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GENERATORS AND INNER AUTOMORPHISM

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Abstract
This paper presents the generators and computation of inner automorphism where the group of order 6 and 12 are used. The symmetry and the dihedral group is obtained through rotation and reflection of triangle and hexagon and the permutations generated by the generators is obtained by taking the products for order 6 and 12 respectively. The multiplication table for cyclic group will be generated and presented with it generators and relation respectively. The multiplication table for $A_4, C_3 \times C_4$, $C_2 \times C_3, C_2 \times C_6$ *is used for the computation of inner automorphism of a symmetry group of order 6 and the dihedral group of order 12 and the images is also shown.*

Key Words: *Generators* **,** ***I****nner automorphism, Permutation, Symmetry.*

Introduction

An automorphism is an isomorphism from a mathematics object to itself. It is, in some sense a symmetry of the object and a way of mapping the object to itself while preserving all of its structure. The set of all automorphisms of an object forms a group called the automorphisms group. It is, loosely speaking, the symmetry group of the object. This work is basically concerned with the commutation of an inner automorphism which is a certain type of automorphism of a group defined in term of a fixed element of the group called the conjugating element. Finally, if G is a group and y is an element of G, then the inner automorphism defined by y is the map f from G to itself defined for all x in G by the formula

$$f_y(x) = y^{-1}xy \qquad \ldots(1)$$

Here we use the convention that group element act on the right. The operation $x = y^{-1}xy$ is called conjugation. Milne (2017). If we said that the conjugation of x by y leaves x unchanged, it means that y and x commute.

$$\mathrm{y^{-1}xy} \Leftrightarrow \mathrm{yx = xy} \qquad \ldots.(2)$$

Therefore, the existence and number of inner automorphism that are not the identity mapping is a kind of measure of the failure of the commutation law in the group.This paper is divided into four sections. In Section two, definitions of some terms used are given and discussion on recent work on this area is presented. In section three, some important and related theorems are stated, while group of order 6 and order 12 are showcased for the computation of inner automorphism in section four.

Section 2

2.1 Symmetry Group: Symmetry group of an object is the group of all transformation under which the object is invariant with composition as the group operation (Rose ,1978).

2.2 Dihedral Group: Dihedral group is the group of symmetry of a regular shape which includes rotation and reflections(Gardiner, 1980).

2.3 Finitely generated group: Finitely generated group is a group G that has some finite generating set S, So that every element of G can be written as the combination (under the group operation) of finitely many elements of the finite set S and of inverses of such elements(Rose, 1978)

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2.4 Finitely generated Abelian group: An Abelian $gp(G,+)$ is called finitely generated if there exist finitely many element $x_{1,}x_{2,}x_3 \dots \dots . . x_6 \; \forall \; x \in G$ can be written in the form $x = n_1x_1 + n_1x_1 + \cdots n_6x_6$ with interger $n_1 \dots n_6$ we said that$\left(x_{1,}x_{2,}x_3 \dots \dots . . x_6\right)$ is a generating set of G.

2.5 Permutation of S_n: Symmetries group is a special type of group. Its characteristic importance lies in the fact that any group can be identified with a subgroup of a symmetry group.

An element of S_n may be regarded as a permutation of integer $\{1,2,3 \dots . n\}$. let α be an element of S_n, it is usual to denote α by

$$\alpha = \begin{pmatrix} 1 & 2 & 3 & \cdots n \\ \alpha_1 & \alpha_2 & \alpha_3 & \alpha_n \end{pmatrix}$$

Where the notation indicates that $\alpha(1) = \alpha_1, \alpha(2) = \alpha_2, \dots \alpha(n)$

If β be an element of S_n, it is usual to denote β by

$$\beta = \begin{pmatrix} 1 & 2 & 3 & \cdots n \\ \beta_1 & \beta_2 & \beta_3 & \beta_n \end{pmatrix}$$

The composition $\beta * \alpha = \begin{pmatrix} 1 & 2 & 3 & \cdots n \\ \beta\alpha_1 & \beta\alpha_2 & \beta\alpha_3 & \beta\alpha_n \end{pmatrix}$

Some of the previous work done by researchers on Automorphism includes

Khurana and Bala (2017), In their article, they proved that the group of inner automorphism of G is isomorphic to some other groups depending upon the number of centralizers n. moreover they said that for some group G, the group of inner aautomorphism $Inn(G)$ has order 6 or 9 then G will be C_5 group and if for some group H, the group of inner automorphism Inn(H) has order 4 then H will be C_4 group and conversely. Jaber and Yasein (2016), showed that for a field K of characteristic not 2 and $A = A_0 + A_1$ be the central simple supperalgebra over K, and they let $*$ to be supperinvolution on A. Nasrabadi and Farimani (2015), study some properties of automorphism group G and give necessary and sufficient conditions on G such that $Aut_i(G) = Inn(G)$. According to Bonanome et al., (2012), they let G be any group for which there is a least j such that $Z_j = Z_{j+1}$ in the upper central series. Cannon and Holt (2003), developed a new method for computing the automorphism group of finite permutation group and for testing two such groups for isomorphism. Araujo, et al (2010), presented an algorithm that can be used to calculate the automorphism group of finite transformation semigroup. The general algorithm employs special method to compute the automorphism group of a finite simple semigroup.

Section 3

Theorem 3.1

Let $Aut(G)$ be the set of all automorphism of a group G. Then $Aut(G)$ is a group

Proof: Let G be a group and let $\theta, \emptyset$ be automorphism of G then $\theta, \emptyset$ are homomorphism.

$\theta \cdot \emptyset(xy) = \theta[\emptyset(x)\emptyset(y)] = \theta \cdot \emptyset(x) \cdot \theta \cdot \emptyset(y)$ so that $\theta\emptyset$ is an homomorphism which is also one to one and onto, hence is a automorphism.

For automorphism $\theta, \emptyset$, associativity of composition of mapping implies that $\theta(\emptyset\theta)$. Also identity mapping on G is an identity automorphism and for each $\theta \in Aut(G)$, $\theta^{-1} \in Aut(G)$ since θ^{-1} is bijective and for all $xy \in G$

$\theta^{-1}(xy) = \theta((xy)^{-1}) = \theta(y^{-1}x^{-1}) = \theta(y^{-1})\theta(x^{-1})$ but

$$\theta^{-1}(xy) = [\theta(xy)^{-1}] = [\theta(x) \cdot \theta(y)] = \theta(y)^{-1} \cdot \theta(x)^{-1}$$

$\therefore \; \theta(y)^{-1}\theta(x)^{-1} = \theta(y^{-1}x^{-1})$. Thus θ^{-1} is an homorphism

Hence, for every $\theta \in Aut(G)$ there is θ^{-1}

Theorem 3.2

An automorphism maps an element of the order n to an element of the same order

Proof: let $f: G \longrightarrow G$ be an automorphism and $x^n = e \; in \; G \; for \; x \in G$ then

$f(x^n) = f(x \times x \times x \times x \dots \dots \dots . \times x) = f(x) \cdot f(x) \cdot f(x) \dots \dots \dots . f(x)$ for f an homomorphism.

$\therefore f(x^n) = \left(f(x)\right)^n = x^n = e$ therefore $f(x)$ has order n. thus any automorphism of a group maps identity to itself
Theorem 3.3. Let $I(G)$ be the set of all inner automorphism on a group G. Then $I(G)$ is a group and is a normal subgroup of $Aut(G)$
Proof: let $I(G) = \{f_a : a \in G\}$ where $f_a(x) = a^{-1}xa \;\forall\; x \in G$ if $f_a, f_b \in I(G)$ then

$$f_a f_b(x) = f_a(b^{-1}xb) = a^{-1}b^{-1}xba = (ba)^{-1}xba = f_{ba}(x)$$
$$\therefore f_a f_b(x) = f_{ba}(x)$$

Thus $f_{ba}(x)$ is an inner automorphism and since $ba \in G$, $f_{ba} = I(G)$, therefore $I(G)$ is close with respect to product of associativity, if $f_a, f_b, f_c \in I(G)$ for all $x \in G$ then

$$f_a \cdot f_b \cdot \left(f_c(x)\right) = f_a \cdot f_{bc}(x) = f_{(ab)c}(x) = f_{a(bc)}(x)$$

$\therefore (f_a \cdot f_b) \cdot f_c = f_a \cdot (f_b \cdot f_c)$ for $a = e$, the identity in G, $f_e(x) = exe = x$ and $f_a f_e(x) = f_{ea}(x) = f_a(x)$, for all $x \in G$ showing that f_e is an identity element in $I(G)$, finally for each $f_a \in I(G)$, then f_a^{-1} is also in $I(G)$ such that
$f_a f_a^{-1}(x) = f_a(axa^{-1}) = a^{-1}axa^{-1}a = x = f_e(x) \implies f_a^{-1}$ is the inverse of f_a. Thus $I(G)$ is a group
Let $g \in Aut(G)$ and $f_a \in I(G)$. Suppose that $g(x) = y \; and \; g^{-1}(a) = b, \forall\, x, y, a, b \in G$ then

$$g^{-1} \cdot f_a \cdot g(x) = g^{-1}\left(f_a(y)\right) = g^{-1}(a^{-1}ya) = g^{-1}(a^{-1})g^{-1}(y)g^{-1}(a)$$
$$\therefore g^{-1}(a^{-1})g^{-1}(y)g^{-1}(a) = \left(g^{-1}(a)\right)^{-1}g^{-1}(y)g^{-1}(a) = b^{-1}xb$$

Hence, since $g(x) \in G$, then $g^{-1}f_a g(x) \in I(G)$ but f_a is arbitrary member of $I(G)$ therefore, if G is abelian for every $a \in G$ we shall have

$$f_a(x) = a^{-1}xa = x, x \in G$$

which is an identity mapping therefore an automorphism (which is the only one possible i.e $|I(G)| = 1$, hence, $I(G)$ is a trivial group but the case is different for non-abelian i.e. if G is non abelian then $I(G)$ is non trivial group.
Lemma 3.1: let G be a group and $g \in G$. Then the map $f_y : G \longrightarrow G$ given by $f_y(x) = y^{-1}xy$, for all $x \in G$ is an inner automorphism.

3.3 Group of order 6

Let G be a group of order 6. There are at least non isomorphism abelian groups of order 6 namely: $C_6, C_2 \times C_3$. if G is abelian it is isomorphism to one of this groups.C_6, the cyclic group of order 6 described via the generator a with relation $a^6 = 1$
S_3, the symmetry group on three elements (position of equilateral triangle)

$$S_3 = \{(1), (123), (132), (12), (13), (23)\}$$

Permutation generated by a, b by taking products
For $e = (1)$ setting $a = (123)$ and $b = (132)(12)$ we have :
$a^2 = (132), a^3 = (1), b^2 = (1), ab = (12), a^2b = (23)$
Now assuming G is non abelian group of order 6 then G has an element of order 3 and no element of order 6
Let $a \in G$ be an element of order 3 and we put $H = gp(a) = \{e, a, a^2, a^3\}$ then $G = H \cup Hb$ for some $b \in G$ that is $G = \{e, a, a^2, \} \cup \{e, b, ab, a^2b\}$ implies that
$G = \{e, a, a^2, , b, ab, a^2b\}$ since $H \triangle G$ and $b^2 \in H$ we have the possibility that $b^2 = e$
If $b^2 = e$. Let $k = gp(b)$, then $H \cap K = e$ and $G = HK$
Now $H \triangle G$ so that $b^{-1}ab \in H$ and since a is of order 3. We have $b^{-1}ab = a \; or \; a^3$ if $b^{-1}ab = a$ then $ab = ba$ implies that G is abelian which is contrary to the assumption so $b^{-1}ab \neq a$, hence, $b^{-1}ab = a^3$
Which implies that $(b^{-1}ab)b = a^2b \implies ba(b^{-1}b) = a^2b$,

$$\therefore ba = a^2b$$

Hence $G = \{e, a, a^2, a^3, b, ab, a^2b\}$ where $ba = a^2b$, $b^2 = a^3 = e$

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3.4 Group of order 12

Let G be a group of order 12. Then there are up to isomorphism, exactly five group of order 12: $C_{12}, C_2 \times C_6, C_3 \times C_3, Alternating\ group\ A_4,\ C_2 \times S_3$. There are two Abelian and three non abelian groups
C_{12}, The cyclic group of order 12 described via the generator a with relation $a^{12} = 1$
Let $a \in G$ be an element of order 6 and we put $H = gp(a) = \{e, a, a^2, a^3, a^4, a^5\}$ then $G = H \cup Hb$ for some $b \in G$ that is

$$G = \{e, a, a^2, a^3, a^4, a^5\} \cup \{e, b, ab, a^2b, a^3b, a^4b, a^5b\} \text{ implies that}$$
$$G = \{e, a, a^2, a^3, a^3, a^4, a^5\ b, ab, a^2ba^3b, a^4b, a^5b\}$$

since $H \triangle G$ and $b^2 \in H$ we have the possibility that $b^2 = e$
If $b^2 = e$. Let $k = gp(b)$, then $H \cap K = e$ and $G = HK$
Now $H \triangle G$ so that $b^{-1}ab \in H$ and since a is of order 6. We have $b^{-1}ab = a\ or\ a^3$ if $b^{-1}ab = a$ then $ab = ba$ implies that G is abelian which is contrary to the assumption so $b^{-1}ab \neq a$ hence $b^{-1}ab = a^5$ which implies that $(b^{-1}ab)b = a^5b \implies ba(b^{-1}b) = a^5$

$$\therefore ba = a^5b. \text{ Hence}$$

$G = \{e, a, a^2, a^3, a^4, a^5\ b, ab, a^2b, a^3b, a^4b, a^5b\}$ where $ba = a^5b, b^2 = a^6 = e, a^2b = a^4b, a^3b = a^3b,$
$C_3 \times C_4$, The semi direct product of a cyclic group of order 4 acting on a cyclic group of order 3 described via the generator $a\ and\ b$ with relation $b^2 = a^3, ab = a^5b, a^6 = e$.

4.0 Computation of Inner Automorphisms

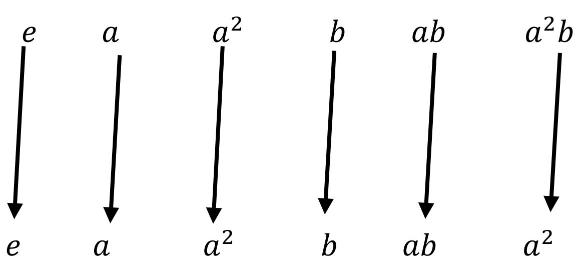
Inner automorphism is a certain type of automorphism of a group define in terms of a fixed element of the group called the conjugating element
If G is a group and y is an element of G, then the inner automorphism defined by y is the map $f: G \longrightarrow G$ defined for all $x \in G$ by the formula
$f_y(x) = y^{-1}xy$

4.1 Inner Automorphism Group of order 6
$G = \{e, a, a^2, a^3, b, ab, a^2b\}$ be a group with the defining relation $ba = a^2b\ \ b^2 = a^3 = e$
Now setting $y = e$ then

$$f_e(e) = e, f_e(a) = a, f_e(a^2) = a^2, f_e(b) = b, f_e(ab) = ab, f_e(a^2b) = a^2b$$

Thus

f_e:



Which is an identity mapping
(ii) Setting $y = a$, we obtain

$$f_a(e) = a^{-1}ea = e, \quad f_a(a) = a^{-1}aa = a^2aa = a, \quad f_a(a^2) = a^{-1}a^2a = a^2a^2a = a^2$$
$$f_a(b) = a^{-1}ba = a^2ba = b, \quad f_a(ab) = a^{-1}aba = a^2aba = a^2b,$$
$$f_a(a^2b) = a^{-1}a^2ba = a^2a^2ba = a^2b = a$$

Thus

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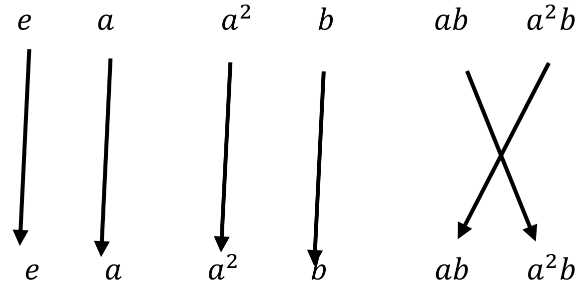
f_a:



(iii) Setting$y = a^2$, we obtain

$f_{a^2}(e) = (a^2)^{-1}ea^2 = aea^2 = e$, $f_{a^2}(a) = (a^2)^{-1}aa^2 = aaa^2 = a$

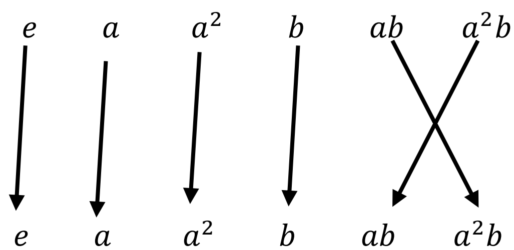
$f_{a^2}(a^2) = (a^2)^{-1}a^2a^2 = aa^2a^2 = a^2$, $f_{a^2}(b) = (a^2)^{-1}ba^2 = aba^2 = b$

$f_{a^2}(ab) = (a^2)^{-1}aba^2 = aaba^2 = ab = a^2b$,

$f_{a^2}(a^2b) = (a^2)^{-1}a^2ba^2 = aa^2ba^2 = a^2b = ab$

Thus:

f_{a^2}:



(v) Setting $y = b$, we obtain

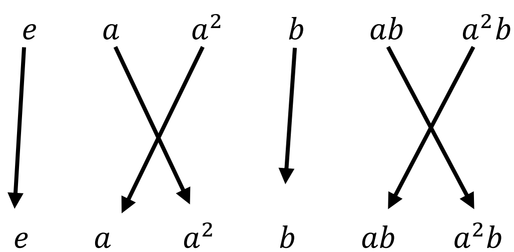
$f_b(e) = b^{-1}eb = e$, $f_b(a) = b^{-1}ab = bab = ba^2 = a$

$f_b(a^2) = b^{-1}a^2b = ba^2b = bab = a$, $f_b(b) = b^{-1}bb = bbb = b$

$f_b(ab) = b^{-1}abb = babb = a^2b$, $f_b(a^2b) = b^{-1}a^2bb = ba^2bb = a^2b = ab$

Thus

f_b:



(v) Setting $y = ab$, we obtain

$f_{ab}(e) = (ab)^{-1}eab = e$, $f_{ab}(a) = (ab)^{-1}aab = a^2baab = a$

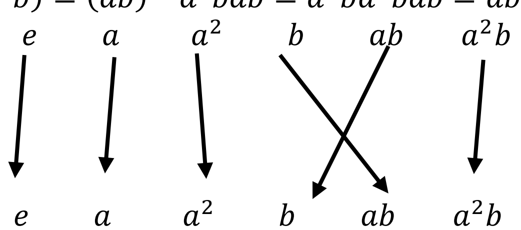
$f_{ab}(a^2) = (ab)^{-1}a^2ab = a^2ba^2ab = a^2$, $f_{ab}(b) = (ab)^{-1}bab = a^2bbab = abbab = a^2b = ab$,

$f_{ab}(ab) = (ab)^{-1}abab = a^2babab = ababab = b$

$f_{ab}(a^2b) = (ab)^{-1}a^2bab = a^2ba^2bab = aba^2bab = ab = a^2b$

Thus

f_{ab}:



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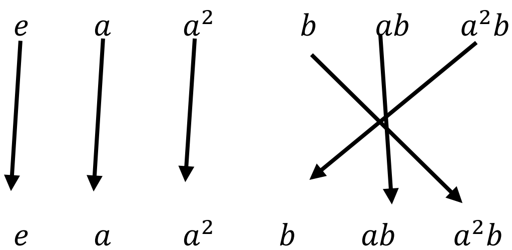
(vi) Setting$y = a^2b$, we obtain

$f_{a^2b}(e) = (a^2b)^{-1}ea^2b = abea^2b = e,\quad f_{a^2b}(a) = (a^2b)^{-1}aa^2b = abaa^2b = a$
$f_{a^2b}(a^2) = (a^2b)^{-1}a^2a^2b = aba^2a^2b = a^2,\quad f_{a^2b}(b) = (a^2b)^{-1}ba^2b = abba^2b = a^2bba^2b = ab = a^2b,\quad f_{a^2b}(ab) = (a^2b)^{-1}aba^2b = ababa^2b = a^2baba^2b = a^2b = ab$
$f_{a^2b}(a^2b) = (a^2b)^{-1}a^2ba^2b = aba^2ba^2b = a^2ba^2ba^2b = b$

Thus

f_{a^2b}:



Therefore

$I(G) = \{f_e, f_a, f_{a^2}, f_b, f_{ab}, f_{a^2b}\}$ Where $f_a = f_{a^2}$

4.2 Inner Automorphism Group of order 12

Let $G = \{e, a, a^2, a^3, a^4, a^5, b, ab, a^2b, a^3b, a^4b, a^5b\}$ be a group with the defining relation $ba = a^5b, b^2 = a^6 = e, a^2b = a^4b, a^3b = a^3b,$

Thus $f_y: G \longrightarrow G \quad \forall\, y \in G$, where $f_y(x) = y^{-1}xy \;\forall\, x \in G$ become the inner automorphism. Hence

(i) Setting $y = e$, we obtain the identity mapping

$f_e(e) = e, f_e(a) = a, f_e(a^2) = a^2, f_e(a^3) = a^3, f_e(a^4) = a^2, f_e(a^5) = a^5, f_e(b) = b, f_e(ab) = ab, f_e(a^2b) = a^2b, f_e(a^3b) = a^3b, f_e(a^4b) = a^4b, f_e(a^5b) = a^5b$. Thus

(ii) Setting $y = a$ we have

$f_a(e) = a^{-1}ea = a^5ea = e,\quad f_a(a) = a^{-1}aa = a^5aa = a$
$f_a(a^2) = a^{-1}a^2a = a^5a^2a = a^2,\quad f_a(a^3) = a^{-1}a^3a = a^5a^3a = a^3$
$f_a(a^4) = a^{-1}a^4a = a^5a^4a = a^4,\quad f_a(a^5) = a^{-1}a^5a = a^5a^5a = a^5$
$f_a(b) = a^{-1}ba = a^5ba = a^5a^5b = a^4b = a^2b,\; f_a(ab) = a^{-1}aba = a^5aba = ab = a^5b,\quad f_a(a^2b) = a^{-1}a^2ba = a^5a^2ba = a^5a^3b = a^2b = a^4b$
$f_a(a^3b) = a^{-1}a^3ba = a^5a^3ba = a^5a^3ba = a^3b,\quad f_a(a^4b) = a^{-1}a^4ba = a^5a^4ba = a^5a^2ba = a^5a^2a^5b = b,\quad f_a(a^5b) = a^{-1}a^5ba = a^5a^5ba = a^5b = ab$

Setting $y = a^2$ we have

$f_{a^2}(e) = (a^2)^{-1}ea^2 = a^4ea^2 = e,\quad f_{a^2}(a) = (a^2)^{-1}aa^2 = a^4aa^2 = a$
$f_{a^2}(a^2) = (a^2)^{-1}a^2a^2\;\; a^4a^2a^2 = a^2,\quad f_{a^2}(a^3) = (a^2)^{-1}a^3a^2 = a^4a^3a^2 = a^3$
$f_{a^2}(a^4) = (a^2)^{-1}a^4a^2 = a^4a^4a^2 = a^4,\quad f_{a^2}(a^5) = (a^2)^{-1}aa^2 = a^4a^5a^2 = a^5$
$f_{a^2}(b) = (a^2)^{-1}ba^2 = a^4ba^2 = b,\quad f_{a^2}(ab) = (a^2)^{-1}aba^2 = a^4aba^2 = ab = a^5b$
$f_{a^2}(a^2b) = (a^2)^{-1}a^2ba^2 = a^4a^2ba^2 = a^2b = a^4b,\quad f_{a^2}(a^3b) = (a^2)^{-1}a^3ba^2 = a^4a^3ba^2 = a^5a^3ba = a^3b,\quad f_{a^2}(a^4b) = (a^2)^{-1}a^4ba^2 = a^4a^4ba^2 = a^4b = a^2b$
$f_{a^2}(a^5b) = (a^2)^{-1}a^5ba^2 = a^4a^5ba^2 = a^5b = ab$

Setting $y = a^3$ we have

$f_{a^3}(e) = (a^3)^{-1}ea^3 = a^3ea^3 = e, \quad f_{a^3}(a) = (a^3)^{-1}aa^3 = a^3aa^3 = a$

$f_{a^3}(a^2) = (a^3)^{-1}a^2a^3 \; a^3a^2a^3 = a^2, \quad f_{a^3}(a^3) = (a^3)^{-1}a^3a^3 = a^3a^3a^3 = a^3$

$f_{a^3}(a^4) = (a^3)^{-1}a^4a^3 = a^3a^4a^3 = a^4, \quad f_{a^3}(a^5) = (a^3)^{-1}a^5a^3 = a^3a^5a^3 = a^5$

$f_{a^3}(b) = (a^3)^{-1}ba^3 = a^3ba^3 = b, \quad f_{a^3}(ab) = (a^3)^{-1}aba^3 = a^3aba^3 = ab = a^5b$

$f_{a^3}(a^2b) = (a^3)^{-1}a^2ba^3 = a^3a^2ba^3 = a^2b = a^4b, \quad f_{a^3}(a^3b) = (a^3)^{-1}a^3ba^3 = a^3a^3ba^3 = a^3b,$

$f_{a^3}(a^4b) = (a^3)^{-1}a^4ba^3 = a^3a^4ba^3 = a^4b = a^2b$

$f_{a^3}(a^5b) = (a^3)^{-1}a^5ba^3 = a^3a^5ba^3 = a^5b = ab$

Setting $y = a^4$ we have

$f_{a^4}(e) = (a^4)^{-1}ea^4 = a^2ea^4 = e, \quad f_{a^4}(a) = (a^4)^{-1}aa^4 = a^2aa^4 = a$

$f_{a^4}(a^2) = (a^4)^{-1}a^2a^4 \; a^2a^2a^4 = a^2, \quad f_{a^4}(a^3) = (a^4)^{-1}a^3a^4 = a^2a^3a^4 = a^3$

$f_{a^4}(a^4) = (a^4)^{-1}a^4a^4 = a^2a^4a^4 = a^4, \quad f_{a^4}(a^5) = (a^4)^{-1}a^5a^4 = a^2a^5a^4 = a^5$

$f_{a^4}(b) = (a^4)^{-1}ba^4 = a^2ba^4 = b, \quad f_{a^4}(ab) = (a^4)^{-1}aba^4 = a^2aba^4 = a^2a^5ba^4 = a^5b = ab,$

$f_{a^4}(a^2b) = (a^4)^{-1}a^2ba^4 = a^2a^2ba^4 = a^2b = a^4b$

$f_{a^4}(a^3b) = (a^4)^{-1}a^3ba^4 = a^2a^3ba^4 = a^3b, \quad f_{a^4}(a^4b) = (a^4)^{-1}a^4ba^4 = a^2a^4ba^4 = a^4b = a^2b,$

$f_{a^4}(a^5b) = (a^4)^{-1}a^5ba^4 = a^2a^5ba^4 = a^2aba^4 = ab = a^5b$

Setting $y = a^5$ we have

$f_{a^5}(e) = (a^5)^{-1}ea^5 = aea^5 = e, \quad f_{a^5}(a) = (a^5)^{-1}aa^5 = aaa^5 = a$

$f_{a^5}(a^2) = (a^5)^{-1}a^2a^5 \; aa^2a^5 = a^2, \quad f_{a^5}(a^3) = (a^5)^{-1}a^3a^3 = aa^3a^5 = a^3$

$f_{a^5}(a^4) = (a^5)^{-1}a^4a^5 = aa^4a^5 = a^4, \quad f_{a^5}(a^5) = (a^5)^{-1}a^5a^5 = aa^5a^5 = a^5$

$f_{a^5}(b) = (a^5)^{-1}ba^5 = aba^5 = a^5ba^5 = a^4b = a^2b, \quad f_{a^5}(ab) = (a^5)^{-1}aba^5 = aaba^5 = a^2ba^5 = a^4ba^5 = a^3b, \quad f_{a^5}(a^2b) = (a^5)^{-1}a^2ba^5 = aa^2ba^5 = a^2b = a^4b$

$f_{a^5}(a^3b) = (a^5)^{-1}a^3ba^5 = aa^3ba^5 = a^4ba^5 = a^2ba^2 = ab = a^5b$

$f_{a^5}(a^4b) = (a^5)^{-1}a^4ba^5 = aa^4ba^5 = aa^4ab = b,$

$f_{a^5}(a^5b) = (a^5)^{-1}a^5ba^5 = aa^5ba^5 = a^5b = ab$

Setting $y = b$ we have

$f_b(e) = b^{-1}eb = beb = e, \quad f_b(a) = b^{-1}ab = bab = a^5bb = a^5$

$f_b(a^2) = b^{-1}a^2b = ba^2b = a^4bb = a^4, \quad f_b(a^3) = b^{-1}a^3b = ba^3b = a^3$

$f_b(a^4) = a^{-1}a^4b = ba^4b = a^2bb = a^2 \quad f_b(a^5) = b^{-1}a^5b = ba^5b = bab = a$

$f_b(b) = b^{-1}bb = bbb = b, \quad f_b(ab) = b^{-1}abb = babb = ab = a^5b$

$f_b(a^2b) = b^{-1}a^2bb = ba^2bb = ba^2bb = a^2b = a^4b, \quad f_b(a^3b) = b^{-1}a^3bb = ba^3bb = a^3b,$

$f_b(a^4b) = b^{-1}a^4bb = ba^4bb = a^4b = a^2b$

$f_b(a^5b) = b^{-1}a^5bb = ba^5bb = a^5b = ab$

Setting $y = ab$ we have

$f_{ab}(e) = (ab)^{-1}eab = a^5beab = e, \quad f_{ab}(a) = (ab)^{-1}aab = a^5baab = abaab = a^3$

$f_{ab}(a^2) = (ab)^{-1}a^2ab \; a^5ba^2ab = aba^2ab = a^4, \quad f_{ab}(a^3) = (ab)^{-1}a^3ab = a^5ba^3ab = aba^3ab = a^5, \quad f_{ab}(a^4) = (ab)^{-1}a^4ab = a^5ba^4ab = a^5a^2bab = a^2$

$f_{ab}(a^5) = (ab)^{-1}a^5ab = a^5ba^5ab = aba^5ab = a, \quad f_{ab}(b) = (ab)^{-1}bab = a^5bbab = a^5ba^5bb = a^4b = a^2b, \quad f_{ab}(ab) = (ab)^{-1}abab = a^5babab = a^5ba^5ba^5b = a^3b, \quad f_{ab}(a^2b) = (ab)^{-1}a^2bab = a^5ba^2bab = a^5a^4ba^5b = a^2b = a^4b$

$f_{ab}(a^3b) = (ab)^{-1}a^3bab = a^5ba^3bab = a^5ba^3ba^5b = ab = a^5b, \quad f_{ab}(a^4b) = (ab)^{-1}a^4bab = a^5ba^4bab = a^5ba^2ba^5b = b, \quad f_{ab}(a^5b) = (ab)^{-1}a^5bab = a^5ba^5bab = a^5b = ab$

Setting $y = a^2b$ we have

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$f_{a^2b}(e) = (a^2b)^{-1}ea^2b = a^4bea^2b = e,\quad f_{a^2b}(a) = (a^2b)^{-1}aa^2b = a^4baa^2b = a^4a^5ba^4b = a,$
$f_{a^2b}(a^2) = (a^2b)^{-1}a^2a^2b = a^4ba^2a^2b = a^2$
$f_{a^2b}(a^3) = (a^2b)^{-1}a^3a^2b = a^4ba^3a^2b = a^4ba^3a^4b = a^5,\quad f_{a^2b}(a^4) = (a^2b)^{-1}a^4a^2b = a^4ba^4a^2b = a^4,\quad f_{a^2b}(a^5) = (a^2b)^{-1}aa^2b = a^4ba^5a^2b = a^2ba^5a^2b = a^3,\quad f_{a^2b}(b) = (a^2b)^{-1}ba^2b = a^4bba^2b = a^4ba^4bb = a^2b = a^4b$
$f_{a^2b}(ab) = (a^2b)^{-1}aba^2b = a^4baba^2b = a^4a^5bba^4b = ab = a^5b$
$f_{a^2b}(a^2b) = (a^2b)^{-1}a^2ba^2b = a^4ba^2ba^2b = a^4ba^4a^4b = b$
$f_{a^2b}(a^3b) = (a^2b)^{-1}a^3ba^2b = a^4ba^3ba^2b = a^4ba^3ba^4b = a^5b = ab$
$f_{a^2b}(a^4b) = (a^2b)^{-1}a^4ba^2b = a^4ba^4ba^2b = a^4b = a^2b$
$f_{a^2b}(a^5b) = (a^2b)^{-1}a^5ba^2b = a^4ba^5ba^2b = a^4baba^4b = a^3b$

Setting $y = a^3b$ we have

$f_{a^3b}(e) = (a^3b)^{-1}ea^3b = a^3bea^3b = e,\quad f_{a^3b}(a) = (a^3b)^{-1}aa^3b = a^3baa^3b = a^3ba^5ba^3b = a^5,$
$f_{a^3b}(a^2) = (a^3b)^{-1}a^2a^3b = a^3ba^2a^3b = a^3a^4ba^3b = a^4$
$f_{a^3b}(a^3) = (a^3b)^{-1}a^3a^3b = a^3ba^3a^3b = a^3,\ f_{a^3b}(a^4) = (a^3b)^{-1}a^4a^3b = a^3ba^4ba^3b = a^3a^2ba^3b = a^2,\ f_{a^3b}(a^5) = (a^3b)^{-1}a^5a^3b = a^3ba^5a^3b = a^3aba^3b = a$
$f_{a^3b}(b) = (a^3b)^{-1}ba^3b = a^3bba^3b = b,\ f_{a^3b}(ab) = (a^3b)^{-1}aba^3b = a^3baba^3b = a^3ba^5ba^3b = a^5b = ab,\quad f_{a^3b}(a^2b) = (a^3b)^{-1}a^2ba^3b = a^3ba^2ba^3b = a^2b = a^4b$
$f_{a^3b}(a^3b) = (a^3b)^{-1}a^3ba^3b = a^3ba^3ba^3b = a^3b,\ f_{a^3b}(a^4b) = (a^3b)^{-1}a^4ba^3b = a^3ba^4ba^3b = a^4b = a^2b,\quad f_{a^3b}(a^5b) = (a^3b)^{-1}a^5ba^3b = a^3baba^3b = ab = a^5b$

Setting $y = a^4b$ we have

$f_{a^4b}(e) = (a^4b)^{-1}ea^4b = a^2bea^4 = e,\quad f_{a^4b}(a) = (a^4b)^{-1}aa^4b = a^2baa^4b = a^4baa^4b = a^3,$
$f_{a^4b}(a^2) = (a^4b)^{-1}a^2a^4b = a^2ba^2a^4b = a^2a^4ba^4b = a^4$
$f_{a^4b}(a^3) = (a^4b)^{-1}a^3a^4b = a^2ba^3a^4b = a^2ba^3a^2b = a,\ f_{a^4b}(a^4) = (a^4b)^{-1}a^4a^4b = a^2ba^4a^4b = a^2ba^4a^2b = a^2,\quad f_{a^4b}(a^5) = (a^4b)^{-1}a^5a^4b = a^2ba^5a^4b = a^5,\quad f_{a^4b}(b) = (a^4b)^{-1}ba^4b = a^2bba^4b = a^2ba^4bb = a^2b = a^4b$
$f_{a^4b}(ab) = (a^4b)^{-1}aba^4b = a^2baba^4b = a^4baba^4b = a^3b,\ f_{a^4b}(a^2b) = (a^4b)^{-1}a^2ba^4b = a^2ba^2ba^4b = a^4ba^4ba^4b = b,\ f_{a^4b}(a^3b) = (a^4b)^{-1}a^3ba^4b = a^2ba^3ba^4b = a^4ba^3ba^4b = a^5b = ab,$
$f_{a^4b}(a^4b) = (a^4b)^{-1}a^4ba^4b = a^2ba^4ba^4b = a^4b = a^2b,\quad f_{a^4b}(a^5b) = (a^4b)^{-1}a^5ba^4b = a^2ba^5ba^4b = a^2baba^4b = ab = a^5b$

(iii) Setting $y = a^5b$ we have

$f_{a^5b}(e) = (a^5b)^{-1}ea^5b = abea^5b = e,\quad f_{a^5b}(a) = (a^5b)^{-1}aa^5b = abaa^5b = a^5baa^5b = a^5,$
$f_{a^5b}(a^2) = (a^5b)^{-1}a^2a^5b = aba^2a^5b = aa^4ba^5b = a^4$

$f_{a^5b}(a^3b) = (a^5b)^{-1}a^3a^3b = aba^3a^5b = a^5ba^3a^5b = a,\quad f_{a^5b}(a^4) = (a^5b)^{-1}a^4a^5b = aba^4a^5b = a^5ba^4a^5b = a^2,\quad f_{a^5b}(a^5) = (a^5b)^{-1}a^5a^5b = aba^5a^5b = a^5ba^5a^5b = a^3,$
$f_{a^5b}(b) = (a^5b)^{-1}ba^5b = abba^5b = a^5bba^5b = a^4b = a^2b,\quad f_{a^5b}(ab) = (a^5b)^{-1}aba^5b = ababab = a^5ba^5ba^5b = a^3b$
$f_{a^5b}(a^2b) = (a^5b)^{-1}a^2ba^5b = aba^2ba^5b = a^5ba^4ba^5b = a^2b = a^4b$
$f_{a^5b}(a^3b) = (a^5b)^{-1}a^3ba^5b = aba^3ba^5b = a^5ba^3ba^5b = ab = a^5b$
$f_{a^5b}(a^4b) = (a^5b)^{-1}a^4ba^5b = aba^4ba^5b = a^5ba^2ba^5b = b$
$f_{a^5b}(a^5b) = (a^5b)^{-1}a^5ba^5b = aba^5ba^5b = a^5b = ab$

Conclusion

Thus, the Inner Automorphism of $C_2 \times C_6$ are
$I(G) = \{f_e, f_a, f_{a^2}, f_{a^3}, f_{a^4}, f_{a^5}, f_b, f_{ab}, f_{a^2b}, f_{a^3b}, f_{a^4b}, f_{a^5b}\}$ where $f_{a^2} = f_{a^3}$

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