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RELATIONSHIP BETWEEN SLUDGE DEWATERABILITY NUMBER AND CARMAN'S SPECIFIC RESISTANCE

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ABSTRACT

A representative of a sludge sample collected from the same source was filtered under the same environmental condition and the result analysed with two different concepts. One method of analysis uses Sludge Dewaterability Number, while the second employed the Carman's Specific resistance concept in sludge filterability. Two plots are employed, one shows a process of specific resistance determination while the other indicates how Sludge dewaterability number can be determined. A critical examination of the two plots reveals that a relationship might exist between the two concepts. Also, the analysis of the filtration results obtained using both the old and the new concepts, confirms that both Sludge Dewaterability Number values and the Specific resistance values decrease with an increase in the conditioner dosages. This is a mark of good coagulation with an increased filterability when conditioner dosages (ferric chloride) are increased in filtration process. The new equations have therefore shown some good relationships between sludge dewaterability number and specific resistance, which are in line with the original concepts. It has also been shown that sludge dewaterability number is directly proportional to the area of filtration and the square root of the specific resistance. This varies according to the diameter of the filtrate collecting cylinder and the area of filtration that is used in the determination of specific resistance (r) values.

Key words: Sludge, Coagulant, Coagulation & Filtration, Sludge Dewaterability Number, and Specific Resistance,

INTRODUCTION

Pollution control poses a lot of problems all over the world and of utmost importance is the disposal and reduction of the sludge volume generated from different sources. The increase in population and rapid industrial growth has worsened the situation by increasing the volume and the nature of the wastewater generated. This wastewater has to be treated to satisfy the health requirement of the society and reduce environmental hazards to the people. Whatever the treatment process chosen to handle the wastes generated, sludge is produced. Sludge handling in waste treatment account for 21-50% of total plant operating and maintenance cost (Randall et al., 1971). The compressible nature and high water content (about 97%) makes sludge handling a difficult task. In view of the high water content, sludges are usually de-watered or filtered prior to final disposal. This de-watering process leads to the development of different sludge filtration theories.

CAKE FILTRATION THEORIES

There are basically three notable cake filtration theories that are based on three different concepts: Sludge filtration resistance, filtration coefficients and Sludge dewaterability number. All the above three theories have their own individual limitations arising either from their method of derivation or experimental procedure. Thus, none of them is without criticism, yet they are employed in sludge de-watering prediction

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because of their relative accuracy in sludge de-watering process.

Carman (1938), with the concept of specific resistance developed an equation of filtration given as:

$$t = \mu rcv^2/2PA_c^2 + \mu RV_f/PA_c \quad \ldots\ldots(1)$$

Where A_f = area of filtration, C_o = mass of dry cake deposited per unit volume of filtrate, r = Specific resistance of the cake, R = Specific resistance of the septum, t = time of filtration, P = filtration pressure, μ = Viscosity of filtrate and V_f = filtrate volume. From Equation (1) the slope of the straight line plot obtained when t / V_f is plotted against V_f gives the value of b in Equation (4)(Fig1).

Equation (1) has been modified several times because of its limitations (Anazodo 1974,Ademiluyi et al; 1982 &1984,). It has various units of measurements and a disparity exist between theoretical and experimental data. The equation is used in predicting sludge dewatering process, because its experimental results give values that are acceptable to some reasonable accuracy for industrial applications/engineering purposes. Another sludge de-waterability number equation that was proposed with a view to solving the numerous problems associated with other filtration equations is the Sludge Dewaterability Number (SDN) equation (Ademiluyi et al, 1987). The SDN equation is given as:

$$SDN = \Delta H(C_o - C_f)/V_i C_c t + H_o/V_i t \quad \ldots\ldots(2).$$

Where C_o = initial sludge concentration, C_c = cake concentration , C_f = filtrate concentration, Ho = initial head loss, ΔH = change in head loss, V_i = approach velocity and t = time of filtration . Equation (2) has its limitations purely from its derivation stand point, but because of its relative accuracy in predicting sludge dewatering process, it is in use moreso as it compares favourably with other filtration theories (Ademiluyi & Eze 1990, Fig3).

To enable the use of SDN equation as a constant parameter over a whole filtration cycle, the equation was modified (Ademiluyi and Eze 1989) to give:

$$SDN = 1/H_f[\Delta H(1 - V_f/Vs) + H_o] \quad \ldots\ldots(3).$$

Where H_f = height of filtrate in the filtrate receiver, and Vs = volume of sludge . A plot of H_f against $\Delta H(1 - V_f/Vs)$ would give a straight line, the reciprocal of the slope (see Fig.2) is the SDN. It has earlier been shown that the foregoing rectilinear relationship holds for all filtration pressures tested for specific resistance and SDN equations with good correlation coefficients (Ademiluyi &Eze, 1990). It has also been shown that both Equations (1) and (3) could be used to determine the effectiveness of sludge conditioners (Ademiluyi & Eze, 1990). Equation (3) has one advantage over all equations so far proposed for sludge filtration modelling. All the parameters can be evaluated quickly in the laboratory within some few minutes of experimentation, whereas in previously proposed equations, the measurement of C_o, C_c, C_f in Equation (2), and C_c in Equation (1) and its modified forms would take at least 24hrs.

Having highlighted on both the theoretical and experimental relationships and the differences between specific resistance and sludge dewaterability number equations, it may be necessary to establish their mathematical relationship possibly for easier and effective sludge de-watering predictions.

RELATIONSHIP BETWEEN SDN AND SPECIFIC RESISTANCE

Specific resistance (r) is used to measure filterability in Carman's filtration equation and is given as :

$$r = 2PbA^2c/\mu C_o \quad \ldots\ldots(4)$$

Also sludge dewaterability number equation is used to measure filterability and is given as :

$$SDN = 1 / H_f \, [\Delta H (1 - V_f / Vs) + H_o] \qquad \ldots\ldots(5)$$

SDN equation was modified to enable it's use as a constant parameter over a whole filtration cycle, and in it's accurate determination, a graphical plot of H_f against ΔH (1 - V_f / Vs) is used, from where the inverse of the slope obtained gives the value of SDN. In this determination, and for an accurate experimental set up, the initial head loss (H_o) is always zero and therefore does not contribute to the value of SDN.This value (H_o) can therefore be ignored in it's mathematical relationship with other concepts. Thus:

$$SDN = 1 / H_f \, [\Delta H (1 - V_f / V_s)] \qquad \ldots\ldots(6)$$

$$H_f = V_f / A_f \qquad \ldots\ldots(7)$$

A_c / A_f = (α) ; ratio of area of cake to area of filtrate collecting cylinder.

Therefore, $A_c = \alpha A_f$ ……(8)

Combining Equation (6) with Equation (7) results as follows:

$$SDN = A_f / V_f \, [\Delta H (1 - V_f / V_s)] \qquad \ldots\ldots(9)$$

$$A_f = (SDN \, V_f) / \Delta H (1 - V_f / V_s) \qquad \ldots\ldots(10)$$

From Equation (4), $A_f = (\mu C_o r)^{1/2} / (2 P b)^{1/2}$ ……(11)

Where A_c is the area of filtration in Carman's Equation, hence making it the subject in Equation(4) which yield Equation (11). Also substituting Equation (10) into Equation (8), we obtain

$$A_c = (SDN \, \alpha V_{f)} / \Delta H (1 - V_f / V_{s)} \qquad \ldots\ldots(12).$$

Equations (11) and (12) are equal because both represent A which is the area of cake in terms of specific resistance (r) and SDN , respectively. Therefore,

$$(SDN \, \alpha \, V_f) / \Delta H (1 - V_f / V_s) = (\mu C_o r)^{1/2} / (2Pb)^{1/2} \qquad \ldots\ldots(13)$$

$$SDN \, \alpha \, V_f = \Delta H (1 - V_f / V_s)(\mu C_o r / 2Pb)^{1/2} \qquad \ldots\ldots(14)$$

$$SDN \, \alpha \, V_f = (\Delta H - \Delta H \, V_f / V_s)(\mu C_o r / 2pb)^{1/2} \qquad \ldots\ldots(15)$$

$$SDN \, \alpha \, V_f = [\Delta H^{1/2} + (\Delta H \, V_f / V_s)^2 - 2 \, \Delta H^2 V_f / V_s] \mu C_o r / 2Pb \qquad \ldots\ldots(16)$$

V_f / V_s is small, $(V_f / V_s)^2$ is very small and therefore all terms containing them can be ignored: $(SDN \alpha V_f) = \Delta H^2 \mu C_o r / 2Pb$ .... (17)

$$SDN^2 = (\Delta H / \alpha V_f)^2 (\mu C_o r / 2Pb) \qquad (18)$$

$$SDN = \Delta H / \alpha V_f (\mu C_o r / 2Pb)^{1/2} \qquad (19)$$

$$SDN = (\Delta H / \alpha V_f) A_c \qquad (20)$$

From Equations (19) and (20), SDN is directly proportional to the area of filtration and the square root of specific resistance in Carman's filtration Equation.

RESULTS AND DISCUSSION

Fig.1 shows how b, used in the determination of (r) can be obtained (Ademiluyi, 1988) while Fig.2 shows how SDN can be determined (Ademiluyi & Eze, 1989). Table 1 shown below is graphically represented in Fig.3. This graph is a prediction of some relationship between SDN and specific resistance when the sludge is conditioned with ferric chloride.

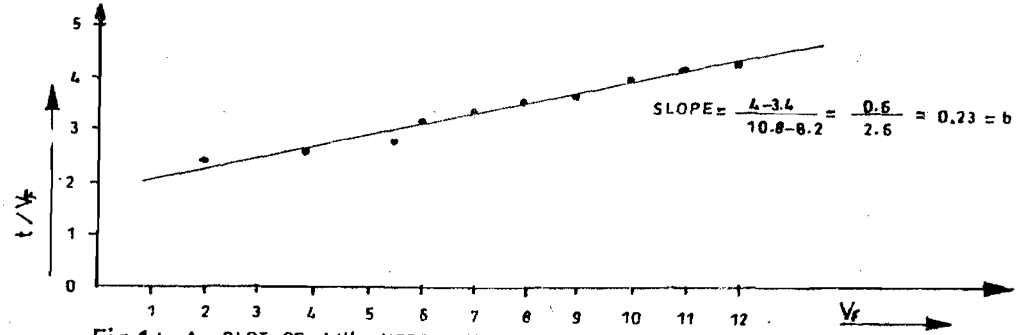

Fig.1: A PLOT OF t/V_f VERSUS V_f TO DETERMINE THE VALUE OF b IN THE CARMAN'S SPECIFIC RESISTANCE EQUATION.

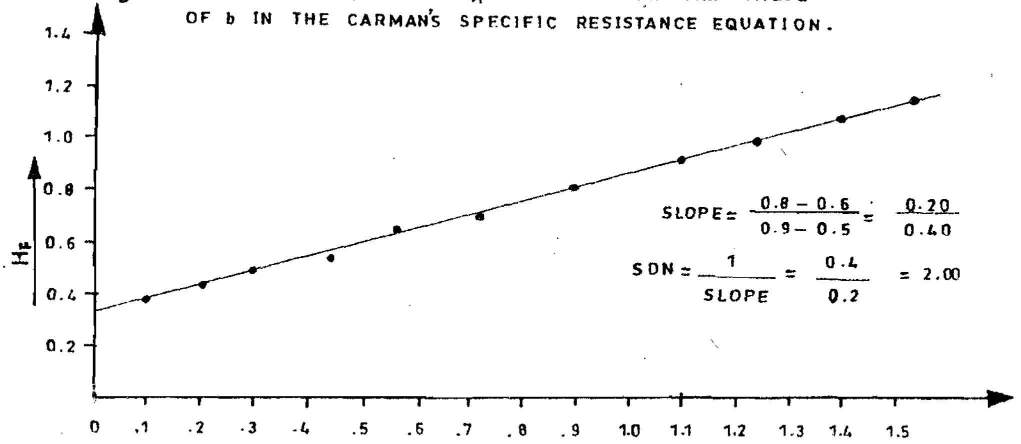

Fig.2: A GRAPH SHOWING THE PLOT OF H_f VERSUS $\Delta H (1 - \frac{V_f}{V_s})$ AS A METHOD FOR FINDING THE VALUE OF SDN ($P = 53.37 kN/m^2$)

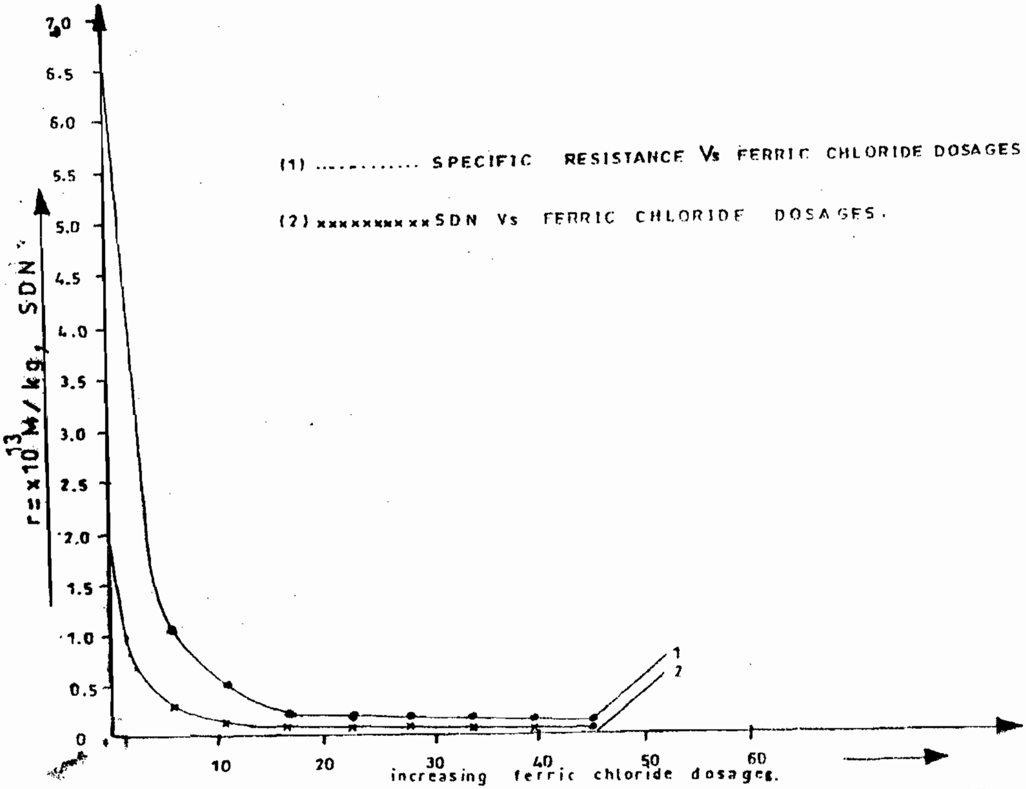

Fig.3: A PLOT OF SPECIFIC RESISTANCE (r), SDN VERSUS PERCENTAGE

Table 1

%Dosages	0	5.64	11.28	16.92	22.56	28.20	33.84	39.48	45.12
SDN	2.00	0.28	0.19	0.10	0.09	0.08	0.07	0.05	0.04
Sp. Res.($r \times 10^{13}$)	6.94	1.07	0.56	0.20	0.19	0.18	0.15	0.11	0.09

The validity of the new equation will depend on the results obtained and their compatibility with **results from** other filtration theories. Table 2 shown [illegible] below compares results from the new concept and **other concepts** using the same experimental results. This means that the same dosages were used to **obtain results**, which were analysed with different concepts for an easier understanding.

Equation (19) can be re-arranged by making (r) the subject, hence:

$$r = (2Pb)(SDN\ \alpha\ V_f)^2 / (\Delta H \mu C_o)\text{, since } = Ac / A_f \text{ and } H_f = V_f / A_f .$$

$$\text{Then, } r = (2Pb)(SDN\ A_C\ H_f) / \Delta H \mu C_o) \qquad \text{.............................(19 b)}$$

Also Equation (20) can be re-arranged by making SDN the subject, hence:

$$SDN = (\Delta H A_c)/(\alpha V_f)\text{, but } \alpha = A_c / A_f\text{, Therefore}$$

$$SDN = \Delta H A_f / V_f \text{ .} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (20\text{ b}).$$

The values for model predictions for both (r) and SDN shown in Table 2 were obtained from the equation of relationship between the two concepts re-arranged as shown above in Equation 19b& 20.
The results obtained in Table 2 were carried out under the same experimental conditions, and, because they decrease with increasing conditioner dosages, the new relationship gives a good prediction for sludge filterability in all sludge dewatering process.

CONCLUSION

The differences between specific resistance and sludge dewaterability number equations have been explained. Both can be used to describe sludge dewatering process despite their drawbacks because of their relative accuracy in sludge dewatering predictions. A relationship between the two have therefore been established to further improve their predictive potentials in sludge dewatering process. Table 4 clearly shows that the new relationship is in line with other sludge filtration theories and can therefore be used in sludge dewatering predictions. It is easy to measure and calculate. It is dimensionless and lacks the inconsistencies inherent in other dewatering equations. Thus, with the knowledge of the change in head

Table 2

%Conditioner. Dosages	SDN from original. Concept	SDN from model. Prediction	Specific Res.From Original Concept. ($\times 10^{13}$)	Specific Res.From. Model Prediction ($\times 10^{9}$)
0	2.00	0.82	6.94	1.43
5.64	0.28	0.41	1.07	1.21
11.28	0.19	0.32	0.56	0.91
16.92	0.10	0.31	0.20	0.81
22.56	0.09	0.28	0.19	0.64
28.20	0.08	0.27	0.18	0.54
33.84	0.07	0.26	0.15	0.49
39.48	0.05	0.21	0.11	0.43
45.12	0.04	0.20	0.09	0.41

loss (ΔH), ratio of the diameter of the filter paper to that of the filtrate collecting cylinder (α), volume of filtrate ($V f$), and the area of the filtration in Carman's equation (A_c), Sludge Dewaterability Number can easily be determined.

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