

MODELING THE TREND OF PATIENT ATTENDANCE IN PRIMARY HEALTH CENTRE USING LEAST SQUARE METHOD

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Abstract

The study used least square model to investigated the patient attendance in Model Primary Health Centres (MPHCs) in Rivers State. With the application of nonlinear regression models, whose parameters were determined with the least-squares method. It was found that a logarithmic nonlinear model proved the best fit for the source data and the parameters $\alpha=40.460$ and $\beta= 1.270$ gave rise to the model as $N(t)= (40.458)(1.265)^t$, revealing an annual increase rate of 26.5%. The model had 26.02% mean error, reflecting 73.98% accurate fit. The model was used to forecast the patients attendance from 2015-2020. An estimate of 346 patient attendance annual increases aimed the research to recommend for government's expansion of health care infrastructures, more health personnel should be adequately trained to meet the manpower need of the health institutions and that there should be public awareness to encourage rural dweller to readily access the Model Primary Health care's because a healthy nation is a wealthy nation and further studies in the area of factors militating against patients' accessibility to MPHCs in Rivers State was advocated.

Key words: *Least square method, modelling, Primary health centre, trend.*

Introduction

The provision of a health facility is one thing, and accessibility is another thing. The Rivers State government embarked on the construction of model Primary Health Centre across the twenty-three local governments, with the intention to provide affordable health care delivery to curb infant and maternal mortality among the rural and urban populace. Since inception, the model primary health centres had received considerable number of patients for medical diagnosis, treatment, childbirth, family planning, and other medical services. This cannot be unconnected to the need for a qualitative service delivery. In order to ensure greater efficiency or effectiveness in the medical records, statistical information about the patient there is need for a model primary health worker to have statistical knowledge of patient trends. To achieve the statistical information of the patient trend, the use of regression models or methods will be required. To determine the patient trend or forecast the patient attendance to the available health centres, regression model is applied to forecast and also determines the patient trend over a period of time. According to Amadi et al., (2013), making any forecast about the future is that of collecting data on the past observations. The ability to forecast optimally, understand dynamic relationship between variables and control optimally is of a great practical important. Forecasts are essential in both short-term and long-term business plans

Udoka (2014) studied the comprehensive medical information on delivery system. He also made mentioned that adequate historical information about their patients' problems and the need for the involvement of more medical records offices to take charge of day-to-day administrative records of primary healthcare delivery system. It is also important to account for seasonal influence on patient

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attendance as large scale cross-sectional studies which measured the individual's problems in different season (Rich et al., 2012). According to Lawrence et al., (2005) least square is defined as procedure for associating a function with an observed physical phenomenon in order to finding the best fits that has mathematical relevance with minimal deviation from the true values. Regression analysis had remained useful mathematical tools in investigating the relationship that exists between the predictor variable and the response variable. Thus, regression analysis as a branch of inferential statistics compares quantities or variable, to discover relationships that exist between them and to formulate those relationships in useful ways. To this end, Carl Fredrick Gauss (1777-1855) remained the father of least square method of regression analysis.

The study aim at formulating a nonlinear regression model that would statistically minimize the error between the observed data and model predictions of the patients attendance from 2000-2014 periods, and advice policy makers in public health administration through the model forecast on future policies be made.

Materials and Methods

Nonlinear Model

Mathematically, nonlinear regression model is defined $N = f(t, \beta) + e$ where β represents nonlinear parameter, N is the dependent variable called the response and t is the independent variable called the predictor and e is the error term (Steven et al., 2010).

Model Formation

The data collected was finite number of patients who has accessed the Model Primary Health Centre (MPHC) in Etche Local Government Area of Rivers State over a finite period of time, measured in years. The expected model is therefore a direct relationship of number of patients and time period, with some parameters.

Let $N(t)$ be the number of patients who had visited the health centre in a particular year t . Then,

$$N(t) = f(t; \alpha, \beta) + e, \quad t \geq 0 \quad (1)$$

Where $\alpha, \beta = 0$ are some parameters N is the response variable, t is the predictor variable, and e is the error term.

Model Specification

The assumed model $N(t) = f(t; \alpha, \beta) + e$ is an additive one. Estimating the regression function $f(t; \alpha, \beta)$ amount to estimating the curve of mean n values. Thus, a scatter diagram suggests that it is not a simple linear mathematical expression for $f(t; \alpha, \beta)$.

On the other hand, the plot of $y = \log_{10} N$ and $x = t \log_{10} \beta$ showed linear relationship. Therefore, $f(t; \alpha, \beta) = \alpha\beta^t$ is a logarithmic function. Hence, the specified model for the study becomes

$$N(t) = \alpha\beta^t, \quad t \geq 0, \quad \alpha, \beta = 0 \quad (2)$$

Mathematical Formulation

From equation (2), taking the logarithm to base 10, of both sides yields.

$$\log_{10} N(t) = \log_{10}(\alpha\beta^t) \quad (3)$$

$$= \log_{10} \alpha + t \log_{10} \beta$$

$$\log_{10} N = t \log_{10} \beta + \log_{10} \alpha \quad (4)$$

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This becomes a simple linear relationship by making the following assumption.

$$\begin{aligned} \text{Let } y &= \log_{10} N \\ x &= \log_{10} \beta \\ \alpha_0 &= \log_{10} \alpha \end{aligned}$$

This implies

$$y = xt + \alpha_0 \tag{5}$$

To determine the values of x and α_0 the least-square method was adopted. This method is the most widely used statistical method of finding the trend. Thus the equation (5) becomes:

$$Y_t = \alpha_0 + xt$$

where Y_t is the estimated trend value for a given time period t .

α_0 = trend line value at $t = 0$, X = slope of the trend or Change of Y_t per unit time (t), t = the time unit.

Now, the method of least-squares requires that the sum of squared deviations of the observed values from the estimated trend values. Y_t be a minimum.

i.e. $\sum(y - Y_t)^2 = \text{minimum}$

Let the minimum be defined as S

That is $S = \sum(y - Y_t)^2 \tag{6}$

But $Y_t = \alpha_0 + xt$

$\therefore S = \sum(y - \alpha_0 - xt)^2 \tag{7}$

Thus, the parameters α_0 and x are determined by solving the normal equations resulting from equation (5).

$$\begin{aligned} S &= \sum(y - Y_t)^2 \\ \Rightarrow S &= \sum(y^2 - 2yY_t - Y_t^2) \\ &= \sum(y^2 - 2\alpha_0 y - 2xty - \alpha_0^2 + 2\alpha_0 xt + x^2 t^2) \\ S &= \sum y^2 - 2\alpha_0 \sum y + \alpha_0^2 + 2\alpha_0 x \sum t - 2x \sum ty + x^2 \sum t^2 \\ \frac{\partial s}{\partial \alpha_0} &= -2 \sum y + 2\alpha_0 + 2x \sum t \\ \frac{\partial s}{\partial x} &= -2 \sum ty + 2\alpha_0 \sum t + 2x \sum t^2 \end{aligned} \tag{8}$$

At minimum value, the critical points are determined when

$$\begin{aligned} \frac{\partial s}{\partial x} = 0 &\Rightarrow \sum y = n\alpha_0 + x \sum t \\ \sum ty &= \alpha_0 \sum t + x \sum t^2, \quad n > 1 \end{aligned} \tag{9}$$

Solving equations (8 - 9) simultaneously yields

$$X = \frac{n \sum ty - \sum y \sum t}{n \sum t^2 - (\sum t)^2} \tag{10}$$

Such that $\alpha_0 = y - xt$

But $\alpha_0 = \log_{10} \alpha$

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Therefore $\alpha_0 = 10$

And

$$X = \log_{10} \beta$$

$$\Rightarrow \beta = 10^x$$

Error Analysis

The kth error is given $|\varepsilon_n k| = \left| a_{k,j+1} - \frac{a_{k,j}}{a_{k,j+1}} \right| \times 100\%$

$$\varepsilon_n k = \left| a_{k,j+1} \right| \frac{a_{k,j}}{a_{k,j+1}}$$

Where $a_{k,j+1}$ is kth observed value, $a_{k,j}$ is a model estimated value.

Forecast

The model having fitted the data 73.94% accuracy, the table below shows the model forecast for patients' attendance in MPHCs from 2015 to 2020

Results

Model Identification

The graph of the plots from complete data of the patients attendance revealed by observation, the pattern of patients' attendance in Akwa Etche from 2000 to 2014

Figure 4.1 scatter plot of patients' attendance from 2000 - 2014

Method of Solution

The method of solution adopted to determining the parameter values was the least square method based on algebraic manipulation of the variables

Table 1: Working table for determination of parameters

T	N	Y=log ₁₀ N	ty	t ²
0	58	1.76	0	0
1	59	1.77	1.77	1
2	63	1.80	3.60	4
3	75	1.88	5.64	9
4	121	2.08	8.32	16
5	137	2.14	10.70	25
6	170	2.23	13.38	36
7	172	2.24	15.68	49
8	187	2.27	18.16	64
9	188	2.27	20.43	81
10	353	2.55	25.50	100
11	353	2.55	28.05	121
12	616	2.79	33.48	144
13	1626	3.21	41.73	169
14	1885	3.28	45.92	196
∑t=105	∑N=6063	∑y=34.82	∑ty=272.36	∑t ² =1015

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Determination of parameters

$$X = \frac{n\sum ty - \sum t \sum ty}{n\sum t^2 - (\sum t)^2} \tag{11}$$

From Table 1,

$$n = 15, \sum ty = 272.36, \sum t = 105, \sum t^2 = 1015, \sum y = 34.82$$

$$\therefore X = \frac{(15)(272.36) - (105)(34.82)}{(15)(1015) - (105)^2}$$

$$\Rightarrow X = 0.102$$

Also $\alpha_0 = \bar{y} - x\bar{t}$

But $\bar{y} = \frac{\sum y}{n} = \frac{34.82}{15} = 2.321$

$$\bar{t} = \frac{\sum t}{n} = \frac{105}{15} = 7.0$$

$$\therefore \alpha_0 = 2.321 - (0.102)(7)$$

$$\Rightarrow \alpha_0 = 1.607$$

Hence $\alpha = 10 = 40.458$

$$\beta = 10^x = 1.265 \tag{12}$$

but $N(t) = \alpha\beta^t, t > 0$

$$\Rightarrow N(t) = (40.458)(1.265)^t$$

The equation is the required nonlinear model estimating the patients attendance in MPHCs in Rivers State, from 2000-2014 under review.

Model Fitting

The observed data and the model values were plotted on the same graph to reveal the extent to which the nonlinear model fitted the true values.

Table 2: Nonlinear model fitting

t(years)	N:OV	N(t):MV	$\epsilon_t\%$
2000	58	40	31
2001	59	51	14
2002	63	65	3
2003	75	82	9
2004	121	104	14
2005	137	131	4
2006	170	166	2
2007	172	210	22
2008	187	265	42
2009	188	336	79
2010	353	425	20
2011	353	537	52
2012	616	679	10
2013	1626	859	47
2014	1885	1087	42

OV=observed value, MV=model value

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Table 3: Nonlinear model fitting

t	N:OV	y	10 ^y
0	58	1.607	40
1	59	1.709	51
2	63	1.811	65
3	75	1.913	82
4	121	2.015	104
5	137	2.117	131
6	170	2.219	166
7	172	2.321	210
8	187	2.423	265
9	188	2.525	336
10	353	2.627	425
11	353	2.729	537
12	616	2.831	679
13	1626	2.933	859
14	1885	3.035	1087

Recall that $y = a_0 + xt$

$$\Rightarrow y = 1.607 + 0.102t$$

But $y = \log_{10} N \therefore N = 10^y = 10^{1.607+0.102t}$

$$\therefore \sum |\varepsilon_{nk}| = 391$$

$$\therefore \varepsilon\% = 26.06\%$$

Table 4: Patients’ attendance forecast

Year	Number of patients
2015	1375
2016	1739
2017	2200
2018	2783
2019	3521
2020	4454

Annual rate of increase $r=26.5\%$

Discussion of Findings

The nonlinear particular model $N(t)=(40.458)(1.265)^t$ fitted the data accurately for eight data points. The mean error was 26.06%, reflecting 73.94% accuracy, which was acceptable. The model $N(t)$ can be expressed as $N(t)= 40.458 (1+0.265)^t$ showing an annual rate of 26.5% patients attendance. The statistical implication revealed an approximate number of 364 patients every year accessing the health facilities in Rivers State. Nonlinear logarithmic regression model has offered an approximate estimate of patents attendance in Model Primary Health Centres.

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Conclusion

The study agreed with the application of nonlinear regression model as applied to the investigation of the relationship between shoulder height and Antler size (Roger, 1983). Therefore, nonlinear models are intrinsically linearly related response and predictor variable-wise. The forecast was valid for only but a short period of years because other factors that could either hamper or encourage patients attendance were not considered. Therefore, the model is fit to monitor growth in the patient attendance record.

Recommendations

From the statistical evidence inherent in the nonlinear regression model with 26.5% annual increase rate, it became pertinent to recommend to the government the following policy trust.

- i. The government should be prepared to expand infrastructures in the public health sector.
- ii. More health personnel should be adequately trained to meet the manpower need of the health institutions.
- iii. There should be public awareness to encourage rural dweller to readily access the Model Primary Health care's because a healthy nation is a wealthy nation.

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