sciences, demography etc. However, in many applied areas such as lifetime analysis, finance and insurance, there is a need to extend these distributions. This is because, there exist some lifetime problems where the real data does not follow any of the classical or standard probability distributions. For that reason, numerous methods for generating new probability distributions have been discussed (Bourguignon, 2014). To handle this, there is strong need to propose useful models for better study of the real-life phenomenon. There has been an enlarged interest in developing new univariate continuous distributions by adding new shape parameters to the baseline model. This induction of parameter(s) has been proved useful in discovering tail properties and also for improving the goodness-of-fit of the proposed generator family (Saboor *et al*., 2015).

Weibull distribution is widely used in analyzing life-time data mainly because in presence of censoring, it is much easier to handle as numerically compared to gamma distribution. It has the two parameter i.e. scale and shape parameter and also has increasing and decreasing failure rates depending on the shape parameter. As one of the disadvantage of the distribution, its maximum likelihood estimators has a very low asymptotic convergence to normality (Bain, 1997). The approach of generalizing probability distribution was first introduced by Mashall and Olkin (1997) suggested by adding one parameter to the survival function $G(x)=1-G(x)$, where $G(x)$ is the cumulative distribution function of the baseline distribution.

Exponentiated-G class of distributions was proposed by Gupta *et al.* (1998) which introduces one parameter to the cumulative distribution function of any univariate continuous probability distribution. Following Gupta *et al.,(*1998), Gupta and Kundu(1999)proposed two parameter generalized-exponential (GE) distribution as generalization of the exponential distribution. The GE distribution is also known as Exponentiated Exponential (EE) distribution. A three-parameter Exponential-Weibull (EW) distribution was proposed by Cordeiro*et al., (*2013)for modelling lifetime data. The mathematical properties of the Exponential-Weibull distribution were investigated and it was observed that the distribution fitted the subject data better than other distributions compared in the literature. Other generalization of exponential distribution can be found in Exponentiated-Exponential by Gupta and Kundu (2001), Exponentiated-Weibull distribution by Pal *et al.,*(2006), A Generalization of the Weibull Probability distribution by Ayal and Tsokos (2011), Transmuted-Exponential distribution by Saboor *et al.,*(2015), and The New Kumaraswamy-Exponential-Weibull distribution by Cordeiro *et al.,*(2016) among others. Inclusion, our interest here is to propose a new probability function which can serve as a generalization of Exponential-Weibull distribution, called Generalized-Exponential-Weibull (GEW) distribution by addition of one shape parameter to the Exponential-Weibull distribution using a generator called Generalized-G proposed by Gupta *et al.,*(1998).

MATERIALS AND METHODS

In this section, the methods used in deriving the Generalized-Exponential-Weibull distribution are clearly explained.

The Modified EW (Generalized Exponential-Weibull) Distribution

Let X be the lifetime random variable representing the minimum of the two random variables for Exponential and Weibull distributions. Hence, we define the CDF and the pdf of the modified EW distribution named Generalized Exponential-Weibull (GEW) distribution as follows:

$$F(x) = [1 - e^{-(\alpha x + \beta x^{\lambda})}]^{\gamma}\,, \tag{5}$$

and

$$f(x) = \gamma(\alpha + \beta\lambda x^{\lambda-1})e^{-(\alpha x+\beta x^{\lambda})}[1 - e^{-(\alpha x+\beta x^{\lambda})}]^{\gamma-1} \tag{6}$$

Respectively, where $\gamma>0$ is the additional shape parameter and $\alpha>0$, $\lambda>0$, $\beta>0$ are the base line parameters.

Special Case:

For $\gamma=1$, $\beta=0$ in equation (5), the proposed distribution reduce to Exponential distribution. For $\beta=0$ in equation (5), it reduce to Generalized Exponential (GE) distribution. For $\gamma=1$ in equation (5), it reduce to Exponential Weibull (EW) distribution.

Figure 1 and 2 provide the CDF and PDF plots of the GEW distribution for some selected values.

Some Basic Properties of the GEW Distribution

The reliability function $S(x)$ and hazard function $H(x)$ are respectively given by

$$S(x) = 1 - F(x) = 1 - [1 - e^{-(\alpha x+\beta x^{\lambda})}]^{\gamma} \tag{7}$$

and

$$H(x) = \frac{f(x)}{1-F(x)} = \frac{\gamma(\alpha+\beta\lambda x^{\lambda-1})e^{-(\alpha x+\beta x^{\lambda})}[1-e^{-(\alpha x+\beta x\lambda)}]^{\gamma-1}}{1-[1-e^{-(\alpha x+\beta x\lambda)}]^{\gamma}} \tag{8}$$

respectively. After further simplification, equation (8) can be written as

$$= \frac{\gamma f_{EW}(x)F_{GEW}(x)}{F_{EW}(x)[1-F_{GEW}(x)]}$$

This clearly shows that the pdf of the proposed distribution can be express as

$$f_{GEW}(x) = \frac{\gamma f_{EW}(x)F_{GEW}(x)}{F_{EW}(x)} \tag{10}$$

Plots of the reliability function and hazard function of GEW distribution for some selected values of the parameters α, β, λ, and γ are given in the figure 3 and 4 below.

Asymptotic Behavior

In this subsection, we observed the behavior of our proposed model given in equation (6) as $x \to 0$ and as $x \to \infty$. This includes considering $\lim_{x\to 0} f(x)$ and $\lim_{x\to\infty} f(x)$.

$$\lim_{x\to 0} f(x) = \lim_{x\to 0}(\gamma[1-e^{-(\alpha x+\beta x^{\lambda})}]^{\gamma-1}(\alpha+\beta\lambda x^{\lambda-1})e^{-(\alpha x+\beta x^{\lambda})}) = 0 \tag{11}$$

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty}(\gamma[1-e^{-(\alpha x+\beta x^{\lambda})}]^{\gamma-1}(\alpha+\beta\lambda x^{\lambda-1})e^{-(\alpha x+\beta x^{\lambda})}) = 0 \tag{12}$$

Moment

Theorem: The rth moment of the random variable $X \sim GEW(\alpha,\beta,\lambda,\gamma)$ is given by

$$E(X^r) = \frac{\sum_{0=j}^{\infty}\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}(-1)^{j+k+m}\Gamma(\gamma)}{j!k!m!\Gamma(\gamma-j)}\alpha^m\{\gamma\alpha\frac{\Gamma\left(\frac{r+k+m+-2\lambda}{\lambda}\right)}{\lambda[\beta(1+j)]^{\frac{r+k+m+1}{\lambda}}} + \beta\frac{\Gamma\left(\frac{r+m-\lambda}{\lambda}\right)}{[\beta(1+j)]^{\frac{r+\lambda+m}{\lambda}}}\}$$

Proof: The rth moment of a random variable *X* with pdf $f(x)$ is defined by

$$\mu_r' = \int_0^{\infty} x^r f(x)dx \tag{14}$$

Substituting equation (6) in equation (14), we have

$$E(X^r) = \int_0^{\infty} x^r\gamma\alpha e^{-(\alpha x+\beta x^{\lambda})}[1-e^{-(\alpha x+\beta x^{\lambda})}]^{\gamma-1}dx + \int_0^{\infty} x^r\beta\lambda x^{\lambda-1}e^{-(\alpha x+\beta x^{\lambda})}[1-e^{-(\alpha x+\beta x^{\lambda})}]^{\gamma-1}$$

$$= \gamma\alpha\int_0^{\infty} x^r e^{-(\alpha x+\beta x^{\lambda})}[1-e^{-(\alpha x+\beta x^{\lambda})}]^{\gamma-1}dx + \beta\lambda\int_0^{\infty} x^{r+\lambda-1}e^{-(\alpha x+\beta x^{\lambda})}[1-e^{-(\alpha x+\beta x^{\lambda})}]^{\gamma-1}dx \tag{15}$$

Applying binomial expansion and further simplification, equation (15) become

$$E(X^r) = \gamma\alpha\int_0^\infty x^r e^{-(\alpha x+\beta x^\lambda)} \sum_{j=0}^\infty \frac{(-1)^j \Gamma(\gamma)}{j!\Gamma(\gamma-j)} \sum_{k=0}^\infty \frac{(-1)^k (\alpha j)^k x^k}{k!} e^{-j\beta x^\lambda} dx +$$

$$\beta\lambda\int_0^\infty x^{r+\lambda-1} e^{-(\alpha x+\beta x^\lambda)} \sum_{j=0}^\infty \frac{(-1)^j \Gamma(\gamma)}{j!\Gamma(\gamma-j)} \sum_{k=0}^\infty \frac{(-1)^k (\alpha j)^k x^k}{k!} e^{-j\beta x^\lambda} dx$$

$$= \sum_{j=0}^\infty \sum_{k=0}^\infty \frac{(-1)^{j+k} \Gamma(\gamma)(\alpha j)}{j!k!\Gamma(\gamma-j)} \gamma\alpha\int_0^\infty x^{r+k} e^{-(\alpha x+\beta x^\lambda)} e^{-j\beta x^\lambda} dx +$$

$$\beta\lambda \sum_{j=0}^\infty \sum_{k=0}^\infty \frac{(-1)^{j+k} \Gamma(\gamma)(\alpha j)}{j!k!\Gamma(\gamma-j)} \int_0^\infty x^{r+\lambda-1} e^{-(\alpha x+\beta x^\lambda)} - j\beta x^\lambda dx \tag{16}$$

but

$$e^{-(\alpha x+\beta x^\lambda)} = e^{-\alpha x} \times e^{-\beta x^\lambda} \tag{17}$$

and using McLaurin's series expansion

$$e^{-\alpha x} = \sum_{m=0}^\infty (-1)^m \times \frac{\alpha^m x^m}{m!} \tag{18}$$

putting equation (18) in equation (16), we have

$$E(X^r) = \sum_{j=0}^\infty \sum_{k=0}^\infty \frac{(-1)^{j+k} \Gamma(\gamma)}{j!k!\Gamma(\gamma-j)} \{\gamma\alpha\int_0^\infty x^{r+k} \sum_{m=0}^\infty (-1)^m \frac{\alpha^m x^m}{m!} e^{-\beta x^\lambda (1+j)} dx +$$

$$\beta\lambda\int_0^\infty x^{r+\lambda-1} \sum_{m=0}^\infty (-1)^m \frac{\alpha^m x^m}{m!} e^{-\beta x^\lambda (1+j)} dx\}$$

$$= \sum_{j=0}^\infty \sum_{k=0}^\infty \sum_{m=0}^\infty \frac{(-1)^{j+k+m} \Gamma(\gamma)}{j!k!m!\Gamma(\gamma-j)} \{\gamma\alpha^{m+1} \int_0^\infty x^{r+k+m} e^{-\beta x^\lambda (1+j)} dx + \beta\lambda\alpha^m \int_0^\infty x^{r+\lambda+m-1} e^{-\beta x^\lambda (1+j)} dx \tag{19}$$

Letting

$$w = \beta x^\lambda (1+j)$$

$$x^\lambda = \frac{w}{\beta(1+j)} \quad \Rightarrow \frac{w^{1/\lambda}}{[\beta(1+j)]^{1/\lambda}} = x$$

$$x = \frac{w^{1/\lambda}}{[\beta(1+j)]^{1/\lambda}}$$

then,

$$\frac{dx}{dw} = \frac{1/\lambda w^{\frac{1-\lambda}{\lambda}}}{[\beta(1+j)]^{1/\lambda}}$$

implying that

$$dx = \frac{w^{\frac{1-\lambda}{\lambda}}}{\lambda[\beta(1+j)]^{1/\lambda}} dw \tag{20}$$

putting equation (20) in equation (19), leads to

$$E(X^r) = \frac{\sum_{0=j}^\infty \sum_{k=0}^\infty \sum_{m=0}^\infty (-1)^{j+k+m} \Gamma(\gamma)}{j!k!m!\Gamma(\gamma-j)} [(\gamma \times \alpha^{m+1} \int_0^\infty \frac{w^{\frac{r+k+m}{\lambda}}}{[\beta(1+j)]^{\frac{r+k+m}{\lambda}}} e^{-w} \times \frac{w^{\frac{1-\lambda}{\lambda}}}{\lambda[\beta(1+j)]^{1/\lambda}} +$$

$$\beta\lambda\alpha^m \int_0^\infty \frac{w + \lambda + m - \frac{1}{\lambda}}{[\beta(1+j)]^{r+\lambda+m-\frac{1}{\lambda}}} e^{-w} \times \frac{w^{\frac{1-\lambda}{\lambda}}}{\lambda[\beta(1+j)]^{\frac{1}{\lambda}}} dw] \tag{21}$$

After further simplification equation (21) become

$$E(X^r) = \frac{\sum_{0=j}^{\infty}\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}(-1)^{j+k+m}\Gamma(\gamma)}{j!k!m!\Gamma(\gamma-j)}\alpha^m\{\gamma\alpha\frac{\Gamma\left(\frac{r+k+m+-2\lambda}{\lambda}\right)}{\lambda[\beta(1+j)]^{\frac{r+k+m+1}{\lambda}}}+\beta\frac{\Gamma\left(\frac{r+m-\lambda}{\lambda}\right)}{[\beta(1+j)]^{\frac{r+\lambda+m}{\lambda}}}\}$$

Hence proved.

Moment Generating Function

The moment generating function (mgf) of the random variable X that follows GEW distribution having probability density function (pdf), $f(x)$ given in equation (6) is given by

$$M_x(t) = E(e^{tx}) = \int_{-\infty}^{\infty} e^{tx} f(x)dx \tag{22}$$

Theorem: the MGF of the random variable $X \sim GEW(\alpha,\beta,\lambda,\gamma)$ is given by

$$E(e^{tx}) = \sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{r=0}^{\infty}\frac{(-1)^{j+k+m}\Gamma(\gamma)}{j!k!m!\Gamma(\gamma-j)}\frac{(t)^r}{r!}\alpha^m\{\gamma\alpha\frac{\Gamma\left(\frac{r+k+m+-2\lambda}{\lambda}\right)}{\lambda[\beta(1+j)]^{\frac{r+k+m+1}{\lambda}}}+\beta\frac{\Gamma\left(\frac{r+m-\lambda}{\lambda}\right)}{[\beta(1+j)]^{\frac{r+\lambda+m}{\lambda}}}\}$$

Proof: The mgf of a random variable X with pdf $f(x)$ is defined by

$$M_x(t) = E(e^{tx}) = \int_{-\infty}^{\infty} e^{tx} f(x)dx \tag{23}$$

This can be obtained by replacing X^r with e^{tx} in equation (19). i.e.

$$E(e^{tx}) = \frac{\sum_{0=j}^{\infty}\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}(-1)^{j+k+m}\Gamma(\gamma)}{j!k!m!\Gamma(\gamma-j)}[(\gamma\times\alpha^{m+1}\int_0^{\infty} e^{tx}x^{k+m}e^{-\beta x^r(1+j)})dx]+ \tag{24}$$

$$[\beta\alpha^m\lambda\int_0^{\infty} e^{tx}x^{\lambda+m-1}e^{-\beta x^{\lambda}(1+j)}dx]$$

but,

$$e^{tx} = \sum_{r=0}^{\infty}\frac{(tx)^r}{r!} \tag{25}$$

Putting equation (25) in equation (24), it leads

$$E(e^{tx}) = \frac{\sum_{0=j}^{\infty}\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}(-1)^{j+k+m}\Gamma(\gamma)}{j!k!m!\Gamma(\gamma-j)}\sum_{r=0}^{\infty}\frac{(t)^r}{r!}[(\gamma\times\alpha^{m+1}\int_0^{\infty} x^{r+k+m}e^{-\beta x^r(1+j)})dx]+ \tag{26}$$

$$[\beta\alpha^m\lambda\int_0^{\infty} x^{r+\lambda+m-1}e^{-\beta x^{\lambda}(1+j)}dx]$$

Letting

$$w = \beta x^{\lambda}(1+j) \Rightarrow \frac{w}{[\beta(1+j)]} = x^{\lambda}$$

$$\Rightarrow \frac{w^{1/\lambda}}{[\beta(1+j)]^{1/\lambda}} = x$$

$$\frac{dx}{dw} = \frac{1/\lambda w^{\frac{1-\lambda}{\lambda}}}{[\beta(1+j)]^{1/\lambda}}$$

$$dx = \frac{w^{\frac{1-\lambda}{\lambda}}}{\lambda[\beta(1+j)]^{1/\lambda}}dw \tag{27}$$

$$E(e^{tx})=\frac{\sum_{0=j}^{\infty}\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}(-1)^{j+k+m}\Gamma(\gamma)}{j!k!m!\Gamma(\gamma-j)}\sum_{r=0}^{\infty}\frac{(t)^r}{r!}[(\gamma\times\alpha^{m+1}\int_0^{\infty}\frac{w^{\frac{r+k+m}{\lambda}}}{[\beta(1+j)]^{\frac{r+k+m}{\lambda}}}e^{-w}\times\frac{w^{\frac{1-\lambda}{\lambda}}}{\lambda[\beta(1+j)]^{1/\lambda}}+$$

$$\beta\lambda\alpha^m\int_0^{\infty}\frac{w+\lambda+m-\frac{1}{\lambda}}{[\beta(1+j)]^{r+\lambda+m-\frac{1}{\lambda}}}e^{-w}\times\frac{w^{\frac{1-\lambda}{\lambda}}}{\lambda[\beta(1+j)]^{\frac{1}{\lambda}}}dw] \quad (28)$$

but

$$\int_0^{\infty}w^{\frac{r+k+m+1-\lambda}{\lambda}}e^{-w}dw=\Gamma\left(\frac{r+k+m+1-\lambda}{\lambda}-1\right)=\Gamma\left(\frac{r+k+m-2\lambda}{\lambda}\right) \quad (29)$$

also,

$$\int_0^{\infty}w^{\frac{r+\lambda+m-1+1-\lambda}{\lambda}}e^{-w}dw=\Gamma\left(\frac{r+m}{\lambda}-1\right)=\Gamma\left(\frac{r+m-\lambda}{\lambda}\right) \quad (30)$$

Putting equation (29) and (30) in equation (28) it gives

$$E(e^{tx})=\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{r=0}^{\infty}\frac{(-1)^{j+k+m}\Gamma(\gamma)}{j!k!m!\Gamma(\gamma-j)}\frac{(t)^r}{r!}\alpha^m\{\gamma\alpha\frac{\Gamma\left(\frac{r+k+m+-2\lambda}{\lambda}\right)}{\lambda[\beta(1+j)]^{\frac{r+k+m+1}{\lambda}}}+\beta\frac{\Gamma\left(\frac{r+m-\lambda}{\lambda}\right)}{[\beta(1+j)]^{\frac{r+\lambda+m}{\lambda}}}\} \quad (31)$$

Hence proved.

Order Statistics

Order Statistics are applied in wide range of problems including robust statistical estimation, characterization of probability model and goodness of fit tests, entropy estimation and analysis of censored samples, reliability analysis and quality control.

Suppose $X_{1:n}, X_{2:n}, ..., X_{i:n}$ is a random sample from distribution with pdf $f(x)$ and let $X_{1:n}, X_{2:n}, ..., X_{i:n}$ denote the corresponding order statistics obtained from this sample. The pdf $f_{i:n}(x)$ of the i[th]order statistics can be express as

$$f_{i:n}(x)=\frac{n!}{(i-1)^!(n-1)^!}f(x)[F(x)]^{i-1}[1-F(x)]^{n-1} \quad (32)$$

$$f_{i:n}(x)=\frac{n!}{(i-1)^!(n-1)^!}\gamma(\alpha+\beta\lambda x^{\lambda-1})e^{-(\alpha x+\beta x^{\lambda})}[1-e^{-(\alpha x+\beta x^{\lambda})}]^{\gamma-1}$$

$$\times[1-e^{-(\alpha x+\beta x^{\lambda})}]^{\gamma(i-1)}\{1-[1-e^{-(\alpha x+\beta x^{\lambda})}]^{\gamma}\}^{n-1} \quad (33)$$

$$=\frac{n!}{(i-1)^!(n-1)^!}\gamma(\alpha+\beta\lambda x^{\lambda-1})e^{-(\alpha x+\beta x^{\lambda})}\{1-[1-e^{-(\alpha x+\beta x^{\lambda})}]^{\gamma}\}^{n-1}\times\sum_{j=0}^{\gamma i-1}\binom{\gamma i-1}{j}(-1)^j e^{-j(\alpha x+\beta x^{\lambda})}$$

but

$$e^{-[(\alpha x+\beta x^{\lambda})+j(\alpha x+\beta x^{\lambda})]}=e^{-(1+j)(\alpha x+\beta x^{\lambda})}=\sum_{k=0}^{\infty}\frac{(-1)^k(1+j)^k(\alpha x+\beta x^{\lambda})^k}{k^!}$$

this implies that equation (19) can be written as

$$f_{i:n}(x)=\frac{n!\gamma}{(i-1)^!(n-1)^!k^!}\gamma(\alpha+\beta\lambda x^{\lambda-1})\{1-[1-e^{-(\alpha x+\beta x^{\lambda})}]^{\gamma}\}^{n-1}$$

$$\times\sum_{j=0}^{\gamma i-1}\sum_{k=0}^{\infty}\binom{\gamma i-1}{j}(-1)^{j+k}(1+j)^k(\alpha x+\beta x^{\lambda})^k \quad (34)$$

Parameter Estimation

In this section, we employ the method of maximum likelihood to estimate the parameter of the GEW distribution. Let $X_1, ..., X_n$ be a random sample from GEW distribution with unknown parameter vector $\omega=(\alpha,\beta,\lambda,\gamma)$. The likelihood function ***L*** is given by

$$L(\omega) = \ln \prod_{i=1}^{n} \{\gamma(\alpha + \beta\lambda x^{\lambda-1}) e^{-(\alpha x + \beta x^{\lambda})} [1 - e^{-(\alpha x + \beta x^{\lambda})}]^{\gamma-1}\} \tag{35}$$

$$= n\ln\gamma + n\ln\alpha + n\ln\beta + n\ln\lambda + (\lambda-1)\sum_{i=1}^{n}\ln x - \sum_{i=1}^{n}(\alpha x + \beta x^{\lambda}) +$$

$$(\gamma-1)\sum_{i=1}^{n}\ln(1 - e^{-(\alpha x + \beta x^{\lambda})})$$

The estimate of the parameters where obtained by taking a partial derivative of $L(\omega)$ with respect to each parameter as respectively given below

$$\frac{\partial\omega}{\partial\gamma} = \frac{n}{\gamma} + \sum_{i=1}^{n}\ln(1 - e^{-(\alpha x + \beta x^{\lambda})})$$

$$\frac{\partial\omega}{\partial\alpha} = \frac{n}{\alpha} - \sum_{i=1}^{n} x + (\gamma-1)\sum_{i=1}^{n}\frac{(\alpha + \beta\lambda x^{\lambda-1})e^{-(\alpha x + \beta x^{\lambda})}}{1 - e^{-(\alpha x + \beta x^{\lambda})}}$$

$$\frac{\partial\omega}{\partial\beta} = \frac{n}{\beta} - \sum_{i=1}^{n} x^{\lambda} + (\gamma-1)\sum_{i=1}^{n}\frac{x^{\lambda}(\alpha + \beta\lambda x^{\lambda-1})}{1 - e^{-(\alpha x + \beta x^{\lambda})}}$$

$$\frac{\partial\omega}{\partial\lambda} = \frac{n}{\lambda} + \sum_{i=1}^{n}\ln x - \sum_{i=1}^{n}\beta x^{\lambda}\ln x + (\gamma-1)\sum_{i=1}^{n}\frac{\beta x^{\lambda}\ln x e^{-(\alpha x + \beta x^{\lambda})}}{1 - e^{-(\alpha x + \beta x^{\lambda})}}$$

The estimate of the unknown parameters can be obtain by setting the vector (ω) to zero i.e $\partial(\hat{\omega}) = 0$ and solving them simultaneously. These equations cannot be solved analytically, statistical software (e.g. R package) can be used to solve the equations numerically by means of iterative technique such as the Newton-Raphson algorithm.

Application

In this section, we provide an application of the developed GEW distribution to show its usefulness to fit real data set. The GEW distribution is compared with other related distributions, namely Exponential-Weibull (EW) and Kumaraswamy Exponential-Weibull (KwEW) distribution whose pdf's are given as follows:

- The density function of the EW distribution is given by

$$f(x) = (\alpha + \beta\lambda x^{\lambda-1})e^{-(\alpha x + \beta X)}$$

$$where\ \alpha > 0, \beta > 0, \lambda > 0$$

- The density function of the KwEW distribution is given by

$$f(x) = \alpha\gamma(\lambda + k\beta x^{k-1})e^{-\lambda x - \beta x^{k}}(1 - e^{-\lambda x - \beta x^{k}})^{-1+\alpha} \times \{1 - (1 - e^{-\lambda x - \beta x^{k}})\}$$

$$where\ \lambda > \beta > 0, k > 0, \alpha > 0\ and\ \gamma > 0.$$

RESULTS AND DISCUSSION

We analyzed the data set which was originally reported by Bain, (1976) and referenced by Mashall and Olkin (1997). The data set is uncensored on the breaking stress of carbon fibers (In Gba). We estimate the unknown parameters of the GEW and other distributions mentioned above using the method of maximum likelihood. Table 1 below provides the MLEs of the model parameters and their standard errors in the parentheses.

Table 1: MLEs and their standard errors (in parentheses) for Breaking Stress of carbon Fiber data set.

Model	Estimates				
	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\lambda}$	$\hat{\gamma}$	$\hat{k}$
EW	0.36113	0.02490	3.28146		
	(0.04380)	(0.01015)	(0.31081)	___	__
GEW	**0.82264**	**0.09651**	**2.43514**	**10.77656**	
	(0.09037)	**(0.04492)**	**(0.33683)**	**(2.28643)**	___
KwEW	0.865898	-0.252877	1.688675	0.080005	0.008235
	(0.47525)	(0.021106)	(0.077757)	(0.009702)	(0.010530)

The model selection was carried out using the Akaike Information Criteria (AIC), Bayesian Information Criteria (BIC), Hannan-Quinn Information Criteria (HQIC) and Consistent Akaike Information Criteria (CAIC).In this selection, the smaller values of goodness-of-fit measures fit the data better. Table 2 shows the numerical values of the -2L(.), AIC, BIC, HQIC and CAIC.

Table 2. The statistics *-2L, AIC, BIC, HQIC and CAIC* for carbon fiber data set

	Goodness-of-fit				
Model	*-2L(.)*	*AIC*	*BIC*	*HQIC*	*CAIC*
EW	457.291	463.291	464.6210	459.3954	465.9259
GEW	**335.4498**	**343.291**	**342.7798**	**337.5542**	**344.0847**
KwEW	487.1036	497.1037	494.4336	489.2076	495.7385

Table 2 above shows that the GEW model has the lowest values for AIC, BIC, HQIC, and CAIC statistics among all the fitted models. Hence, we have strong evidence that GEW model could be chosen to be the best fitted model for the subject data.

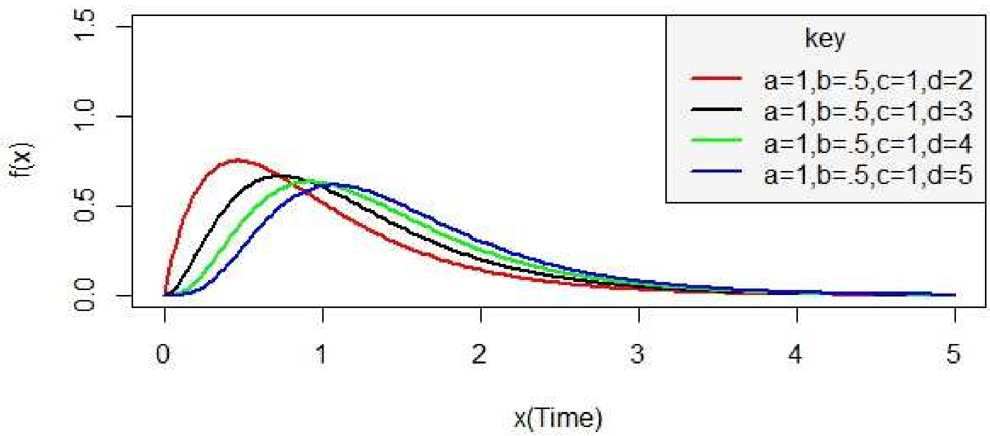

Figure 1: The PDF of GEW distribution for some selected values of the parameters.

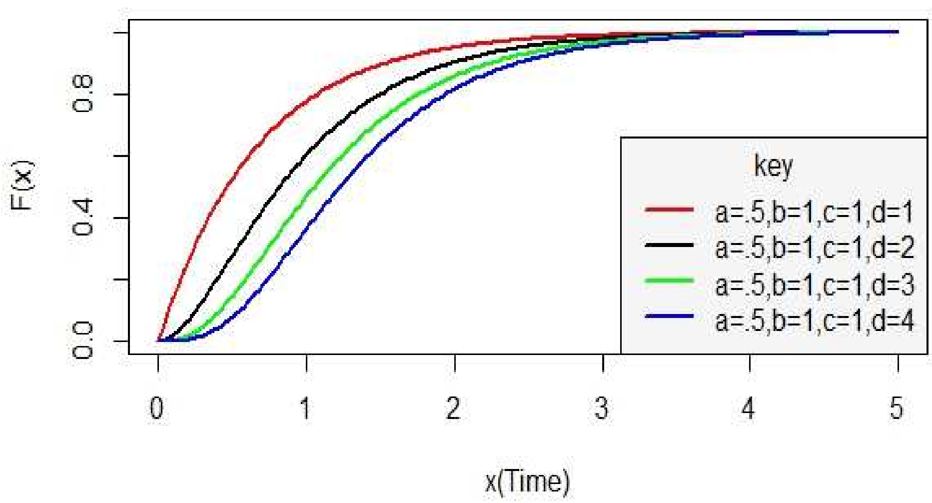

Figure 2: The CDF of GEW distribution for some selected values of the parameters.

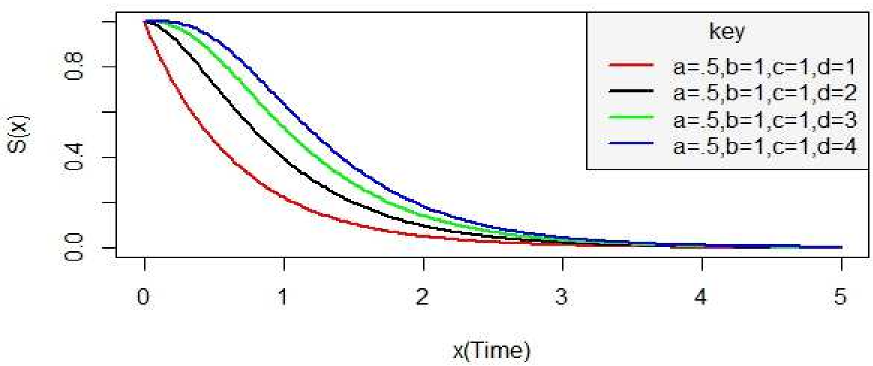

Figure 3: The Reliability rate function of GEW distribution.

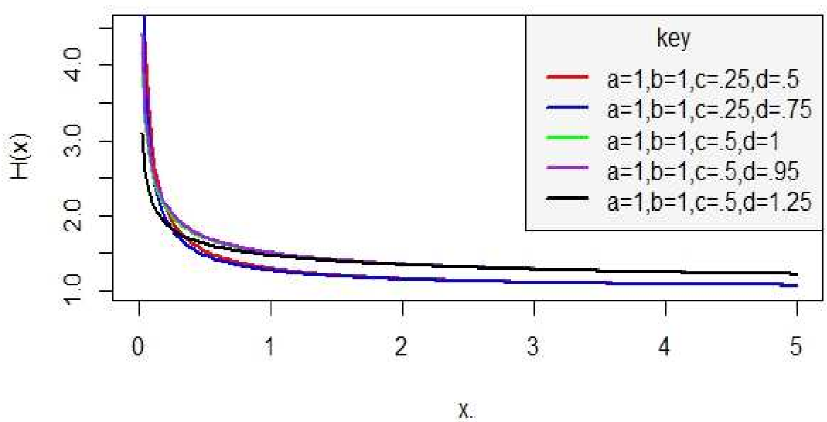

Figure 4: The Hazard rate function of GEW distribution.

It is observed that the hazard rate function is very flexible, indicating the adequacy of the proposed distribution in modelling increasing, decreasing and constant failure rates behavior.

CONCLUSION AND RECOMMENDATIONS

In this paper, we proposed a new four-parameter distribution, called the Generalized Exponential-Weibull (GEW) distribution which extend the Exponential-Weibull (EW) distribution introduced by [5]. Strong evidence shows that the proposed distribution provide more flexibility in analyzing real life data. Statistical properties including moment, moment generating function, survival and hazard function, order statistics of the distribution were also derived. We estimate the parameters of the proposed distribution using the method of maximum likelihood and also obtained the asymptotic behavior of the distribution. We compared our distribution with other related distribution from the literature to obtain the goodness-of-fit of the new distribution using AIC, CAIC, HQIC and BIC criteria. We applied the proposed distribution to a real life data set. The GEW provides a better fit than the other distributions when compared. We hope that the proposed distribution will give a room for wider application in different arears of research such as engineering, reliability analysis, medicine and economics among others.

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