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Mathematical Modeling and Analysis of Kidnapping as a Communicable Disease

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Abstract

Kidnapping affects most African countries, including Nigeria, and has become a pressing national issue. This study develops and analyzes a mathematical model of kidnapping dynamics, treating it as a communicable process. The model incorporates key compartments including susceptible individuals, kidnappers, abductees, and rescued individuals. The basic reproduction number R0R_0 is derived and used as a threshold parameter. Stability analyses is carried out using linearization and Lyapunov functions are performed to determine the conditions under which the kidnapping-free and endemic equilibria are stable. Sensitivity analysis identifies the most influential parameters on R0R_0, notably the direct recruitment rate of kidnappers. Numerical simulations validate the model and reveal that when R0<1R_0 < 1, all kidnapping-related compartments diminish over time, indicating that kidnapping can be eradicated under appropriate conditions. Conversely, when R0>1R_0 > 1, kidnapping becomes endemic. The results underscore the significance of reducing the recruitment rate of kidnappers and enhancing rescue operations as effective strategies to combat the spread of kidnapping. These findings provide valuable insight for policymakers and security agencies to design informed and impactful interventions.

INTRODUCTION

kidnapping has become endemic in the northwest of the West African as roving gangs of armed men abduct people from villages, highways and schools and demand ransom money from their relatives, Reuters (2022). Kidnapping is among the current major social problems in Nigeria, affecting the free movement of people and the socio-economic development of the country. The few people that are benefiting from the act are known as conflict entrepreneurs who are gaining directly or indirectly. Kidnapping has led to insecurity, loss of lives, poverty in the areas affected. Okoli and Agada (2014) Kidnapping is organized and systematic than armed robbery but highly profitable Dodo (2010) Kidnapping is a criminal offense that occurs when a person abducts another person without his consent for the main purpose of financial ransom, political objectives, or other nefarious objectives. Inyang and Abraham (2013) Kidnapping is the unlawful detention of a person through the use of force, threats, fraud or enticement. The purpose is an illicit gain; economic or material, in exchange for liberation. It may also be used to pressure someone into doing something or not doing something. Nigeria has one of the world's highest rates of kidnap for-ransom cases. Other countries high up on the list include Venezuela, Mexico, Yemen, Syria, the Philippines, Iraq, Afghanistan and Somalia, etc. Aloegba (2015. Since 2019, there have been 735 mass

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abductions in Nigeria, according to socio-political risk consultancy firm, SBM intelligence (Sears Business Monitor) e. It said between July 2022 and June 2023, 3620 peoples were abducted in 582 kidnapping cases with about 5billion Naira (3,878,390Doller) paid in ransoms. SBM Intelligence (2024).

In the past three years, a sharp rise in insecurity has led to gangs kidnapping hundreds of peoples for ransom in Nigeria, and staff of prosperous agricultural enterprises have been particularly targeted, forcing many farmers to abandon or reduce operations. More than 350 farmers were kidnapped or killed in 12 months up to June 2022 according to a Nigerian security tracking website. Nwaubani, (2023). According to a report by SBM intelligence between July 2023 when the Bola Tinubu assumed charge as president, and June 2024 at least 7,568 peoples were abducted in Nigeria, with kidnappers demanding approximately 10.9 billion in ransom and receiving about 1.05billion. Naira and also 2308 deaths were recorded due to various violent incidents, including attacks from terrorists, bandits, and kidnappers. SBM Intelligence, (2024); Aliyuidowu, (2024); Alli, (2024); Udi, (2024); Aro,(2024). In the Northern States banditry and kidnapping is rampant in states such as Zamfara, Katsina, Sokoto, Niger, Kaduna, Kebbi, Niger, Abuja and Kogi. The kidnapping groups usually operate both in the rural and urban areas in Northern Nigeria, and often demand for high ransom payment from the families of the victims, and consequently leave the families and victims financially bankrupt, in many instances bandits enforce levies to many communities in the rural areas Faruk, and Abdullahi, (2022); Daily Trust, (2025); PremiumTimes,(2024). In some cases, kidnappings are meticulously planned operations targeting high-profile individuals or wealthy families, while in others, they may be opportunistic crimes committed for financial gain or as acts of retaliation or intimidation Okoli,and Agada,(2014). Kidnapping is a significant societal challenge with far-reaching consequences for individuals, communities, and governments. Understanding the dynamics of kidnapping incidents is crucial for devising effective prevention and intervention strategies. Inyang & Abraham, 2013. However, a great number of people have been abducted and kidnapped by criminals for a variety of reasons and purposes, such as rape, illegal sexual relations, selling human parts for ritual sacrifice, political retaliation, slavery, ransom-bargaining, marriage, murder or assassination, unlawful activities, and other reasons. Oyewole, 2016.

Nasidi (2024) introduced a fractional optimal control model to study the dynamics of the ransom kidnapping epidemic, applying fractional calculus to better capture the memory and hereditary properties of the system. His approach differs from traditional integer-order models by considering the non-local characteristics of real-world kidnapping scenarios, where past events influence current behaviors and outcomes. The model incorporates key compartments such as potential victims, kidnappers, and law enforcement, with control variables representing interventions like security reinforcement, public awareness, and negotiation policies. Faruk and Abdullahi (2022) provide a detailed empirical assessment of the socio-economic impacts of armed banditry and kidnapping in Katsina State, Nigeria. Their study reveals that kidnapping, alongside banditry, has disrupted agricultural productivity, trade, and education across several local government areas. Ogundoyin et al. (2020) developed a strategic anti-kidnapping model aimed at enhancing the security of travelers, particularly in high-risk regions. Their work addresses the increasing trend of abductions targeting individuals in transit, such as on highways and inter-city routes, which are common in Nigeria's current security landscape. The model integrates surveillance technology, predictive analysis, and real-time data to assess risk and provide alerts, offering a proactive mechanism to prevent potential kidnapping incidents. The current level of insecurity has current level of insecurity has severely hampered the country's security architecture, so more information is needed to address the recent surge in violent criminal attacks in Nigeria. The combination of terrorist and banditry attacks in the country has not only put a significant financial strain on fiscal policy but has also compelled an increase in the amount of money the national government spends on security (Aina, 2024; Ogunyale, 2024; Sahara Reporters, 2024).. Along with cattle rustling, kidnapping and kidnapping for ransom were embraced as new tactics. Many impoverished young people from many communities served as informants, providing valuable intelligence to large financial interests. Okoli, and Agada, (2014); AFP, (2016). According to a summary of the statistics, Abia State led the group with 110 kidnapping cases overall, 58,109 arrests, 41 prosecutions, and one fatality. Akwa Ibom registered 40 kidnapping instances, 418 arrests, 11 prosecutions, and 43 releases. Delta recorded 44 kidnapping cases, 27 arrests, 31 prosecutions, and one death. The study also stated that kidnappers stole over 600 million dollars between July and September 2008 and July 2009. Also in Zamfara over 5000 kidnapping cases were recorded. But beyond statistics being available, it is a known fact that most kidnapping cases are never reported to the police authority for fear of murdering the victims; hence, most families prefer to pay ransom than to lose one of their own, The Nation, (2009); U.S. Department of State, (2010); Aminu,(2014).

The Existing Model

The human population satisfies the equation

$N = K_s + K_{p_a} + K_{p_e}$.

$\frac{dK_s}{dt} = \lambda N + wK_{p_e} - \gamma K_{p_a}\frac{K_s}{N} - \mu K_s - \beta K_{p_a}\frac{K_s}{N}$

$\frac{dK_{p_a}}{dt} = \gamma K_{p_a}\frac{K_s}{N} - \mu K_{p_a} - a_1 K_{p_a}K_{p_e}$

$\frac{dK_{p_e}}{dt} = \beta K_{p_a}\frac{K_s}{N} - \mu K_{p_e} - a_2 K_{p_a}K_{p_e} - wK_{p_e}$

$\frac{dN}{dt} = (\lambda - \mu)N - (a_1 + a_2)K_{p_a}K_{p_e}$ Okrinya, 2018.

Model Formulation

Kidnappers are mainly encouraged through negotiations and ransom payment. Rescue of victims by security agents without payment of ransom may make kidnapping less attractive. The timescale in which a victim remains in the kidnappers' den before rescue is a very important determinant of the safety and survival of the victim.

The model population is divided into four subgroups, namely:

- Susceptible individuals (S),
- Kidnappers' individuals (K),
- Abductee individuals (A),
- Rescue individuals (R).

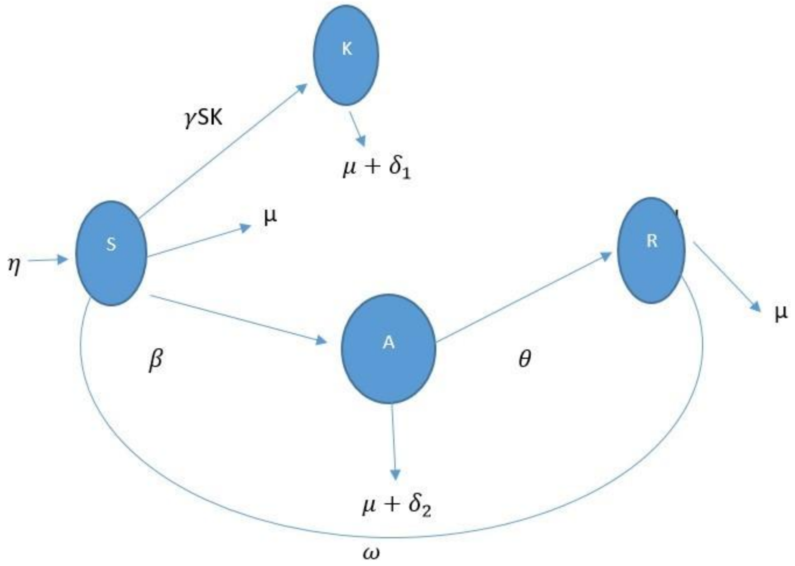


Figure 1: schematic diagram of the model

The population of the susceptible compartment (S) increases due to daily recruitment, either through birth or immigration, at the rate η, and movement from the rescue compartment (R) to the susceptible compartment at the rate ω. It decreases due to natural death at the rate μ, and due to kidnapping at the rate $\gamma SK/N$. The probability of a susceptible person meeting a kidnapper is assumed to be proportional to K/N, leading to movement to the abductee compartment (A) at the rate $\pi SK/N$.

The population of kidnappers (K) is generated from the susceptible population at the rate $\gamma SK/N$, and decreases due to natural death at the rate μ, as well as the death of kidnappers during operations at the rate δ_1.

The abductee population (A) is generated from the susceptible compartment (S) at the rate $\pi SK/N$. It decreases due to natural death at the rate μ, and also due to the death of abductees during operations at the rate δ_2.

The rescue compartment (R) increases due to the rescue rate θ, and decreases due to movement from the rescue compartment (R) to the susceptible compartment at the rate ω. It also depreciates due to natural death at the rate μ. Where:

- η is the recruitment rate.
- μ is the natural death rate.
- γ is the rate of kidnapping by direct conversion.
- π is the rate of abduction.
- δ_1 is the death rate of kidnappers during operations.
- δ_2 is the death rate of abductees during operations.
- θ is the rescue rate.
- ω is the rate of rescued individuals returning to the susceptible population.

Model Equations

The model equations are derived as follows:

$$\frac{dS}{dt} = \eta - \frac{\gamma SK}{N} - \frac{\pi SK}{N} - \mu S + \omega R \quad (1)$$

$$\frac{dK}{dt} = \frac{\gamma SK}{N} - (\mu + \delta_1)K \quad (2)$$

$$\frac{dA}{dt} = \frac{\pi SK}{N} - (\mu + \delta_2 + \theta)A \quad (3)$$

$$\frac{dR}{dt} = \theta A - (\mu + \omega)R \quad (4)$$

Equilibrium Points

We find the equilibrium points of the model by equating the right hand side of the model equations to zero. They are as follows:

Kidnapped-Free Equilibrium

At this particular equilibrium, all the kidnapping compartments are equal to zero that is to say $A = 0$, $K = 0$,

Thus, to find the kidnapped-free equilibrium we proceed as follows:

$$\frac{dS}{dt} = \eta - \frac{\gamma SK}{N} - \frac{\pi SK}{N} - \mu S + \omega R = 0. \quad (5)$$

This implies that,

$$\eta - \mu S = 0. \quad (6)$$

Thus,

$$S = \frac{\eta}{\mu}. \quad (7)$$

Now, the kidnapped-free equilibrium is given by:

$$E_0 = \left(\frac{\eta}{\mu}, 0, 0, 0\right). \quad (8)$$

Endemic Equilibrium

The endemic equilibrium point is obtained by equating the right-hand side of model equations to zero and solving for *S,A,K,andR*. Thus, from the second equation, we have,

$$\frac{dK}{dt} = \frac{\gamma SK}{N} - (\mu + \delta_1)K = 0. \quad (9)$$

Solving for S gives:

$$\frac{\gamma S}{N} - (\mu + \delta_1) = 0, \quad (10)$$

$\implies$

$$S = \frac{N(\mu + \delta_1)}{\gamma}. \quad (11)$$

Substituting into the equation for abductees population, we have:

$$\frac{dA}{dt} = \frac{\pi SK}{N} - (\mu + \delta_2 + \theta)A = 0, \quad (12)$$

$\implies$

$$A = \frac{\pi(\mu + \delta_1)}{\gamma(\mu + \delta_2 + \theta)}K. \quad (13)$$

Substituting into the equation for rescued equation, that is to say:

$$\frac{dR}{dt} = \theta A - (\mu + \omega)R = 0, \tag{14}$$

we have,

$$R = \frac{\theta\pi(\mu + \delta_1)}{\gamma(\mu + \delta_2 + \theta)(\mu + \omega)} K. \tag{15}$$

Solving for *K*, we obtain:

$$K^* = \frac{(\mu N(\mu + \delta_1) - \eta\gamma)(\mu + \delta_2 + \theta)(\mu + \omega)}{(\mu + \delta_1)[\omega\theta\pi - (\gamma + \pi)(\mu + \delta_2 + \theta)(\mu + \omega)]}, \tag{16}$$

$$A^* = \frac{\pi(\mu N(\mu + \delta_1) - \eta\gamma)(\mu + \omega)}{\gamma[\omega\theta\pi - (\gamma + \pi)(\mu + \delta_2 + \theta)(\mu + \omega)]}, \tag{17}$$

And

$$R^* = \frac{\theta\pi(\mu N(\mu + \delta_1) - \eta\gamma)}{\gamma[\omega\theta\pi - (\gamma + \pi)(\mu + \delta_2 + \theta)(\mu + \omega)]}. \tag{18}$$

Thus, the endemic equilibrium is given by:

$$E_1 = \left(\frac{N(\mu + \delta_1)}{\gamma}, K^*, A^*, R^*\right), \tag{19}$$

Where K^*,A^*,R^* are as derived above.

Boundedness of Solution

Since our compartments are sub-divided into population sizes, when we prove the total population is bounded, then the remaining sub-populations are also bounded.

Theorem 1: The solution of the system of equations is bounded for $t \geq 0$ if they enter the invariant area

$$\Omega = \{(S, K, A, R) \in \mathbb{R}^4 \mid S, K, A, R \geq 0, N(t) \leq \frac{\eta}{\mu}\} \tag{20}$$

Proof: Suppose $(S,K,A,R) \in R^4$ be any solution of the system with a non-negative initial condition. The total population size is given by:

$$N(t) = S(t) + K(t) + A(t) + R(t). \tag{21}$$

Differentiating N with respect to t we get:

$$\frac{dN(t)}{dt} = \frac{dS(t)}{dt} + \frac{dK(t)}{dt} + \frac{dA(t)}{dt} + \frac{dR(t)}{dt}. \tag{22}$$

Substituting the system equations we have:

$$\frac{dN(t)}{dt} = \eta - \mu(S + K + A + R) - \delta_1 K - \delta_2 A. \tag{23}$$

Since $N = S + K + A + R$,

$\Longrightarrow$

$$\frac{dN(t)}{dt} = \eta - \mu N - \delta_1 K - \delta_2 A. \tag{24}$$

This implies that,

$$\frac{dN}{dt} \leq \eta - \mu N, \tag{25}$$

$\Longrightarrow$

$$\frac{dN}{dt} + \mu N \leq \eta. \tag{26}$$

Using the integrating factor method and solving the inequality we have,

$$N(t) \leq \frac{\eta}{\mu} + Ce^{-\mu t}, whereCisaconstant. \tag{27}$$

At $t = 0$:

$$N(0) \leq \frac{\eta}{\mu} + C. \tag{28}$$

Thus, as $t \to \infty$, $e^{-\mu t} \to 0$, giving

$$N(t) \leq \frac{\eta}{\mu}. \tag{29}$$

Hence, all feasible solutions of the model system enter the region

$$\Omega = \{(S, K, A, R) \in \mathbb{R}^4 \mid S, K, A, R \geq 0, N(t) \leq \frac{\eta}{\mu}\}. \tag{30}$$

Thus, the system is positively invariant under the model and is mathematically well-posed in the domain.

Positivity of Solutions

The following result guarantees that the kidnappers model (which is a population of human beings) governed by the model equations has positive solution.

Theorem 2: Let $S(0) > 0$, $K(0) > 0$, $A(0) > 0$, and $R(0) > 0$ be the initial population. Then, $S(t) > 0$, $K(t) > 0$, $A(t) > 0$, and $R(t) > 0$ for all t. *Proof* **:** From

$$\frac{dS}{dt} = \eta - \frac{\gamma SK}{N} - \frac{\pi SK}{N} - \mu S + \omega R, \tag{31}$$

The solution is given by

$$S(t) = S(0)e^{\int_0^t (\eta - \mu s)ds}$$

Now, the exponential function is always positive, thus $S(t) > 0$ since from our assumption $S(0) > 0$. Applying similar methods we verify that

$$K(t) \geq 0, \quad A(t) \geq 0, \textit{ and } \quad R(t) \geq 0 \textit{ for all } t. \tag{32}$$

Basic Reproduction Number

In this section, we use the next-generation matrix method developed by Rufai,,et al (2024)., to compute the basic reproduction number, denoted by R_0. The procedure is as follows: We obtain from our model

$$F = \begin{bmatrix} \frac{\gamma SK}{N} \\ \frac{\pi SK}{N} \end{bmatrix} \quad \text{and} \quad \mathcal{F} = \begin{bmatrix} \frac{\gamma S(N-K)}{N^2} & 0 \\ \frac{\pi S(N-K)}{N^2} & 0 \end{bmatrix}$$

$$V = \begin{bmatrix} -(\mu+\delta_1)K \\ -(\mu+\delta_2+\theta)A \end{bmatrix} \quad \mathcal{V} = \begin{bmatrix} -(\mu+\delta_1) & 0 \\ 0 & -(\mu+\delta_2+\theta) \end{bmatrix}$$

Then, differentiating partially we have and

.

At disease-free equilibrium, $K = 0$, $A = 0$ and $R = 0$, thus, from $N = S + K + A + R$ implies $S = N$. Therefore,

$$\mathcal{F} = \begin{bmatrix} \gamma & 0 \\ \pi & 0 \end{bmatrix}.$$

Computing the inverse of V we have,

$$\mathcal{V}^{-1} = \begin{bmatrix} \frac{1}{-(\mu+\delta_1)} & 0 \\ 0 & \frac{1}{-(\mu+\delta_2+\theta)} \end{bmatrix}.$$

The next generation matrix is given by FV^{-1}. Thus,

$$\mathcal{F}\mathcal{V}^{-1} = \begin{bmatrix} \frac{\gamma}{(\mu+\delta_1)} & 0 \\ \frac{\pi}{(\mu+\delta_1)} & 0 \end{bmatrix}.$$

The eigenvalues of FV^{-1} are $\lambda_1 = \frac{\gamma}{(\mu+\delta_1)}$ and $\lambda_2 = 0$. However, the basic reproduction number R_0 is the dominant eigenvalue of the next generation matrix FV^{-1} which is equal to

$$R_0 = \frac{\gamma}{(\mu + \delta_1)}.$$

Stability Analysis

In this section, we conduct the stability analysis of the equilibrium points. We first compute the Jacobian matrix at (S,K,A,R). Thus,

$$J = \begin{bmatrix} -\frac{\gamma K(N-S)}{N^2} - \frac{\pi K(N-S)}{N^2} - \mu & -\frac{\gamma S(N-S)}{N^2} - \frac{\pi S(N-S)}{N^2} & \frac{\gamma SK}{N^2} + \frac{\pi SK}{N^2} & \frac{\gamma SK}{N^2} + \frac{\pi SK}{N^2} + \omega \\ \frac{\gamma S(N-S)}{N^2} & \gamma S - (\mu + \delta_1) & \frac{\gamma SK}{N^2} & \frac{\gamma SK}{N^2} \\ \frac{\pi S(N-S)}{N^2} & \frac{\pi S(N-S)}{N^2} & \frac{\pi SK}{N^2} - (\mu + \delta_2 + \theta) & \frac{\pi SK}{N^2} \\ 0 & 0 & \theta & -(\mu + \omega) \end{bmatrix}$$

Theorem 3: The kidnapped-free equilibrium point is locally asymptotically stable if $R_0 < 1$.
Proof. At the equilibrium E_0,

$$J(E_0) = \begin{bmatrix} -\mu & 0 & 0 & \omega \\ 0 & \gamma - (\mu + \delta_1) & 0 & 0 \\ 0 & \pi & -(\mu + \delta_2 + \theta) & 0 \\ 0 & 0 & \theta & -(\mu + \omega) \end{bmatrix}.$$

The equilibrium points of $J(E_0)$ are

$$\lambda_1 = -\mu$$
$$\lambda_2 = \gamma - (\mu + \delta_1)$$
$$\lambda_3 = -(\mu + \delta_2 + \theta)$$
$$\lambda_4 = -(\mu + \omega)$$

We can clearly observe that λ_1, λ_3, *and* λ_4 are all negative. However, $\gamma - (\mu + \delta_1) < 0$, if

$$\frac{\gamma}{\mu + \delta_1} - 1 < 0.$$

This implies that,

$$R_0 - 1 < 0.$$

We can observe that $R_0 - 1 < 0$ iff $R_0 < 1$. Now, all the eigenvalues are negative if $R_0 < 1$. Thus E_0 is locally asymptotically stable when $R_0 < 1$.

Theorem 4: The endemic equilibrium point is locally asymptotically stable if $R_0 > 1$.
Proof. Similar to the proof of Theorem 3.

Theorem 5: The kidnapped-free equilibrium point is globally asymptotically stable if $R_0 < 1$.
Proof. Define a Lyapunov function $L : \mathbb{R}^4 \to \mathbb{R}$ as:

$$L(S, K, A, R) = \frac{1}{2}[S - S^* + K + A + R]^2.$$

We can observe that the function $L > 0$. Also, at kidnapped-free equilibrium,

$$L(S^*,K^*,A^*,R^*) = 0.$$

Thus, $L(S,K,A,R) \geq 0$, which implies that L is positive definite. Let $M = S - S^* + K + A + R$, thus,

$$L = \frac{1}{2}M^2.$$

Now, differentiating L we have,

$$\frac{dL}{dM} = M.$$

Thus,

$\Longrightarrow$ $$\frac{dL}{dt} = \frac{dL}{dM}\left(\frac{dS}{dt} + \frac{dK}{dt} + \frac{dA}{dt} + \frac{dR}{dt}\right).$$

Hence,

$$\frac{dL}{dt} = [S - S^* + K + A + R] - [-\eta + \mu S]$$

$$\frac{dL}{dt} = -[S - S^* + K - K^* + A - A^* + R - R^*][-\eta + \mu S]$$

This implies

$$\frac{dL}{dt} \leq 0.$$

Therefore, the function L is non-increasing, which supports stability. **Theorem 6:** The endemic equilibrium point is globally asymptotically stable if $R_0 > 1$.

Proof. Define a Lyapunov function $L : R^4 \to R$ as:

$$L(S, K, A, R) = \frac{1}{2}(S - S^* + K - K^* + A - A^* + R - R^*)^2. \tag{33}$$

Observe that,

$$L(S^*,K^*,A^*,R^*) = 0 \text{ and } \qquad L(S,K,A,R) \geq 0. \tag{34}$$

Hence, L is positive definite. Let $Y = S - S^* + K - K^* + A - A^* + R - R^*$, then, $L = \frac{1}{2}Y^2$.

Taking the time derivative we have, $\frac{dL}{dY} = Y$. Now,

$$\frac{dL}{dt} = \frac{dL}{dY}\left(\frac{dS}{dt} + \frac{dK}{dt} + \frac{dA}{dt} + \frac{dR}{dt}\right). \tag{35}$$

Substituting the equations of the model in (35) we have,

$$\frac{dL}{dt} = (S - S^* + K - K^* + A - A^* + R - R^*)(\eta - \mu N - \delta_1 K - \delta_2 A). \tag{36}$$

Then,

$$\frac{dL}{dt} = (S - S^* + K - K^* + A - A^* + R - R^*) - (-\eta + \mu N + \delta_1 K + \delta_2 A). \tag{37}$$

$\Longrightarrow$

$$\frac{dL}{dt} = -(S - S^* + K - K^* + A - A^* + R - R^*)(-\eta + \mu N + \delta_1 K + \delta_2 A). \tag{38}$$

Thus,

$$\frac{dL}{dt} \leq 0. \tag{39}$$

This implies that the endemic equilibrium point is globally asymptotically stable.

Sensitivity Analysis

Sensitivity analysis is conducted to investigate the sensitivity and influence of each parameter with respect to the basic reproduction number R_0 of the model. This analysis is obtained using the forward sensitivity index approach. Assuming θ is a parameter, the formula for the forward sensitivity index is given by

$$\beta_{\theta}^{R_0} = \frac{d\mathcal{R}_0}{d\theta} \times \frac{\theta}{\mathcal{R}_0}.$$

Now, **Sensitivity index of** γ is given by:

$$\beta_{\gamma}^{R_0} = \frac{dR_0}{d_\gamma} \times \frac{\gamma}{R_0} \tag{40}$$

$$\frac{1}{(\mu+\delta_1)} \times \frac{\gamma(\mu+\delta_1)}{\gamma} = 1 > 0 \tag{41}$$

Sensitivity index of μ:

$$\beta_{\mu}^{R_0} = \frac{dR_0}{d_{\mu}} \times \frac{\mu}{R_0} \tag{42}$$

$$-\frac{\gamma}{(\mu+\delta_1)^2} \times \frac{\mu(\mu+\delta_1)}{\gamma} \tag{43}$$

$$-\frac{\mu}{(\mu+\delta_1)} < 0 \tag{44}$$

Sensitivity index of δ_1:

$$\beta_{\delta_1}^{R_0} = \frac{d\mathcal{R}_0}{d\delta_1} \times \frac{\delta_1}{\mathcal{R}_0} \tag{45}$$

$$-\frac{\gamma}{(\mu+\delta_1)^2} \times \frac{\delta_1(\mu+\delta_1)}{\gamma} \tag{46}$$

$$-\frac{\delta_1}{(\mu+\delta_1)} < 0 \tag{47}$$

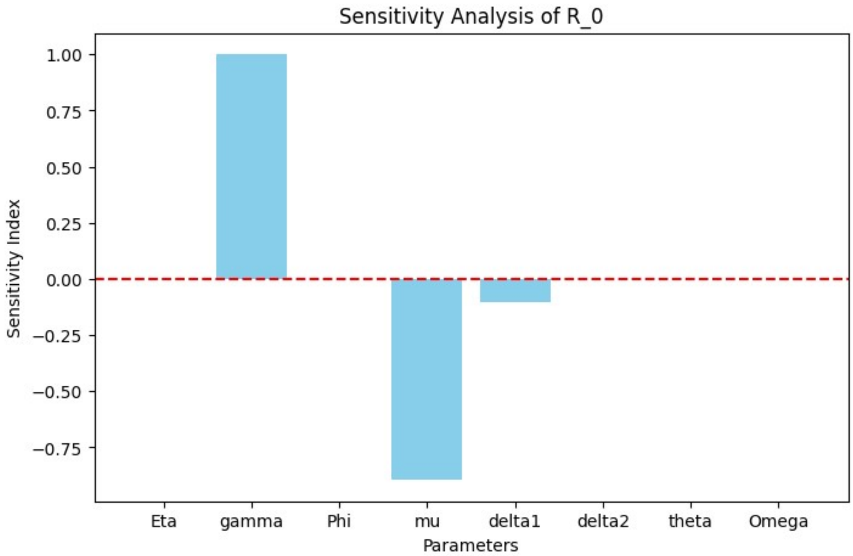


Figure 2: Sensitivity analysis of R_0

From the results above, the most sensitive parameter is γ, which is positive, indicating that an increase in γ will directly increase R_0 and also decrease in γ will directly decrease the basic reproduction number R_0. Meanwhile, μ and δ_1 have negative sensitivity indices, implying that they are less sensitive in R_0. The graph of the sensitivity analysis of R_0 with respect to the parameters is given in Figure 2. Parameter values used are given in section 3.1 below

Parameter Estimation

This section presents parameter values and estimation based on reported data, the estimated model parameters are as follows:

$\eta = 9.77 \times 10^{-3}$, $\gamma = 2.65 \times 10^{-3}$, $\pi = 2.5 \times 10^{-2}$,

$\delta_1 = 3.41 \times 10^{-3}$, $\delta_2 = 9.21 \times 10^{-6}$, $\omega = 8.0 \times 10^{-3}$,

$\theta = 6.33 \times 10^{-4}$, $\mu = 2.96 \times 10^{-2}$.

Numerical Simulations

In this section, we provide the numerical simulations of our mathematical model using Matlab R2015a and parameter values from section 4.1. The aim is to present the graphs of from the simulations to support the analytic part of the model.

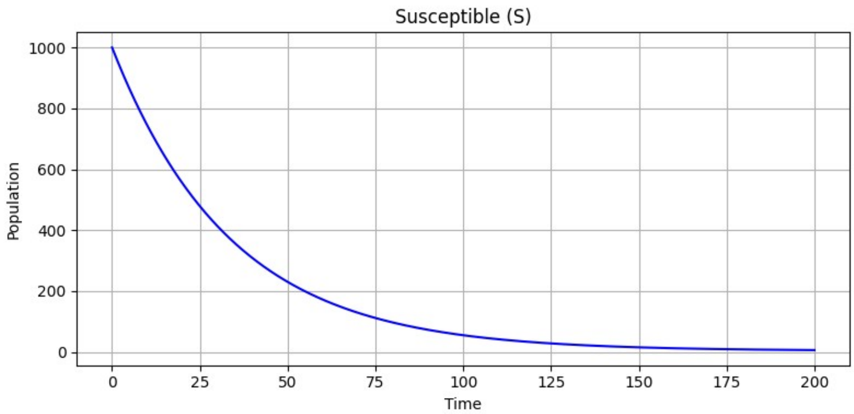


Figure 3: Time series of the susceptible Individuals

Figure 3 represents the dynamics of the susceptible population over time in the kidnapping model. This compartment typically includes individuals who are at risk of being kidnapped. At the initial stage, the population is very high, indicating a large number of unprotected individuals. As time progresses, the number of susceptible individuals decreases exponentially. This decline reflects the transition of individuals from the susceptible class to other compartments, such as abducted or rescued. By t = 200, the susceptible population approaches zero. This implies that nearly every individual has either been abducted or rescued, and the pool of individuals vulnerable to kidnapping has been effectively depleted.

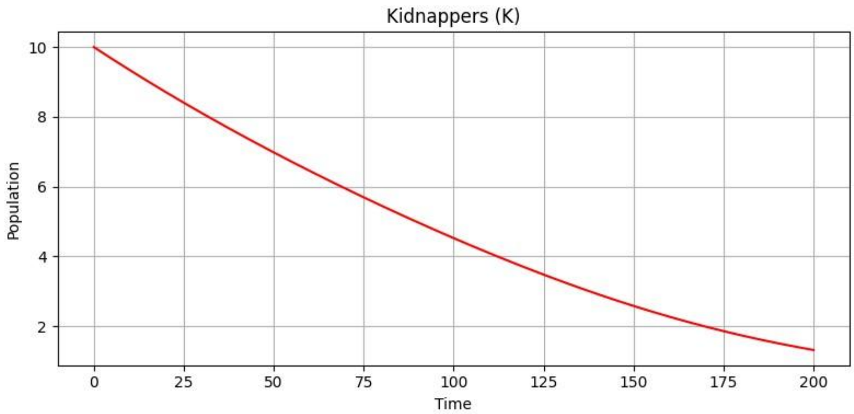


Figure 4: Time series of the kidnappers Individuals

The kidnapper's population presented in Figure 4 suggests changes over time. This compartment typically includes individuals in kidnapping activities. It started at beginning with 10 individuals (Kidnappers). This mean that the single person cannot operate the activity on less group or gang of many peoples Then over time it decrease steadily approaching 1.3 by t = 200.which means it may reduce by both natural death and death due to operation or arrest by security

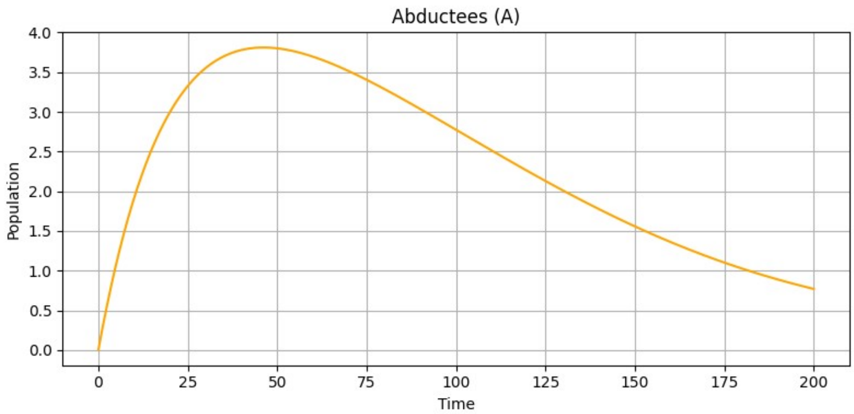


Figure 5: Time series of the abductees Individuals

In figure 5, at initial stage the number of abductee increases which means the kidnapping is occurring frequently, and after reaching the peak then it decreases steadily either by natural death, successful rescue effort, paying ransom, natural escape or release or death by the kidnappers.

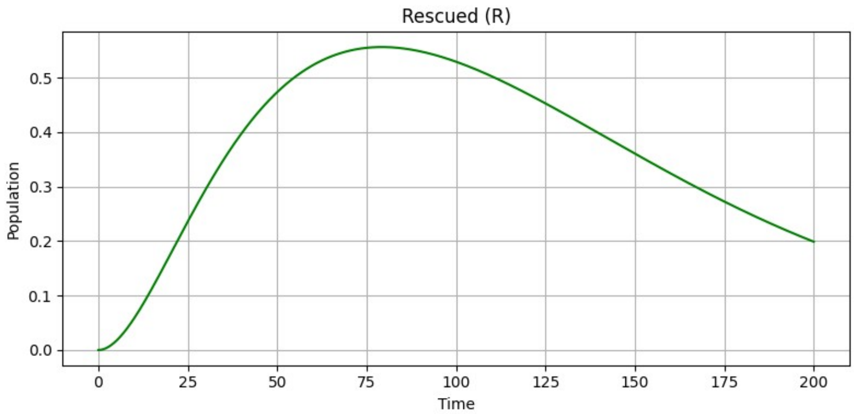
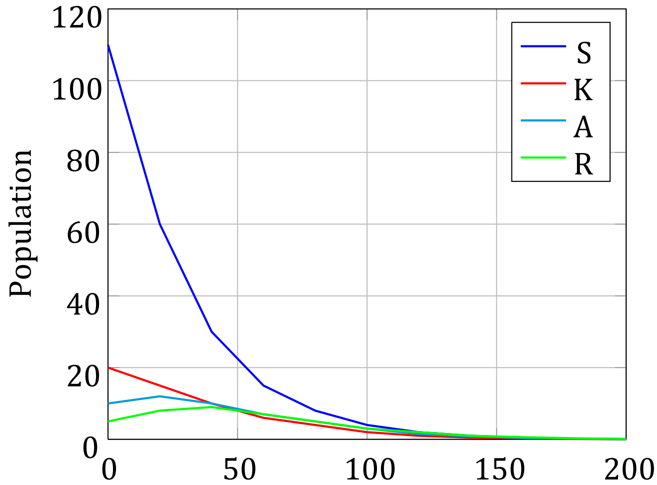


Figure 6: Time series of the rescue compartments

Figure 6 shows that, at the beginning, the rescued population increases steadily, and later decreases over time.

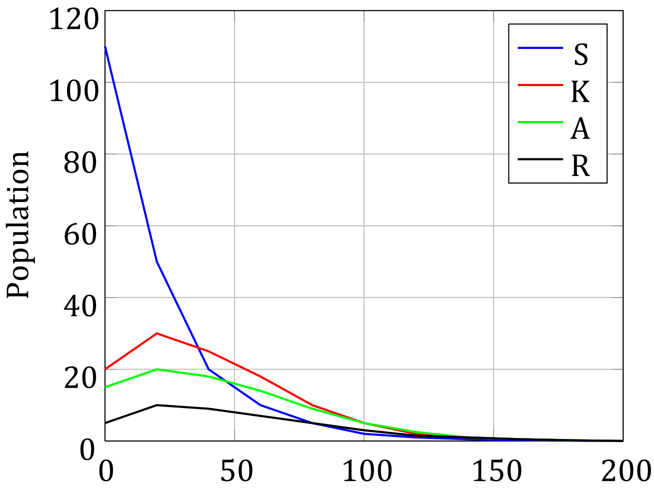


Time

Figure 7: Evolution of kidnapping dynamics for $R_0 < 1$

Figure 7 illustrates the evolution of kidnapping dynamics when $R_0 < 1$. The number of susceptible individuals (S) declines over time, while the number of kidnapped individuals (K) peaks early and then decreases. The abductee group (A) follows a similar trend, peaking early before declining. Rescued individuals (R) initially increase but later approach zero.

Since all variables converge, this indicates that kidnapping will end in a short time when $R_0 < 1$.



Time

Figure 8: Evolution of kidnapping dynamics for $R_0 > 1$

In this case, susceptible individuals (S), abductees (A), and rescued individuals (R) decrease over time, but the number of kidnappers (K) does not decline quickly. This suggests that kidnapping remains endemic and difficult to eliminate in a short period.

CONCLUSION

In this study, we developed and analyzed a mathematical model to understand the dynamics of kidnapping and evaluate strategies for eradication the menace. The key information we obtained from the analysis and simulations of the model is that there is the possibility of eradicating kidnapping by discouraging the recruitment of kidnappers and enhancing speedy rescue of kidnapped victims. Kidnapping has become endemic in the northwest of the West African country as roving gangs of armed men abduct peoples from villages, highways and schools and demand ransom money from their relatives. Subsequent to establishing well-posedness for the positivity and boundedness, the two feasible equilibria (kidnap - free and persistence of the model are found to be globally asymptotically stable if kidnap propagation parameter $R(0)$ is respectively less or bigger than unity

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