Assessment for learning in teacher education: A case study on mathematization through formative strategies in learning triangles

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Abstract

Mathematization and understanding are essential for learning geometry. This study explored assessment practices and their influence on learners' mathematization. The study was conducted at Robe College of Teacher Education, Ethiopia and focused particularly on how the use of assessment for learning contributed to pre-service teachers' mathematization in learning triangles. The sample consisted of 27 pre-service teachers who were learning plane geometry and five mathematics teachers. The study employed a descriptive exploratory method with classroom observation as a data collection instrument, and emphasised the utilization of informal dialogue and argumentation as mechanisms of assessment for learning strategies. The findings revealed that teacher educators predominantly used assessment for learning methods which were found to be insufficient and ineffective in evaluating the mathematization process of pre-service teachers as they were limited to capturing the depth of mathematical thinking and understanding. However, incorporating a variety of assessment-for-learning strategies helped to create the necessary conditions for triangle congruence, fostered mathematization through justified reasoning, supported the learning of mathematical concepts that involved abstraction and advanced cognitive processing. The study concludes that adopting enhanced assessment practices, particularly with a focus on assessment for learning, can address the complexities inherent in some mathematical concepts.

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Introduction

Mathematics is an essential discipline in most educational systems around the world (Chand et al., 2021) that requires appropriate mathematization and concept construction meaningful to real situations. It encompasses analytical and critical competence, meaning making, and problem-solving. To this effect, Freudenthal (1968) noted that mathematics

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teachers should prioritize teaching the process of mathematization rather than simply presenting mathematical concepts. Conceptual understanding is important in learning mathematics which has a significant impact on learners' mathematics performance (Ncube & Luneta, 2025). Obunge and Omeodu (2024) noted that integrating real-world applications into mathematics education is essential for enhancing students' understanding, motivation, and ability to apply mathematical concepts in diverse contexts.

Mathematizing succeeds when it begins with contextual problems and connecting new information to previously learned concepts (Gravemeijer, 1994). For mathematization, learners are encouraged, under close guidance from their teachers, to reimagine and generate mathematical ideas from these contextual problems. Through guided reinvention and problem-solving in real-world scenarios, mathematization fosters the development of students' mathematical concepts and enhances their understanding of the relevance and application of mathematics in real-life contexts. In this regard, assessment serves as a critical tool through which learners can monitor their progress in mathematization (Ngunjiri, 2022), provide feedback to teachers on their lessons (Mahlambi et al., 2022), or evaluate students' overall performance in mathematics (Ozan & Kıncal, 2018). Assessment enables teachers to meet students' individual needs (Tlale, 2013), engages learners in developing conceptual understanding (Lai et al., 2019), and supports them to improve their problem-solving skills (Ukobizaba et al., 2021). However, its implementation varies significantly depending on real-world conditions, teacher competence, student needs, and the overall teaching-learning context. As no single type of assessment is adequate for evaluating all mathematical concepts and skills (Zhai & Nehm, 2023), various assessment formats are needed to evaluate students' abilities to solve problems, justify conclusions, and make mathematical connections and representations (Suurtamm et al., 2016). Other factors include the nature of mathematical knowledge, and the specific skills learners must develop and acquire (Bleiler et al., 2014). Scholars such as Kontorovich (2021) advocate interviews and mathematical discourse along with argumentation (Jonassen & Kim, 2010) as tools for assessing learners' problem-solving and critical thinking skills. The use of assessment for learning progressively builds learners' understanding of mathematics (Mahlambi et al., 2022) and links mathematization with appropriate contextual strategies (Pielsticker, 2022). These approaches positively influence learners' academic performance and confidence in triangle concepts (Gaylo & Masinading, 2022). Moreover, the opportunities these approaches provide for mathematizing contextual problems and developing learners' arguments indicate the paramount importance of dealing with these concepts. Numerous studies underscore the broad benefits of assessment for learning (Black & Wiliam, 1998).

Despite this, in Ethiopia, the predominant method of assessment largely remains traditional, relying primarily on paper-and-pencil tests or exams, which are summative in nature (Gebremariam & Gedamu, 2023; Melaku et al., 2023; Dawit, 2023). Research further indicates that mathematics teacher educators primarily use assessment for summative rather than formative purposes, limiting opportunities for improving student learning (Dagnachew & Sewagegn, 2020). The adoption of assessment for learning among teacher educators in Ethiopian colleges and universities is also notably minimal (Moges, 2018; Oli & Olkaba, 2020), hindering the promotion of mathematization skills. Despite the valuable nature of paper-pencil tests in effectively measuring knowledge and understanding, these may not be sufficient in all contexts, especially for cultivating mathematization skills (McClelland, & Cuevas, 2020).

It is, therefore, vital to examine the interrelationship between assessment for learning conceptions and practices and mathematization. The lack of research on mathematization in the Ethiopian context further highlights the importance of this study. This study aimed to examine the assessment conceptions and practices employed by mathematics educators in a teacher education college to foster the mathematization process among pre-service mathematics teachers. Given the expansive nature of mathematics learning, this study will focus specifically on triangles as part of geometry learning, as this provides an effective framework for connecting mathematical concepts to real-world scenarios. Accordingly, the study sought to address the following research questions:

1. What assessment practices do teacher educators utilize to promote mathematization among pre-service mathematics teachers?
2. What assessment-for-learning strategies are used to assess the mathematization process of pre-service mathematics teachers, and how are these strategies appropriate?

Theoretical Framework

Geometry learning could be well captured when it is integrated with real contexts, and when students connect theoretical knowledge with practice. Hence, schoolteachers need to plan and employ hands-on approaches with the help of manipulatives and models in the teaching and learning of geometry to help their students better understand the concepts and develop problem-solving skills (Tessema et al., 2024). This indicates that geometry learning could best be entertained when it is integrated within the framework of Realistic Mathematics Education (RME), to help students actively engage to construct meaning. Kant and Sarikaya (2021) argue that the primary objective of mathematics in RME is to mathematize real-world situations. Such an approach not only helps pre-service teachers to relate theoretical knowledge of geometry to practice but also explain the real essence of mathematizing geometry concepts in their surroundings. Therefore, using an approach that relates triangle concepts with real life applications is important since such an approach gives an opportunity for teachers themselves to develop awareness, interest, skill, and understanding in using locally available materials from their surroundings.

In so doing, assessments which reflect different purposes are often organized around the notions of assessment as learning (AaL), assessment for learning (AfL), and assessment of learning (AoL) (Schellekens et al., 2021). While AaL and AfL are categorically formative in nature, AoL is summative. AoL is used basically for evaluation purposes and for ensuring accountability, which is conducted at the end of a unit or time span. Formative assessment has an ongoing nature, and it supports improvement of students' learning. It positions both teachers and students as key agents of educational change (Lysaght & O'Leary, 2013), functioning as a bridge between teaching and learning (Lysaght & O'Leary, 2013).

In establishing a guiding theoretical framework for the use of assessment, Zou et al. (2024) offer practical strategies for implementing assessment to enhance teaching and learning where AfL serves as a pedagogical framework for a systematic support for understanding learner progression. Abo and Gursha (2023) and Oli and Olkaba (2020) also provide practical challenges and innovations in assessment modalities that help us ground AfL in real classroom contexts. With the premise of enhancing pre-service teachers' learning and understanding, assessment is expected to reveal their abilities in terms of understanding concepts, procedures, communication, reasoning, and problem-solving (Jaelani & Hasbi, 2022). Assessment strategies also play a crucial role in teaching and learning, and rational assessment strategies can provide a comprehensive understanding of students' abilities, facilitate instructional improvement, and contribute to the teaching and learning process and student development (Zou, 2024).

In learning geometry, one of the ideals is relating geometry concepts with daily life applications. Ethiopia in general and the study area in particular have rich workplace mathematics, and localities provide mathematical applications in action. Such an offer entails mathematizations that are explainable by the users. The baking of *injera*, a local staple food made of *teff* grains, involves pouring batter on an open oven starting from the outermost of the oven and moving to the center. One of the possible mathematizations is the issue of heat and heat transfer; when making pots, there is also mathematization of size and volume. When ploughing, farmers apply concepts of gradient, areas, angles, etc. These are some real-life examples which illustrate the contribution of the daily activities of the local community to mathematization. Teachers in schools are expected to apply geometry concepts in daily life applications or bring locally available mathematical concepts into class. Therefore, it is imperative to develop mathematization skills in pre-service teachers.

Mathematization has diverse conceptualizations. It is viewed as a theory for curriculum design (Isoda, 2018), the application of mathematics in previously unexplored domains and an epistemic phenomenon of historical and philosophical significance (Dijksterhuis, 2010), and a teaching approach that emphasizes the relevance of mathematics in everyday life by enabling students to apply mathematical tools to solve real-world problems (Mumcu, 2016). In addition, mathematization is considered a pedagogical tool (Wheeler, 2001) and a mechanism for expressing or modeling real-life phenomena mathematically (Ferreira & Silva, 2020).

Ferreira and Silvia (2020) provided the mathematization thesis and Widada et al. (2020) demonstrated how realistic mathematics education foster mathematization. In support of this, Isoda (2018) provided a framework for mathematization that connects theory and practice and shows how mathematization can best support contextualizing abstract concepts like triangles. To understand triangle concepts better, therefore, it is fundamental to link the concepts learned with local knowledge and practices of mathematization by pre-service teachers.

Enhancing mathematization skills demands the application of formative assessment strategies which is elaborated by Swan and Foster (2018), who provided practical formative assessment approaches in mathematics classrooms. Pielsticker (2022) also provided lessons on mathematical knowledge of triangle concepts in real contexts. Triangulating geometry teaching-learning, assessment for learning strategies, and enhancement of mathematization is of paramount importance for which Thompson and Kaur (2011) highlighted the importance by employing a multi-dimensional approach to assessing mathematical knowledge, that provides ways into how AfL and mathematization intersect in geometry, especially with triangle concepts.

Methods

Research Design

To explore situations where variables are unclear, Yin (1994) recommends an exploratory case study. This is particularly useful for understanding complex phenomena in real-life contexts. Yin (1994) further emphasized that case study research, including exploratory designs, is appropriate when a researcher has little control over events, and the focus is on contemporary issues within a real-world context. It is, therefore, useful to apply this in educational research like mathematization, where detailed observations and flexible data collection methods allow for a deeper understanding of how assessment for learning can influence the development of conceptual learning. Accordingly, this study employed a single case study design. Embedded were also units of investigation such as strategies teacher educators used in class, pre-service teachers' deliberations, and views on both assessment and mathematization.

Participants

The participants of this study comprised pre-service mathematics teachers and mathematics teacher educators from Robe College of Teacher Education. Comprehensive sampling strategy was used on a total of 27 pre-service teachers to participate in the study. Five mathematics teacher educators were also selected using an availability sampling technique to provide accessibility during the data collection phase and based on their willingness to participate in the study. However, one of these educators, who was teaching the plane geometry course, was purposively selected for an in-depth examination of classroom discourse.

Data Collection Instruments

Data were gathered through field notes and semi-structured interviews to capture data on the assessment for learning practices used by mathematics teacher educators. Observations were also employed to cross-check the educators' self-reported practices against their actual classroom behaviours, and how the reality-based mathematical applications were integrated with classroom instruction to influence mathematization.

Trustworthiness and Credibility

Trustworthiness was ensured by adhering to the key criteria of qualitative research: credibility, transferability, dependability, and confirmability. Credibility was achieved through data triangulation and transferability by providing sufficient context for other educators to determine whether the findings could be applied in different settings. However, given the reality-based mathematization and variations among contexts, the issue of transferability seeks critical scrutiny. Dependability was ensured through rigorous documentation, the creation of an audit trail, and the maturation of information. Confirmability was strengthened by cross-referencing the findings with existing literature, as well as through peer review and feedback from the participants during debriefing sessions. Additionally, the emergent themes and interpretations were validated through rigorous comparison with the established literature, further enhancing the study's credibility and trustworthiness.

Data Analysis

This study employed thematic analysis to uncover issues that addressed the research questions and broader problem under investigation. The process went beyond data summarization where key thematic areas were identified and cross-referenced with emergent cases to re-conceptualize assessment for learning through mathematization. Data from interviews were transcribed verbatim. Observations and field notes served as reference of actions. Open coding was used to identify relevant codes, reviewed, and refined by the research team. Multiple readings and interpretations of the raw data, aligned with the research questions, formed the basis for the analysis. This led to the identification of assessment for learning strategies employed by teacher educators, the mathematization process of pre-service teachers, and the collaborative emergence of themes.

Ethical Considerations

The researchers adhered strictly to all ethical guidelines throughout the study. Ethical approval was granted by the Institutional Review Committee of the College of Education and Behavioral Studies at Addis Ababa University, and the core ethical principles of justice, autonomy, beneficence and non-maleficence were upheld at every stage of the research. We secured permission to conduct the study, and informed consent and voluntary participation. The participants were also informed about confidentiality and anonymity, and that they would not receive any direct benefit from participating in the study. Moreover, they were assured that no personally identifiable information would be used in the research report and that all data collected would be treated with strict confidentiality.

Results

Assessment Mechanisms Practiced in Teacher Education

The assessment mechanisms of the teacher educators under investigation revealed three central themes: conceptualization of assessment, assessment practices, and provision of feedback.

Conceptualization of Assessment

The results revealed that teacher educators have a basic understanding of assessment types and their purposes. One interviewee had the following to say:

> I conceptualize assessment as a means of gathering information about students' understanding. It helps us determine whether they have internalized a given concept. Formative assessment is used during the teaching-learning process, while summative

> assessment is conducted at the end of a program or a unit of study. (Interviewee, MTE1)

This indicates an established view of assessment and his mention of "gathering information" signals assessment not just about measuring outcomes but also understanding student learning. However, the true effectiveness depends on how they are implemented to inform instruction and support learners.

Another interviewee said:

> I assess my students to determine whether they have met the expected learning outcomes. If they do not meet these expectations, I revise the lesson plan accordingly. To achieve this, I use various assessment tools such as assignments, project work, and labs, although these methods are less commonly used in mathematics. (Interviewee, MTE2)

This view emphasizes predefined goals, ensuring assessments are purposeful and potential for opportunities for broader, exploratory learning and problem-solving in learning mathematics. The educator demonstrates flexibility by revising lesson plans which further underscores a commitment to addressing diverse student needs and fostering meaningful learning outcomes. This may allow them to address misconceptions or gaps before advancing to more complex topics. Employing a range of assessment tools also reflects an awareness of diverse assessment methods. These offer opportunities for deeper learning and promote mathematization, especially when aligned with learning objectives. Another interviewee also conceptualized assessment as a means of identifying learners' weaknesses and strengths in learning. He indicated that:

> Our current understanding of assessment is incomplete, as it is often shaped by the way we were previously taught. Instead of using assessment as a tool to improve instructional methods and enhance student learning, we primarily use it for grading purposes, with gaps in the knowledge on how to effectively assess students. (Interviewee, MTE3)

This educator indicates causes of conceptions and uses of assessment depicting assessments are not fully realized in practice. This understanding of assessment suggests that some educators may not be fully prepared to implement strategies covering all domains of learning, entailing the need for integrating all learning domains and aligning assessments with educational objectives. Prior experience shapes understanding of assessment with influence of using traditional assessment methods that are often summative, focusing on grading. Teacher education programs should promote reflective practice and encourage exploration of innovative formative assessment methods that support continuous improvement in student learning. Without adequate expertise in designing and implementing assessments, educators may struggle to create tools that accurately reflect student understanding or foster meaningful learning experiences.

The interviews highlight several challenges that include an incomplete understanding of assessment and the influence of traditional grading-focused practices. Addressing these challenges requires professional development emphasizing formative assessment. The inclusion of mechanisms to assure the process, such as mathematization which builds knowledge restructuring is also worth noting. Therefore, there is the need to focus on equipping educators with the skills to create, implement, and interpret a variety of assessments, with particular emphasis on formative methods that support student learning.

Practices of Assessment

Teacher educators' assessment practices vary, with the dominance of traditional paper-and-pencil assessments, such as tests, mid-term exams, final exams, and assignments, primarily for grading purposes. One interviewee says:

> Most of the time, we do not incorporate diverse assessment techniques aimed at improving learning, such as peer assessment, self-assessment, or portfolios, apart from oral questioning and observation. Although we use oral questions and observations, we fail to analyze the results to enhance our instructional approach. The most common assessments we employ are mid-term and final exams, primarily for summative purposes. (Interviewee, MTE3)

Another interviewee noted:

> In teacher education colleges, assessment for learning is not widely practiced to enhance student learning or improve teachers' instructional strategies. However, it is uncommon to provide quizzes, observe student participation in group work, or identify students' strengths and weaknesses during the teaching and learning process. (Interviewee, MTE4)

This highlights a limited use of diverse and formative assessment techniques which are useful to promote deeper learning, critical thinking, and reflective practices. Another interviewee also remarked:

> Since the courses we teach are extensive and new to learners, my colleagues and I often lack the time to incorporate peer assessment, self-assessment, or portfolios as assessment tools to track students' learning progress. Instead, we focus on summative assessments for grading. (Interviewee, MTE1)

This also highlights critical issues related to assessment practices. When educators use summative assessment, students may not receive the necessary guidance to address gaps in understanding. This may also limit opportunities for students to engage in self-regulation and reflection. Incorporating diverse assessments, hence, fosters a more comprehensive understanding of students' abilities and promotes improved learning outcomes. Otherwise, limited use of assessment for learning could hinder both student learning and the improvement of teachers' instructional strategies. Addressing this is essential, as implementing AfL can create more dynamic, responsive teaching and learning environments where both teachers and students thrive. By fostering a culture of continuous assessment and feedback, educators can better prepare future teachers to incorporate these strategies in their own classrooms, ultimately improving education at all levels.

While summative assessments are essential for grading, formative assessments play a critical role in fostering students' learning and understanding. These require institutional support, professional development, and creative approaches to balance the demands of teaching extensive courses with the need for continuous student feedback.

Feedback Provision

Feedback is useful when teachers use formative assessment as it gauges learner progress toward the intended learning outcomes. However, mathematics teacher educators revealed that providing feedback is not consistently practiced by teacher educators, despite variations among them. Some mentioned that feedback isn't frequently given, especially in content-heavy courses such as algebra or geometry, while others indicated its implementation during individual or group presentations and reflections, in courses like practicum and subject-area methodology. An interviewee educator stated:

> Feedback practices vary depending on the individual educator and the courses taught. It is commonly provided in subject area methodology courses and during teaching practice or practicum. However, in courses like calculus, algebra, and similar subjects, feedback is rarely given. This is because students are not often invited to engage in practical exercises in these courses, and the feedback process is uncommon. (Interviewee, MTE3)

Another educator noted:

> The provision of feedback to students is inconsistent—some educators deliver it effectively, while others do not. Feedback is usually given during presentations and tends to focus on misconceptions and weaknesses. It is common in certain subjects but less so in others. Nevertheless, some teachers occasionally provide feedback, both orally and in writing, for individual or group work. (Interviewee, MTE1)

These statements provide valuable insights into the assessment and feedback practices employed by mathematics teacher educators at teacher education colleges. Several key points emerge, reflecting both the diversity of and challenges in feedback provision across different teaching contexts.

The educator's distinction, in employing feedback, between subject-area method courses and content-based courses like calculus and algebra is alarming. The absence of feedback in content courses raises concerns about the quality of the learning experience. Although these courses focus heavily on the acquisition of mathematical knowledge, the lack of formative assessment opportunities may hinder deep learning. Effective feedback can help students reflect on their understanding, identify misconceptions, and strengthen their mathematization and mathematical reasoning skills. Addressing this gap requires a pedagogical shift toward more active learning strategies in content courses, where formative feedback can enhance students' learning experiences and outcomes. Similarly, attempts to primarily focus on completing the course rather than emphasizing understanding and student learning progress, is a pivotal challenge that seeks consideration.

The inconsistent provision of feedback may leave students feeling discouraged in learning some subjects. A more balanced form of feedback that highlights strengths alongside specific recommendations for improvement would likely be more effective. So, the inconsistencies in feedback delivery indicate that the formative assessment process may not be fully realized, and students may struggle to gauge their progress and identify areas for improvement. Addressing these issues could enhance assessment practices and create a more supportive learning environment.

Assessing the Mathematization Process

Mathematization requires formative assessment and several strategies could enforce these; some of the strategies identified were informal dialogue, observation, probing during presentations, and argumentation.

Informal Dialogue

One mathematics teacher educator, who was responsible for teaching plane geometry was purposefully selected for further investigation and targeted implementation. Observations revealed that he implicitly used interviews, observations, and argumentation to assess the mathematization process of pre-service mathematics teachers. Below is a discourse between the teacher educator, who employed these methods, and the pre-service teachers who worked on determining the necessary and sufficient conditions for triangle congruency.

The instructor designed a game in which two groups participated. One group constructed a triangle secretly. The group provided one condition at a time for the second group to construct a corresponding triangle. At the end of the game, the students compared the triangles for congruency. The instructor guided, observed, and assessed the pre-service teachers' progress by interviewing them as they worked to establish congruency criteria.

One group proposed SSA (Side-Side-Angle) as a criterion for triangle congruency, believing that equality of two sides and a non-included angle suffice congruence. However, they didn’t achieve congruent triangles under this condition. This sparked a discussion, mediated by the teacher educator. Through this discourse, the students realized that while SSA might work in some cases, it is not a universally valid criterion for triangle congruency. The following discourse between the instructor and the pre-service teachers highlights this learning process:

Instructor: I noticed differences in the outcomes between the groups. Could you explain the issue?

PMT4: At the start of the activity, we were provided with the length of one side of the triangle, which was 2 cm. With this information, despite multiple attempts, we were unable to duplicate the triangle constructed by the other group.

Instructor: What observations did you make when comparing the triangle your group constructed with that of the other group?

PMT3: We realized that only one criterion or condition is insufficient to guarantee congruence of two triangles.

Instructor: What additional information did you receive during your second attempt?

PMT1: We were given the length of the second side, which was 3 cm. However, with the two sides, we were still unable to construct an exact replica of the opponent's triangle.

Instructor: Why did this occur and what inferences can you draw from these repeated attempts?

PMT2: Although two sides of a triangle may correspond with the sides of another triangle, this alone does not ensure congruence.

Instructor: What information did the other group provide in the final round?

PMT2: They provided us with the measure of a non-included angle, which was 40 degrees. We applied the Sine Law to determine the angle opposite the side measuring 3 cm.

Instructor: What were the results of your calculations?

PMT2: Our calculations gave us Sin(x) = 0.964, and we determined that x equals 74 degrees.

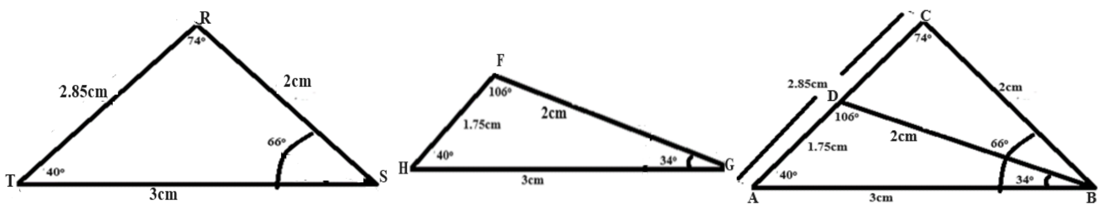
Instructor: Is 74 degrees the only possible value for angle x? Consider what would happen if the angle was 106 degrees. Do you recall the Sine value for the difference between two angles, particularly Sin (180° - x°)?

PMT4: If we let y = 180° - x°, then sin (180° - x°) = sin180°cosx° - cos180°sinx° = 0 - (-1). In our case, if x = 74 degrees, then sin 74° = sin (180° - 74°) = sin106°.

PMT3: We determined that one possible value for the angle was approximately 74 degrees. The third angle was calculated to be 66 degrees, and the third side measured around 2.85 cm. Despite this, we were unable to produce an exact match with the opponent's triangle. When we attempted to superimpose our ΔABD on their ΔABC, they did not coincide. (The first group constructed the triangle on the left, while the lead group initially constructed the middle triangle. The third figure illustrates the result obtained when the opposing group attempted to align their triangle with the original.)

Figure 1

Two Triangles Having Two Equal Sides and One Equal Angle but Different in the Remaining Side and Angles (constructed by pre-service teachers)



Instructor: What could cause this discrepancy?

PMT4: I suspect the discrepancy arises from the differences in the side lengths and angle measurements obtained by our group compared to those of the opposing group.

PMT2: Since sin(x) = 0.964, the angle x could either be 74 degrees or 106 degrees. These two possibilities affect the measurement of the third angle. If we use 74 degrees, the third angle becomes 66 degrees. However, if we use 106 degrees, the third angle would be 34 degrees. This could explain the difference.

Instructor: Can you state the interior angles of a triangle if one of the angles measures 106 degrees?

PMT2: The interior angles would be 40 degrees, 106 degrees, and 34 degrees. This is the triangle constructed by the opposing group earlier.

Instructor: What angles did your group obtain?

PMT3: We calculated 40 degrees, 74 degrees, and 66 degrees.

Instructor: What conclusion can you draw from this?

PMT1: We can conclude that the side-side-angle (SSA) criterion, when involving arbitrary angles, may not guarantee congruence of triangles.

The dialogue provided inquiry-based problem-solving, mathematical reasoning, and advancement of the concept of triangle congruency. From the conversation, it becomes clear that a significant lesson was learned regarding the insufficiency of one condition (SSA Ambiguity) for establishing triangle congruence.

PMT4's group struggled to construct a congruent triangle from just one given side. This opened an essential teaching moment, highlighting the limitations of using only one side to establish congruence of triangles. PMT3's realization that a single side cannot guarantee congruence reflects the group's growing awareness of how multiple conditions must interact to establish congruence. This marks their shift toward understanding more robust congruence conditions.

PMT1 notes that even after receiving the second condition (3 cm), the group couldn't establish congruence. This demonstrates that the pre-service teachers were beginning to recognize the necessity of understanding the relationships between sides and angles, rather than simply relying on isolated measurements. The group's recognition that the angle between two sides was a critical factor (PMT2's inference) shows their advancement in understanding congruence. They were starting to grasp the significance of angle-side relationships and why the SSA (Side-Side-Angle) condition did not guarantee congruence.

When PMT2 applies the Sine Law to calculate the angle opposite a given side, it illustrates their problem-solving skills and understanding of trigonometric principles. However, this also introduces ambiguity, as the Sine of an angle has two possible values (e.g., $\sin(74°) = \sin(106°)$). The instructor's question about whether 74° is the only possible angle prompts the group to reflect on the dual possibilities inherent in the sine function (i.e., $\sin(180° - x) = \sin(x)$). This discussion emphasizes the mathematical nuances of dealing with non-included angles.

PMT2 correctly identifies the ambiguity in their solution, noting that $\sin(x) = 0.964$ could yield either 74° or 106°. This affected the remaining angles, leading to differing triangle configurations depending on which angle they chose. The group's inability to superimpose their triangle over the opponent's reveals a deeper understanding of how SSA ambiguity can result in two distinct, non-congruent triangles. This insight connects their theoretical understanding with practical construction challenges. The discussion culminates with PMT1 concluding that SSA does not guarantee triangle congruence. This marked a pivotal moment where the learners solidified their understanding that some configurations (like SSA) were inconclusive for congruence, necessitating the use of more reliable conditions such as SAS or ASA.

The dialogue encouraged the PMTs to reflect on their process and infer principles rather than seeking direct answers. This is a hallmark of inquiry-based learning, where students discover concepts through exploration and reflection. The conversation among the PMTs also demonstrates active peer learning. They build upon each other's ideas, correct misconceptions (e.g., regarding the use of angles), and collaboratively construct understanding, which is critical in developing deep mathematical knowledge. The dialogue further exemplifies how instructor's questioning and the students' responses serve as formative assessment in action, helping both the instructor and the students gauge understanding throughout the process. The dialogue also illustrates how inquiry and problem-solving promote mathematization - students are not merely memorizing rules, but are actively constructing, testing, and refining mathematical concepts. The PMTs' difficulties in reconstructing the triangles fostered deeper mathematical reasoning, showing how formative assessment can create space for productive struggle, which is essential for meaningful learning in mathematics.

Observation

During his observation, the instructor encountered some unusual concepts while a group of pre-service mathematics teachers attempted to construct a triangle with sides measuring 5 cm and 7 cm, along with a non-included angle of 60 degrees. He engaged in a discussion with the group to address the issue, with some excerpts from the discourse presented below.

Instructor: To begin the exercise, you constructed a triangle with side lengths of 7 cm and 5 cm, along with a non-included angle of 60 degrees, opposite the 5 cm side. After constructing the triangle, you gave it to the opposing group for verification. In their attempt to construct a congruent triangle, they calculated the measure of the angle opposite the 7 cm side using the "Sine law." However, their result seemed unusual. Could you verify the calculation using the Sine law? What was your outcome?

PSMT1: No, sir, we can check it now. Let's denote the angle opposite the 7 cm side as *x*. We are given that the angle opposite the 5 cm side is 60 degrees: By applying "Sine law", $\frac{\text{sinx}}{7\text{cm}} = \frac{\text{sin}60^{\text{o}}}{5\text{cm}}, \Rightarrow \text{sinx} = \frac{7\text{cmsin}60^{\text{o}}}{5\text{cm}} = 1.212 \Rightarrow \text{sinx} = 1.212.$

Instructor: You found a value of 1.212 for sin(x). Is it possible to have this value for the sine of an angle? What is the maximum possible value of the sine function? What does this result indicate? And what can you learn from this?

PSMT2: The sine of an angle is always between -1 and 1. This value suggests there was an issue, either with the side lengths or the angle measurement of the triangle. From this, we learned that when constructing a triangle, it is important to maintain the correct relationship between the side lengths and the internal angles.

Instructor: Suppose you are given two sides and a non-included angle. Under what conditions would the sine law yield a valid triangle? Why?

PSMT3: Let a and b be the sides of the given triangle and let A and B be the angles opposite sides a and b, respectively. In applying the Sine law, we will get the triangle if and only if $0 < \mathrm{sinA} = \frac{\mathrm{asinB}}{\mathrm{b}} \leq 1,\ 0 < \mathrm{sinB} = \frac{\mathrm{bsinA}}{\mathrm{a}} \leq 1$ and the degree measure of the two angles need not be 90 degrees simultaneously.

This conversation highlights several key insights into both mathematical content and the formative role of assessment in promoting deeper understanding among pre-service mathematics teachers (PSMTs). The dialogue underscores the critical thinking processes involved in exploring the congruence of triangles through the sine law, offering a valuable case study for examining how assessment for learning (AfL) strategies can facilitate the mathematization process.

The primary error identified by the PSMTs occurred during the application of the sine law. Their calculation yielded sin(x)=1.212, an impossible value. This error likely stemmed from either incorrect side lengths or a misinterpretation of the angle. The instructor adeptly used this opportunity to prompt students to reflect on the conceptual meaning of the sine function, steering the conversation towards a deeper understanding of the properties of triangles and trigonometric functions. This exemplifies effective formative assessment, where the instructor identifies a misunderstanding and uses it as a learning moment for the students.

As PSMT2 reflected on the error, the group shifted the discussion to how side lengths and angle measures interacted during triangle construction. This discussion reinforced an important geometric concept: the dependency of a triangle's shape on the specific combination of its side lengths and angles. The lesson learned here emphasizes the necessity of critical evaluation and reflection during problem-solving, which is essential for developing a strong mathematical foundation in prospective teachers.

In the final exchange, the instructor asked PSMT3 to generalize the conditions under which the sine law produces a valid triangle when given two sides and a non-included angle. The student provided a thoughtful response, correctly outlining the sine conditions that must be satisfied. This generalization, together with the previous reflection on the incorrect result, demonstrates a progression from specific error diagnosis to a broader conceptual understanding of when the sine law can and cannot be applied.

This dialogue illustrates how assessment can be woven into instruction to promote mathematization of concepts related to triangles—the process of making sense of the world through a mathematical lens while learning the geometry concept of triangles. The instructor's role is not only to correct mistakes but also to guide students in deriving general principles from specific cases. This fosters the mathematization process, as students engage in reflective thinking, reason abstractly about relationships between quantities, and apply their insights to solve broader classes of problems. Through this iterative process, the PSMTs moved from procedural knowledge (applying the sine law) to conceptual understanding (knowing the limitations and conditions for its use).

This case study highlights the importance of creating learning environments where errors are viewed as opportunities for deeper learning. By encouraging students to explore their misconceptions and arrive at correct conclusions, instructors can help develop a more robust understanding of the material. This approach is particularly relevant in teacher education, where PSMTs must develop the ability to not only solve problems but also anticipate and address the types of misconceptions their future students might encounter. In conclusion, this dialogue serves as a powerful example of how effective AfL strategies can support the mathematization process in teacher education, particularly in the context of understanding triangle congruence through trigonometric principles.

Probing During Presentation

In another discussion, the instructor asked a group to present a scenario involving two and three given angles, focusing on the angle-angle (AA) or angle-angle-angle (AAA) criteria for triangle congruence. A pre-service mathematics teacher presented his group's (P) work, and the following dialogue was observed between the instructor and the presenting group.

Instructor: Is the Angle-Angle (AA) criterion sufficient for the congruence of two or more triangles?

PSMT7: No, we've attempted it several times, but we couldn't produce an exact replica of the triangle our opponents constructed. Another pre-service teacher, P1, pointed out that the congruence of two or three corresponding angles alone does not guarantee the congruence of triangles.

Instructor: Why does the AA criterion fail to guarantee congruence in triangles, and how does the AAA criterion compare?

PSMT8: The issue lies in the side lengths of the triangles. For example, consider an equilateral triangle with side lengths of 2 cm and another equilateral triangle with side lengths of 4 cm. Both triangles have identical interior angle measures of 60 degrees, but their side lengths differ, making the triangles non-congruent. This also applies to the AAA criterion.

Instructor: Based on this, what conclusion can we draw?

PSMT6: The congruence of two or three angles of one triangle with the corresponding angles of another triangle is neither a necessary nor sufficient condition for triangle congruence. However, if a corresponding side—either included or non-included—is also congruent, then the triangles will be congruent, provided that the angles are correspondingly congruent.

The conversation helped to explore the criteria for triangle congruence and provided key insights into pre-service mathematics teachers' (PSMTs) understanding of geometric principles. It reflects how future teachers should approach the critical reasoning required in geometry, particularly when assessing triangle congruence.

The instructor questioned and the responses from the PSMTs indicated a clear understanding that angles alone were insufficient for determining congruence. PSMT7 and PSMT6 expressed doubts about the AA criterion, correctly stating that congruent angles do not guarantee congruent triangles. This was reinforced by PSMT8's example. The inclusion of the equilateral triangle example provided a concrete illustration of why the AAA criterion also fell short in ensuring congruence. The exchange highlighted that the PSMTs recognized the limitations of angle-based congruence criteria and understood the side length's role. PSMT6 emphasizes the necessity of including side information, showing an understanding of more robust criteria like SAS (Side-Angle-Side) and ASA (Angle-Side-Angle). Their responses reflect a shift from basic reasoning to a more nuanced comprehension of geometric properties, aligning with deeper mathematization processes.

This conversation also served as an assessment tool for the instructor. By engaging students in dialogue about their reasoning, the instructor effectively gauged their grasp of fundamental concepts while fostering a deeper understanding through discussion. These exchanges reflect formative assessment in action, where the instructor identifies gaps in knowledge and encourages PSMTs to refine their understanding.

In conclusion, the dialogue reinforces that congruence cannot be inferred from angles alone and offers valuable insights into how PSMTs process and articulate geometric concepts, with implications for both teaching and learning in the context of geometry.

Argumentation

Argumentation serves as a means of assessing the mathematization process of pre-service mathematics teachers. The researchers also observed the claim, data, and warrant the pre-service mathematics teachers provided while establishing the necessary and sufficient condition for the congruency of triangles. An example is provided below.

Figure 2

Toulmin's Model of Argumentation on Congruency Criteria of Triangles

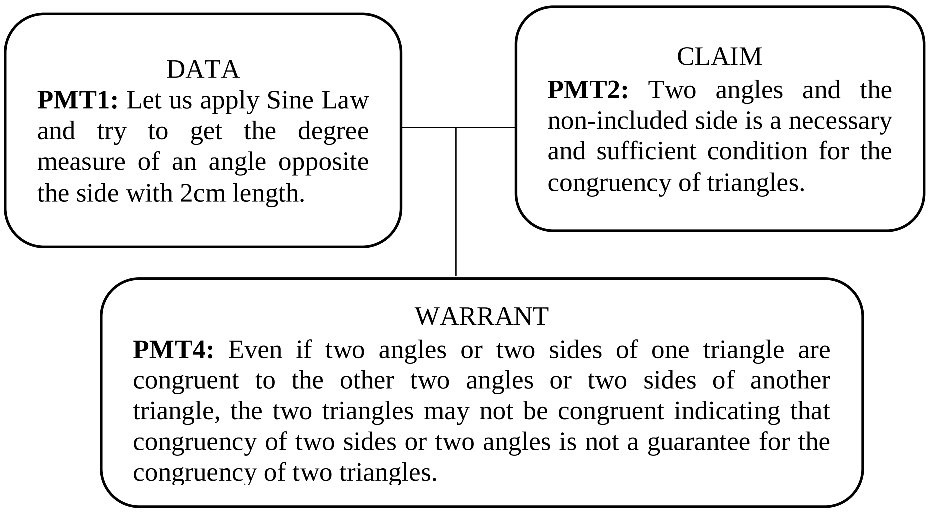
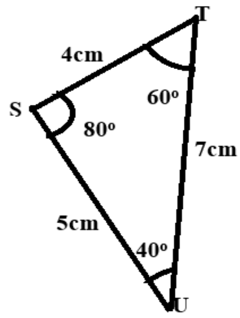


Figure 2 illustrates the claim, data, and warrant provided by the pre-service teachers while establishing the conditions for triangle congruence. The argumentation helped them formulate claims supported by relevant data and warrants that strengthened their justification. During the interview, the instructor explained that this approach allowed him to offer constructive feedback to the pre-service mathematics teachers. However, while establishing the congruence criteria, the pre-service teachers mistakenly constructed a triangle without considering the relationship between the side lengths and the interior angle measures. The instructor further mentioned that, upon applying the Cosine Law as demonstrated below, the pre-service teachers recognized their error in constructing the triangle.

Figure 3

A Triangle Constructed by Pre-Service Teachers Having Misalignment between the Length of Sides of a Triangle and the Degree Measure of Interior Angles



Application of the Cosine Law:

$$(7m)^2 = (4cm)^2 + (5cm)^2 - 2 \times 5cm \times 4cmcos80^{\circ}$$

$$\Rightarrow \quad 49cm^2 = 16cm^2 + 25cm^2 - 40cos80^{\circ}.$$

$$\Rightarrow \quad 49cm^2 = 41cm^2 - 40 \times .174cm^2.$$

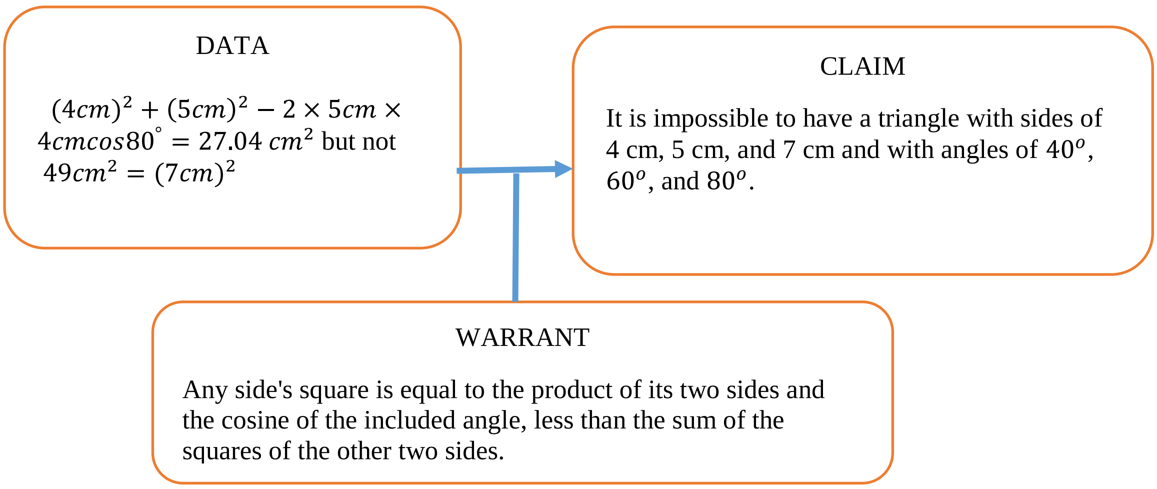
$$\Rightarrow \quad 49cm^2 = 41cm^2 - 6.96cm^2$$

$$\Rightarrow \quad 49cm^2 = 27.04\ cm^2 \ \Rightarrow \Leftarrow (Contradiction).$$

The pre-service mathematics teachers concluded that it is impossible to construct a triangle with side lengths of 4 cm, 5 cm, and 7 cm and with angle of degree measurements of 40, 60, and 80 degrees. They came to this generalization through argumentation, as depicted below.

Figure 4

Toulmin's Model of Argumentation on Constructing Triangle Congruence



Different Strategies in Assessing the Mathematization Process of Pre-Service Teachers

Regarding the types of assessment for learning utilized, the teacher educator explained:

> I utilized observation, interview, presentation, and argumentation as a means of assessment for learning. Interview, argumentation, and presentation had been better to

> assess the mathematization process of pre-service teachers as compared to observation since they are dialogic in nature. These provided an opportunity to assess the reasoning skills, justification, argumentative skills, thinking skills, and mathematical communication skills. Of course, classroom observation also helped me to use further probing questions to uncover the thinking processes of the pre-service mathematics teachers. (Interview with classroom instructor)

This excerpt emphasizes the use of diverse assessment approaches—specifically observation, interviews, presentations, and argumentation—to enhance learning process. These methods are inherently dialogic, fostering dynamic interaction between the teacher educator and the PMTs. This engagement allows for a deeper exploration of students' mathematical reasoning. Interviews, for example, facilitate personalized exchanges, enabling the educator to probe individual PMTs' thought processes, making it a tailored tool for understanding each student’s unique mathematical journey. Argumentation, on the other hand, involves the articulation of mathematical concepts and solutions through logical reasoning. This not only fosters critical thinking but also encourages PMTs to construct, justify, and evaluate mathematical arguments, promoting higher-order thinking skills and mathematical communication. Presentations require PMTs to organize and publicly convey their mathematical understanding, assessing both content mastery and the ability to communicate complex ideas effectively.

The focus on dialogic methods reflects their potential to foster the mathematization process, where PMTs move beyond rote learning toward constructing and applying mathematical knowledge, and engage deeply with mathematical problems, justify their thinking, and develop conceptual understanding, along with the ability to apply mathematical principles to novel situations. These approaches also provide deeper insights into students' thinking processes and foster essential skills such as reasoning, justification, and communication. Although observation is less dialogic, it remains valuable for real-time assessment and can be enhanced with probing questions to better understand students' mathematical thinking.

In using a test or quiz to assess the mathematization process of pre-service teachers, the teacher educator stated that:

> I have been using tests, quizzes, and oral questions to assess the performance of my students. However, I was not certain that a single test could assess all mathematical concepts. We cannot fully understand the thinking process of our students regarding some mathematical concepts using tests or quizzes. Therefore, it is better to use two or more assessments for learning strategies like mathematical discourse in addition to a test or quiz to assess the thinking process of our students. What I understood from this is that mathematical discourse like argumentation and interviews do not only serve as teaching and learning strategies, but also as assessments for learning to better understand learners’ levels of thoughts. (Interview with Classroom Instructor)

While acknowledging the widespread use of tests and quizzes, the teacher educator noted that these methods may not adequately support diagnosis of deeper levels of understanding, especially when dealing with complex mathematical concepts. This aligns with the view that mathematization involves more than procedural knowledge; it requires conceptual understanding and the ability to engage in mathematical discourse. Mathematical discourse, argumentation, and interviews serve not only as teaching and learning techniques but also as effective "assessments for learning" (AfL) approaches. These reflections suggest that employing multiple assessment strategies, including discourse-based methods, enable a more holistic assessment of students' mathematical thinking and foster their mathematization. This deliberated approach is consistent with contemporary educational theory, which advocates for a variety of assessment tools to ensure comprehensive evaluation. The inclusion of argumentation and interviews as assessment strategies allows educators to observe the cognitive processes students undergo as they develop their mathematical thinking, making these strategies particularly effective in promoting and assessing mathematization.

In summary, the interview results underscore the value of integrating diverse assessment strategies, such as mathematical discourse, into the assessment of pre-service teachers' mathematization processes. By combining both traditional methods (tests, quizzes, etc.) with alternative approaches (discourse or argumentation), instructors can gain a more complete understanding of students' mathematical thinking and provide more targeted feedback to support their learning. This balanced approach ensures that assessments are not merely summative but also formative, fostering deeper engagement with the process of mathematization. Incorporating these reflections into assessment design can greatly enhance the effectiveness of teaching and learning in mathematics education.

Discussion

Teacher educators generally conceptualize assessment by categorizing it into formative and summative types, each serving distinct purposes. Formative assessment is perceived as a tool for monitoring students' progress and understanding, allowing educators to identify learning needs and adjust their instruction accordingly. On the other hand, summative assessment is viewed to measure students' achievement of predetermined standards, often used for grading or certification. Although the educators in this study displayed a solid understanding of these distinctions, some of them did not fully implement formative assessment, leaning more towards summative methods. The interview excerpts '*assessment for learning is not widely practiced by educators to enhance student learning or improve teachers' instructional strategies.' (Interviewee, MTE4),* and *'I have been using tests, quizzes, and oral questions to assess the performance of my students. However, I was not certain that a single test could assess all mathematical concepts'* (Interviewee classroom teacher), reveal the understanding on the usefulness of using flexible approaches to assessment while indicating gaps in fully employing formative assessment. This tendency reflects findings from previous studies (Hunduma et al., 2023; Oli & Olkaba, 2020; Sewagegn, 2019).

While summative assessments like final exams, mid-terms, tests, and assignments dominate, some educators did occasionally employ formative techniques such as oral questioning, microteaching, and presentations with integrated feedback, especially in practice-oriented courses like subject area methodology and practicum. However, in more theoretical subjects, such as Calculus, Algebra, and Geometry formative approaches like presentations, microteaching, peer assessment, and self-assessment were conspicuously absent.

Several factors contributed to this limited use of formative assessment techniques, including the educators’ own educational experiences and the content-heavy nature of some courses like Calculus, Algebra, and Geometry. Consequently, pre-service mathematics teachers struggled to develop essential skills such as conceptual understanding, problem-solving, critical thinking, and mathematization. These findings align with those of Rahman et al. (2021), emphasizing the ongoing challenges in shifting away from summative practices.

The overreliance on lecture-based teaching and summative assessments undermines the potential benefits of formative assessment, particularly in providing timely feedback. Without adequate feedback, students miss opportunities to reflect on their mathematical understanding, improve problem-solving abilities, and address misconceptions. This overemphasis on summative assessments promotes rote memorization rather than fostering deeper learning, limiting the mathematization process—a concern echoed by Swan and Foster (2018) that ultimately defies the potential of developing mathematization process skills and thought-provoking efforts.

The study further revealed that argumentation and mathematical discourse enabled pre-service mathematics teachers to gain a more profound understanding of geometric concepts, such as the criteria for triangle congruency. Through discussions, they were able to construct and justify why certain criteria, like AA or AAA, do not guarantee congruency, while others, such as SSA, require further examination. These dialogues helped pre-service teachers improve their reasoning, generalization, and communication skills, all of which are crucial for mathematization. These findings also underscore the value of argumentation and dialogue, not only as instructional strategies but also as effective assessment tools. By fostering discussions that encourage reasoning and justification, educators can better assess students' mathematical thinking and generalization abilities. This aligns with the work of Uygun and Akyüz (2019), who demonstrated that collective argumentation enhances pre-service teachers' reasoning and generalization skills in mathematics.

Similarly, Jonassen and Kim (2010) emphasized that argumentation serves as both a pedagogical tool and an alternative assessment strategy, particularly for evaluating problem-solving skills. Using interviews or informal dialogues to assess the mathematization process, rather than relying solely on traditional assessments, is consistent with other studies (Ardiyani et al., 2018; Araújo & de Lima, 2020; Deniz & Kabael, 2017; Upu et al., 2017; Widada et al., 2019). These researchers demonstrated the effectiveness of informal dialogues in assessing learners' mathematization processes.

Considering these insights, re-evaluating the role of argumentation, dialogue, and feedback in assessment practices could significantly enhance the mathematization process for pre-service mathematics teachers. By more consistently integrating formative assessment strategies, even in more theoretical subjects like algebra, educators can create opportunities for deeper engagement, more meaningful feedback, and an overall improvement in students' mathematical understanding.

A notable discrepancy exists in the feedback practices between subject area methodology/practicum courses and more theoretical subjects. Shifting toward more student-centred approaches, such as active learning, group work, or problem-based learning in theoretical courses, could increase the demand for feedback. This shift would encourage both educators and students to view feedback as a critical tool for academic growth, even in content-heavy subjects like calculus and algebra.

Conclusions

Mathematization is essential for developing conceptual understanding and problem-solving skills, which pre-service teachers need to develop. The role of teacher educators in building mathematization is an area of emphasis as mathematization is indicated to be a process of translating real world situations into mathematical form. Assessment approaches need to build pre-service teachers mathematization, and this demands the use of all forms of assessment. This study has highlighted the over-reliance on traditional, summative assessment methods in teacher education particularly in geometry learning. The findings indicate that formative assessment methods, such as argumentation, mathematical discourse, and presentations hold significant potential for fostering deeper engagement with mathematical concepts. These strategies encourage reasoning, justification, and communication skills, which are crucial for promoting mathematization among pre-service teachers. However, the limited implementation of these approaches, especially in non-practicum and non-subject area teaching methods courses, impedes the development of essential skills in learners which will have a cascading effect on school-based teaching-learning.

To improve the effectiveness of assessment practices in teacher education, there is a need for a paradigm shift towards more formative and student-centred approaches. Teacher educators should integrate diverse assessment methods, including informal dialogue, peer assessment, self-assessment, and argumentation, into their pedagogical practices. This shift would enable a more comprehensive evaluation of students' mathematical thinking and provide more opportunities for timely feedback that promotes mathematization.

Moreover, enhancing teacher educators' knowledge and skills in formative assessment through professional development programs is crucial. These programs should emphasize the importance of diverse assessment strategies and their role in promoting higher-order thinking and mathematization in pre-service mathematics teachers. By adopting these practices, educators can better prepare future teachers to implement similar strategies in their classrooms, ultimately improving mathematics education in schools across Ethiopia.

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