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SIMULATION MODEL FOR DENTAL ARCH SHAPES

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ABSTRACT

Objective: **To develop a simulation model for dental arch shapes.**
Design: **Analysis of measurements of dental casts to determine a general second degree equation for the dental arches.**
Setting: **Department of Human Anatomy and School of Computing and Informatics, University of Nairobi.**
Subjects: **The measurement of dental casts, 30 (15M and 15F) each from three Kenyan ethnic groups (Maasai, Kalenjin, Kikuyu), aged 12 years.**
Results: **The arches change their shapes from a parabola to an ellipse, governed by the boundary conditions at the position of the canine tooth, based on the general second degree equation for the conic sections.**
Conclusion: **The simulation model graphically confirms the change from parabolic to elliptic shapes of dental arches with boundary conditions at the canine. This could be used to show the changes in dental arches for other ethnic groups.**

INTRODUCTION

The mathematics of conic sections has been used to study various aspects in different disciplines. In life sciences, the studies of dental arch shapes in humans and other mammals have been carried out using the theory of conic sections. The search for a mathematical description of biological forms (size and shape) and the method of quantification of changes in form began during the first quarter of the last century(1-3). Several approaches have been developed depending on the nature of the investigated structure and on the hypothesis being tested(4).

Rudge(5) provided a detailed review of the literature dealing with dental arch form and described the ideal arch as being constructed upon an equilateral triangle with slight modification. The six anterior teeth were thought to be arranged on the arch of a circle whose radius was determined by the combined width of the incisors and canines, with the premolars and first permanent molars arranged in a straight line and the second and third molars turning in toward the midline.

McConnail and Scher(6) suggested that the ideal curve for the dental arch would correspond to a catenary curve, which is formed when a fine chain is suspended at both ends. Scott(7) and Musich and Ackerman(8) claimed that a catenary curve best represented a good average fit for arch forms and that any variation from this form was a consequence of alveolar-process growth beyond the normal range. There is considerable variation in the shape of the dental arch, the upper arch being elliptical and the lower parabolic(9). The arch form as defined by the centre of occlusal form of the teeth is parabolic, while the arch form has been described as "U" shaped and tapered(10). Baz(11) determined normal arch size by a "geometric construction" based on measurements taken directly on the patient's face. Remsden(12) studied various methods of predetermination by comparing them with arches of a sample of "normal" occlusions and found inaccuracies in all methods but concluded that a parabola best represents the anterior curvature of the dental arch, although an arch that fitted a precise pattern was to be regarded as an exception rather than the rule. Kato et *al.*(13) examined the relative positions of teeth and average arch form by establishing landmarks on the teeth of the study models and concluded that there was no great disparity between the arches.

Using computer analysis of occlusion Lu(14) claimed that the dental arch could be satisfactorily described by a polynomial equation of the 4th degree. Currier(15) tried to evaluate the arch shape in a selected sample by comparing them with several geometric curves using mathematical analyses and electronic data processing equipment. Sanin et *al.*(16) attempted mathematical functions such as exponential, logarithmic, elliptical, parabolic, hyperbolic and polynomial to fit the dental arches, using computer analysis of point coordinates and concluded that the 4th degree polynomial was among the easiest curves to fit while still adequately describing the dental arch. Pepe(17) analysed models of normal occlusion for seven children by digitisation of a full-size occlusal photograph to Cartesian coordinates which were then compared with mathematically derived curves by a computer. Faber *et al.* (18) made use of a computerised cephalometric study model analysis, stating that predetermination of arch form permits the teeth to be arranged along the curve and arch length of available space to be calculated. Brustone(19) described a procedure for case analysis by digitising points on a 1:1 occlusal photograph of study models to give measurements and tooth position on a visual display unit. Lestrel(3) developed two categories of procedures, homologous - point representation and boundary representation. Several other algorithms for dental arch shapes have been developed: superimposition methods(20), finite element analysis(21), biorthogonal grids(1,3) and Euclidean - distance matrix analysis(21). Pokhariyal(22) has considered a general second degree equation for the entire dental arch, with the boundary condition at the canine where a parabola changes to an ellipse.

The aim of this study was to develop a simulation model for dental arch shapes using a suitable computer program on measurements from dental arches of dental casts for three ethnic groups of Kenya.

MATERIALS AND METHODS

Dental casts for Maasai, Kalenjin and Kikuyu children aged 12 years from Kajiado, Baringo and Muranga districts of Kenya, respectively, were prepared and used for measurements. Dental casts of 15 males and 15 females, from each ethnic group, with normal dentitions having all second permanent molars erupted were used for measurement and analysis. Each child selected had both parents from the same ethnic group. The casts were prepared during several field trips in 1986 - 1989 when intra-oral examinations were also carried out. Permission for the study was obtained from the Office of the President and District education offices.

The measurements (in millimeters to the nearest decimal) for the maxillary and mandibular arches were done using vernier calipers, dividers and a ruler. The X and Y axes were fixed on the dental cast and for each tooth, x-left and x-right values for outer, middle and inner regions and the y-values were measured to fit the equations (Figure 1).

In the Maasai sample, 50% had undergone traditional extraction of mandibular permanent central incisors(23). Hence the measurements required standardisation at the midpoint between the lateral incisors. The analysis was carried out using the general second degree equation for the conic sections given by

$$AX^2+BXY+CY2+DX+EY+F = 0 \quad .... (1)$$

where A, B, ..., F are constants such that A, B and C are not all zero. This was used as the model for the dental arch shape. The use of a general second degree equation, can give various conic sections based on the relationship between the coefficients of the equation. The curve represented by equation (1) is classified according to the values of discriminant as:

Parabola, when $B^2- 4AC = 0$ ...(2)

Ellipse, when $B^2- 4AC< 0$ ...(3)

Two conic sections (a parabola and an ellipse) have been widely used to study dental arch shapes. A parabola is the set of points in a plane that are equidistant from a given fixed point (called a focus) and fixed line (called a directrix) in the plane. An ellipse is the set of points in a plane whose distances from two fixed points (called foci) in the plane have a constant sum.

The data were tabulated for each dental cast recording the name, ethnic group, code, gender, age and the respective dental measurements of dental arches. The program was designed using pseudocode, taking into consideration modularity. This included design of files, input and output screens, code and user interface. The program was tested, debugged and integrated. A test data set were used to reset the overall system. The shapes were then simulated using the computer program and the respective graphs were plotted.

RESULTS

The data measurements for ethnic groups, gender and the total sample were used to simulate the dental arch shapes. The simulated results are shown in Figures 2, 3 and 4 respectively, based on the tabulated data included in these figures. The arches change shapes from a parabola to an ellipse governed by the boundary conditions at the canine tooth, based on the general second degree equation for the conic sections. Figure 2 shows that in the simulated shapes based on the ethnic groups, the parabolas are almost similar for all the three ethnic groups.

Figure 1

Measurements of the inner (*), middle (-) and outer (.) values of x and y on mandibular and maxillary arches measured on a dental cast of a 12 year old Kikuyu boy

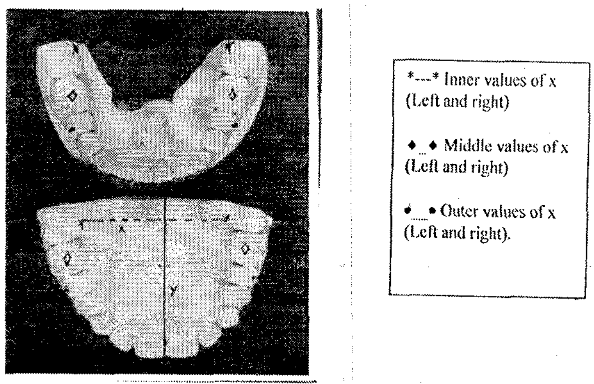


Figure 2

Simulated shape based on the ethnic groups, Kikuyu, Maasai, and Kalenjin, based on the included tabulated data

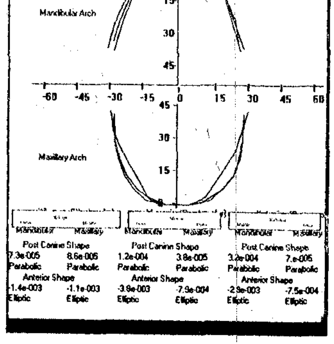


Figure 3

Simulated shape of the dental arches for males and females based on the included tabulated data

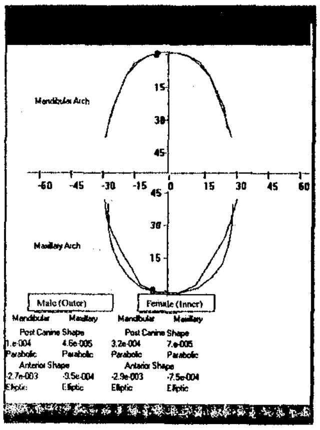
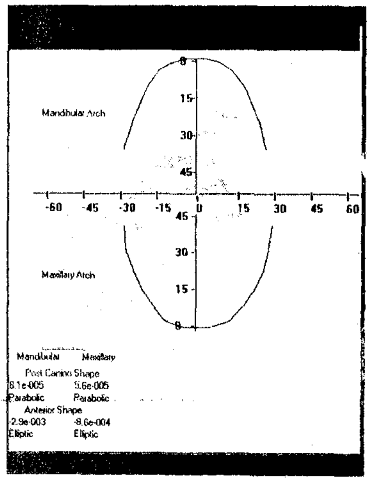


Figure 4

Shapes of the dental arches change from a parabola to an ellipse for the total sample, based on the included tabulated data



The ellipse shapes are almost similar for Maasais and Kikuyus (with slightly wider shapes for Maasais) and are relatively narrower for Kalenjins, in the maxillary arch. However, the situation changes in the mandibular arch. Figure 3 shows that, in the simulated shapes for males and females, the dental arch shape for females is relatively narrower compared to males in the maxillary arch. However, the shapes are almost the same in the mandibular arch. Figure 4 shows that the simulated shape changes from a parabola to an ellipse for the total sample.

DISCUSSION

It is observed that all dental arch shapes are parabolic in the beginning up to the canine and then change to an ellipse for the outer and middle region measurements. This confirms our postulate for the change in shape occurring at the canine, where equation(2) and(3) are accordingly satisfied. The measurements for the inner region show that the dental arch shape remains parabolic throughout without changing to elliptic at the canine. Similar findings were reported by Remsden(12), that a parabolic curve best represents the anterior curvature of the dental arch shape. The differences can be linked to the observed facial traits of various ethnic groups on the lower portion of their faces(24). It was reported that no significant differences in the shape were demostrated while male arches proved to be slightly bigger than female arches. The gender differences were found especially in the maxillary arch. This finding also agrees with the results of a previous study(26).

In all attempts of previous studies to fit curves to the dental arch, researchers wanted to fit one single curve to the entire arch and the conduct error analysis or ad hoc polynomial fits of various degrees to arrive at the best fit. This can be avoided if one understands the fact that the dental arch can have one shape up to a certain point and then take another shape after that point. Elementary knowledge of geometry suggests that the general second degree equation provides a mechanism for shifting from one curve to another with a sound mathematical foundation. This shift is governed by fulfilling conditions at that point. However, the observed phenomena for the outer and middle dental arch shape suggest that the change in the shape from a parabola to an ellipse occurs at a definite point. This is postulated to happen at the canine. The recorded observations for three Kenyan ethnic groups as well as gender are used to obtain the values of A, B,...F in equation(1) for each set of observations recorded from the respective dental casts. Condition(2) and(3) are confirmed for the curves at canines for the outer and middle measurements whereas inner measurements continue to be represented by a parabola throughout without changing to an ellipse.

Currier(15) found that the buccal surfaces of the maxillary arch conform more closely to an ellipse than to a parabola but that neither an ellipse nor a parabola exhibited a significant fit to the lingual curves of the arches. Pepe(17) found that in nearly every arch, some of the point sets representing molars or premolars produced a disproportionately large mean square error to the fit curves. Of all the curve generating equations studied in this manner, catenary was found to be the least descriptive of the arch forms and was found to be inferior even to the quadratic curve fit. The polynomial was not a good fit and Pepe(17) concluded that although this curve may be used to describe the arch form, it is not suffficiently accurate to use for arch-wire construction. The assumed drawback of the Faber *et al*(18) simulation seemed to be that the mathematical curve shape fitted to x-y coordinates is usually either parabolic or a trifocal ellipse and it is yet to be shown that the ideal arch form actually approaches these mathematically derived curves.

In their study of dental arches Richards et *al*(27) used the fourth order polynomial model. The mixed elliptical plus the parabolic model has been suggested by Ferrario *et al*(25) in which the standardised photographs of the models were obtained using a Polaroid 3 inch CU-5 camera. In both photographs the midpoints of incisal edges and canine cusps, and the buccal and lingual cusps of premolars, first and second molars were traced. Maxillary and mandibular arches were then interpolated by a mixed model and a polynomial model. The mixed model suggested by Ferrario *et al*(25) concludes that the shape of dental arch changes from an ellipse to a parabola at canine. In our study the change of the shape occurs from a parabola to an ellipse. This change in shape is also confirmed by the change in values of the discriminant B^2- 4AC.

This study proposes one integrated equation (model) for the entire dental arch which changes its shape at one particular point, the canine. An ellipse can also be interpreted as the departure from circularity and the degree of this departure is measured by eccentricity, which lies between 0 and 1 for the ellipse. The extreme values 0 and1 give a circle and a parabola respectively. The focus-directrix property unites a parabola and an ellipse. Mathematically we can have a common equation for both an ellipse and a parabola, which changes with respect to the conditions being satisfied. The second degree equation can be satisfied by the two respective roots represented by left and right values on the dental arch models. The basic idea explored in(22) has been fully developed in this paper.

The dental arch model can thus be used for accurate arch-wire construction and preparation of dental prostheses. This model can be applied to investigate the differences in the dental arch shapes of other selected groups, so as to provide some insight into their facial traits based on the eccentricity of the simulated curves.

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