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DEVELOPMENT OF A MAINTENANCE SCHEDULING MODEL FOR A GROUP OF MACHINES

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ABSTRACT

This paper presents the development of an efficient method for use in obtaining the best time between coordinated overhauls for a group of machines. The method is capable of stating the overhaul frequency of each machine within this time period. For convenience in the formulation of the model, maintenance schedule was divided into two parts, namely minor and major maintenance schedules. During a minor maintenance, an individual machine is checked and reset, while during a major maintenance, all machines are checked and reset simultaneously. Generally, their model determines the best value of the time between major maintenance and frequencies of minor maintenance, for each machine within this time span. In order to minimize the total cost of repairs and production. A numerical application of this development model in a case study is presented.

Key words: Maintenance, modeling, scheduling, optimization.

INTRODUCTION

In planning for maintenance in an industry where production is continuous, the question of how frequent a machine should be renewed is important. This is because almost all industrial equipment deteriorate with age or usage, unless action is taken to renew them. Renewal can be carried out on a single machine or entire group of machines simultaneously. The advantage of performing maintenance on the entire group of machines simultaneously is very obvious. By carrying out a coordinated maintenance on a group of machines, we could save on the total fixed cost associated with individual overhauls of machines. There is a rich literature on renewal scheduling. The vast majority of them solve problems in a single machine case. For the single machine maintenance case see Federgruen and So (1990) Handlarski (1980), and malik (1979). There are also some attempts on a group maintenance planning. For the group maintenance see Christer and Doherty (1977) and Sule and Harmon (1979). This research will consider how decisions on group maintenance scheduling should be made in the industries.

In the research, an economic model for the determination of group maintenance scheduling frequencies for a number of machines in an industry using exclusive enumeration procedure is developed. This model follows an approach similar to that of Sule and Harmon (1979). However, it is modified by incorporating the effects of breakdown and downtime.

FORMULATION OF THE GROUP MAINTENANCE MODEL

An economic model which minimizes the total operation cost of a group of machines in a manufacturing

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concern is developed. The total cost consists of the repair and the production costs. The cost of repair in the model will consists of:-

1. The major preventive maintenance cost which is the cost of checking and resetting all the machines simultaneously.
2. The minor preventive maintenance cost which is the cost of checking and resetting each machine individually.
3. The breakdown maintenance cost which is the cost of carrying out emergency repair on the machines as they occur.
4. The downtime cost which is due to the inactivity of the machines on account of breakdown. It is actually the penalty paid for not running a machine on account of breakdown.

The cost of production in the model consists of :-

1. Time dependent cost which varies with time from the previous repair. For example the number of defective units produced in a machine is a function of many time dependent variables. These may include distortion of setting, tool ware, change in physical characteristics of operating entities such as lubricating fluid, depreciation of components, etc.
2. Fixed cost which is independent of time from previous repair. They include salaries of management and staff, rents rates and insurance, legal expenses, cost of security staff, stamps, telephone, staff traveling expenses consultant fees, etc.

GROUP MAINTENANCE REPAIR COST FUNCTION

The model assumes that after an interval of every T time units, the cycle time, a major preventive maintenance is performed at a cost C. The average cost of minor preventive maintenance on machine i is dencted as C. For group maintenance to be justified at all in the first place, the cost minor repair performed on all machines independently and without coordination should be greater than cost of major repair. That is

$$\sum_{i=1}^{N} C_i > C$$

Where N is the number of machines in the group. Let B_{ij} be the expected number of breakdown type j in machine i per cycle and let S_{ij} be the cost of rectifying breakdown type j in machine i per cycle. Let T_i be the downtime factor which is defined as the ratio of the time the machine is not expected to run on account of breakdown to down to the time the machine could have been running assuming there was no breakdown. Let P_i be the inactivity cost of machine i per cycle. Assuming that there are M number of breakdown types in machine i per cycle and that there are $(K_i - 1)$ number of minor preventive maintenance performed on machine i in a cycle then the repair cost per cycle RC is given as:

$$RC = \sum_{i=1}^{M} \sum_{i=1}^{N} B_{ij}S_{ij} + \sum_{i=1}^{N} (K_i - 1)\,C_i + \sum_{i=1}^{N} \lambda_i P_i + C \qquad 1$$

$\sum_{i=1}^{M} \sum_{i=1}^{N} B_{ij}\, S_{ij}$ is the total cost of all the machine breakdown;

$\sum_{i=1}^{M} \sum_{i=1}^{N} B_{ij}\, S_{ij}$ is the total cost of all the minor repairs per cycle;

$\sum_{i=1}^{N} \lambda_i P_i$ is the total cost of the machines downtime per cycle and C is the major preventive maintenance cost which is the cost of checking and resetting all the machines simultaneously.

GROUP MAINTENANCE PRODUCTION COST FUNCTION

The frequencies of major and minor maintenance affect not only the repair cost, but also the production cost as it influences the number of defective units produced. Since the number of defective units produced is a function of many time dependent variables such as tool wear, distortion of setting, depreciation, etc, it will, following Malik [1977] function, be assumed that between two successive repairs the unit cost of production at time t of a good unit on machine i is given,

$$f_i(t) = a_i + b_i t \qquad 2$$

Where t is the continous production time previous repair, a_i is the fixed cost of production for machine i and b_i is the wear out factor for machine i. Following Malik [1979], to take care of non-linearity, we change f (t) to,

$$f_i(t) = a_i + b_i t^{n_i} \qquad 3$$

Where n_i is the degree of cost polynomial for machine i.

The function given by equation 3 will normally be valid over a finite time interval, say $0 \leq t \leq t_1$ in practical cases, after which the cost per good unit becomes prohibitive and overhaul would be carried out. The parameters of the function are determined by the type and condition of the machine, but in all cases $a_i \geq 0$, $b_i \geq 0$ and $n \geq 1$.

The cost function of equation 3 has been verified with data from a photocopying business here in Nsukka and found fairly accurate. The business concern has, among other things, four different types of photocopying machines. A particular type of machine is specialized each for photocopying document for official use, textbook for student use, hand out [poor quality material] for student and enlargement work. The defective units produced in each machine were not the same because of the differing qualities standard imposed on the various machines by the kind of work they are specialized for. In addition, the deterioration on each machine from previous servicing could vary depending on its make and age. Table 1 shows the cost data for each of the four photocopying machines. The cost data was collated (Stroud, 1986) on each photocopying machine and an equation of the type $f(t) = a + bt^n$ was developed.

Allowing the powers of this production cost function n to vary from machine to machine will lead to computational difficulties. Hence, for a group of machines n will be calculated for the various machines and then an average n will be taken to be the value of the degree of cost polynomial for all the machines in the

Table 1: Production cost data for the photocopying machine

t in weeks	f(t) in Naira			
	Machine 1	Machine 2	Machine 3	Machine 4
0	8	13	12	11
0.5	9	13	13	12
1.0	11	22	26	17
1.5	21	47	70	36
2.0	43	105	170	79
2.5	83	212	358	159
3.0	149	388	557	292

group. For the problem under consideration n for machine 1 = 3.4, machine 2 = 3.1 machine 3 = 3.8 and machine 4 = 3.7. thus the average

$$n = \frac{(3.4 + 3.1 + 3.8 + 3.7)}{4} = 3.5$$

using n = 3.5, the cost data in table 1 was again collated on each machine and an equation of the type $a + bt^{3.5}$ was developed.

Table 2, shows the recalculated values of a's and b's using n = 3.5 for the four photocopying machine.

TABLE 2 PRODUCTION COST DATA FOR THE PHOTOCOPYING MACHINE				
Machine	1	2	3	4
A	8.6	14.0	12.0	11.0
b	3.0	8.0	14.0	6.0

In all practical cases, if we supposed that the machine time-dependent production cost exists only when it works [while the fixed production cost is always there whether the machine is under a breakdown or not], then we can assume that between two successive repairs the unit cost of production at time t of a good unit of machine 1 is given by:

$$f_i(t) = a_i + b_i[1 - \lambda_i]^n t^n \qquad 4$$

where λ_i is the fraction of time machine i is not working or the downtime factor for machine i. Figure 1 shows the production cost as a function of time for preventive maintenance programme of photocopying machine 1.

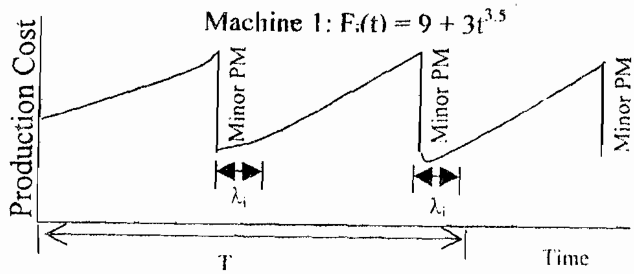

Figure 1: A Preventive Maintenance (PM) Programme for Photocopying Machine 1.

Thus in all practical cases, we can assume that the production cost per cycle, is given by the area under a production cost function similar to that of figure 1. since $(k_i - 1)$ is the number of minor repairs performed on machine i in a cycle and T is the time per cycle then the time between minor repairs will be given as T/k_i. Again, since λ_i is the fraction of time machine I is not working (or the downtime factor for machine interval it can be assumed that the machine did not work for a period given as $\lambda_i T/k_i$.

From figure 1 it can be seen that the production cost under this time period is simply the rectangular area

$a\lambda_i T/k_i$, since the time dependent portion of the production cost exists only when a machine works. Thus, the production cost per cycle, PC, can be expressed as:

$$PC = \sum_{i=1}^{N} ki \int_0^{T/ki} [a_i + b_i[1 - \lambda i]^n t^n] dt + \sum_{i=1}^{N} (a_i \lambda_i \frac{T}{k_i}) k_i \qquad 5$$

Which on performing the integration and simplification, becomes

$$PC = \sum_{i=1}^{N} a_i T + \sum_{i=1}^{N} \frac{b_i [1 - \lambda_i]^n T^{n+1}}{[n+1] k_i^n]} + \sum_{i=1}^{N} a_i \lambda_i T \qquad 6$$

GROUPING MAINTENANCE TOTAL COST FUNCTION

The total annual cost is equal to the sum of the annual repair and production costs. These costs can easily be calculated by multiplying the cycle costs by the number of cycles per year. The number of cycles per year is 1/T, since T is the time per cycle as previously defined. The total cost per year, TC, is then given as

$$TC = 1/T[RC + PC] \qquad 7$$

Where RC and PC are the repair and production cost respectively. Substituting equations 1 and 16 in equation 17 the total annual cost expression becomes.

$$TC = \frac{1}{T}\left(\sum_{j=1}^{M}\sum_{i=1}^{N} B_{ij}S_{ij} + \sum_{i=1}^{N} K_i C_i - \sum_{i=1}^{N} C_i + \sum_{i=1}^{N} \lambda_i P_i + C\right) + \sum_{i=1}^{N} a_i + \frac{T^n}{n+1}\sum_{i=1}^{N}\frac{b_i[1-\lambda_i]}{K_i^n} + \sum_{i=1}^{N} a_i\lambda_i \qquad 8$$

Differentiating the total cost function TC with respect to the cycle time T, setting it to zero and solving for T, gives the best value of the cycle time T, provided the total cost function is convex with respect to T (i.e. $d^2TC/dT^2 > 0$)

$$\frac{dTC}{dT} = -\frac{1}{T^2}\left(\sum_{j=1}^{M}\sum_{i=1}^{N} B_{ij}S_{ij} + \sum_{i=1}^{N} K_i C_i - \sum_{i=1}^{N} C_i + \sum_{i=1}^{N} \lambda_i P_i + C\right) + \frac{nT^{n-1}}{n+1}\sum_{i=1}^{N}\frac{b_i[1-\lambda_i]}{K_i^n} \qquad 9$$

setting dTC/dT = 0 and solving for T, we get

$$\frac{n}{n+1} T^{n-1} \sum_{i=1}^{N}\frac{b_i[1-\lambda_i]}{K_i^n} = \frac{1}{T^2}\left(\sum_{j=1}^{M}\sum_{i=1}^{N} B_{ij}S_{ij} + \sum_{i=1}^{N} K_i C_i - \sum_{i=1}^{N} C_i + \sum_{i=1}^{N} \lambda_i P_i + C\right) \qquad 10$$

$$T^{n+1} = (n+1)\frac{\left(\sum_{j=1}^{M}\sum_{i=1}^{N} B_{ij}S_{ij} + \sum_{i=1}^{N} K_i C_i - \sum_{i=1}^{N} C_i + \sum_{i=1}^{N} \lambda_i P_i + C\right)}{n\sum_{i=1}^{N}\frac{b_i[1-\lambda_i]}{K_i^n}} \qquad 11$$

The minimum or best values of T provided $d^2TC/dT^2 > 0$ as:

$$Tm = \left(\frac{n+1}{n}\left(\frac{\sum_{j=1}^{M}\sum_{i=1}^{N} B_{ij}S_{ij} + \sum_{i=1}^{N} K_i C_i - \sum_{i=1}^{N} C_i + \sum_{i=1}^{N} \lambda_i P_i + C}{\sum_{i=1}^{N}\frac{b_i[1-\lambda_i]}{K_i^n}}\right)\right)^{\frac{1}{n+1}} \qquad 12$$

Table 3 total cost data for group preventive maintenance of the four photocopying machine				
N = 3.5	C = 700			
Parameter	Machine			
	1	2	3	4
A	9	14	12	11
b	3	8	14	6
c	345	96.	198	98
λ	.012	008	006	.008
p	200	250	300	200
B1	.20	25	20	.15
B2	.20	0	15	0
B3	0	0	0	0
S1	70	104	80	60
S2	40	0	60	0
S3	0	0	0	0

$$\frac{d^2TC}{dT^2} = \frac{2}{T^3}\left(\sum_{j=1}^{M}\sum_{i=1}^{N} B_{ij}S_{ij} + \sum_{i=1}^{N} K_iC_i - \sum_{i=1}^{N} C_i + \sum_{i=1}^{N} \lambda_i P_i + C\right) + \left(n\frac{(n-1)}{n+1}T^{n-2}\sum_{i=1}^{N}\frac{b_i[1-\lambda_i]}{K_i^n}\right) \qquad 13$$

since B, S, C, λ, P and b are positive, with the condition $n \geq 1$ expression 13 is always positive, showing that the total cost function is convex with respect to T. hence Tm is an optimum and thus the best value of T. Substituting Tm in expression 12, for T in expression 8, and simplifying, gives the minimum total cost TC at optimum T value. That is:

$$TC = \left(\left(\frac{n}{n+1}\right)^{1/n+1} + \frac{1}{n+1}\left(\frac{n+1}{n}\right)^{n/n+1}\right)\left|\left(\sum_{j=1}^{M}\sum_{i=1}^{N} B_{ij}S_{ij} + \sum_{i=1}^{N} K_iC_i - \sum_{i=1}^{N} C_i + \sum_{i=1}^{N} \lambda_i P_i + C\right)^n \left(\sum_{i=1}^{N}\frac{b_i[1-\lambda_i]}{K_i^n}\right)\right|^{1/n+1} + \sum_{i=1}^{N} a_i[1+\lambda_i] \qquad 14$$

so far we **have only** derived expressions for Tm the optimal frequency of major maintenance and TC the total cost **associated** with the optimal frequency of major maintenance. But we still have to find the optimal frequencies of minor maintenance K_i for the various machines using the TC associated with the optimal frequency of major maintenance Tm. The expression for the optimal frequency of major maintenance Tm is given by equation 12 while the expression for the minimum total cost TC which is associated with Tm is given by equation 14.

Understanding the mathematical nature of equation 14 which gives the total cost with respect to K's is difficult. Hence a numerical Method similar to that of Goyal [1973] is presented.

ENUMERATION SOLUTION PROCEDURE

Optimization here will imply determining the optimal value of T, the cycle time, and the frequencies of the minor maintenance K_i for each machine within T to minimize the total cost of repairs and production TC which is given by equation 14. this will involve.

(1) Carrying out exhaustive analysis of the total cost function with respect to K_i which, implies computation of the total cost TC for the various possible K-Combination.
(2) Selecting the K-combination which gives the minimum total cost TC, and
(3) Determining the optimal value of T from equation 12 using the k-combination giving the minimum total cost TC.

Table 7 below shows the total [production and repair] cost data for each of the four photocopying machines. These data will be used to demonstrate the implication of the exhaustive enumeration solution procedure.

Exhaustive analysis: Suppose that N machines are to be placed on a group preventive maintenance programme and that none of the machines will be maintained at a minor preventive maintenance frequency of more than P in a cycle, then for a system with M = 2 and p = 2, there are 2^2 [that is 4] possible k – combination namely [$k = 1, k_2 = 1$]; [$k_1 = 1, k_2 = 2$]; [$k_1 = 2, k_2 = 1$] and [$k_1 = 2, k_2 = 2$]. For a system with N = 3 and p = 2, there are 2^3; [that is 8] possible k – combinations namely [$k_1 = 1$ $k_3 = 1$]; [$k_1 =1, k_2 = 1, k_3 = 2$] ; [$k_1 = 1, k_2 = 2, k_3 = 2$]; [$k_1 = 2, k_2 = 1, k_3 = 1$]; [$k_1 = 2, k_2 = 1, k_3 = 2$]; [$k_1 = 2, k_3 = 1$]; and [$k_1 = 2, k_2 = 2, k_3 = 2$]. For our present consideration we have a system with N = 4 and p = 6 where the group maintenance of the four photocopying machines is being considered and it is assumed that none of the machines will be maintained at a minor maintenance frequency of more than six in a cycle. In this system where N = 4 and p = 6, there are 6^4 [that is 1296] possible k – combinations. This combination is so large that it can only be conveniently generated by a computer. Exhaustive analysis using this example implies that for all possible combinations of K_i, TC is calculated using equation 14 and substituting values from Table 3. **Table 4 below.** shows the total cost TC for all the possible combinations of K_i in the group preventive maintenance programme of the four photocopying machines. Note that we have only been able to show the table for $K_4 = 3$. for tables showing values for $K_4 = 1, 2, 4, 5, 6$ see Offiong (1996)

From table 3.8 notice that the combination [$K_1 = 4, K_2 = 3, K_3 = 2, K_4 = 4$]; [$K_1 = 5, K_2 = 2, K_3 = 4, K_4 = 3$] and [$K_1 = 2, K_2 = 6, K_3 = 5, K_4 = 3$] have their values of TC as 480.89, 650.05 and 45.20 respectively. These values are dotted asterisk in table. A computer programme has been written for the generation of all the possible k-combination and computation of the associated total cost TC [see Offiong, 1996]. The programme as it is now can only handle systems with N = 4 and P = 6. However, it can easily be tailored to handle systems with both lower and higher combinations of N and P.

Selecting the optimum k-combination: After the generation of all possible k-combinations and computation of the associated total cost TC the next task in the numerical analysis is that of selecting the k-combination which gives the minimum total cost. Figure 2 shows plots of the total cost TC against the values 1, 2, ..., 6 of some k for fixed values of other k's for the system presently under consideration where there are four photocopying machines and it is assumed that none of the machines will be maintained at a minor maintenance frequency of more than six in a cycle. Due to limited space and for clarity only six plots have been made out of the 216 plots that could have resulted from this system. However, sampling has shown that all the plots will be similar in shape to the six sample plots. The K-combinations of the six sample plots are:-

1. $K_4 = 1, 2, \ldots, 6$ for fixed values of K_3 at 1, K_2 at 3 and K_1 at 2;
2. $K_4 = 1, 2, \ldots, 6$ for fixed values of K_3 at 2, K_2 at 3 and K_1 at 2;
3. $K_4 = 1, 2, \ldots, 6$ for fixed values of K_3 at 3, K_2 at 3 and K_1 at 2;
4. $K_4 = 1, 2, \ldots, 6$ for fixed values of K_3 at 4, K_2 at 3 and K_1 at 2;
5. $K_4 = 1, 2, \ldots, 6$ for fixed values of K_3 at 5, K_2 at 3 and K_1 at 2;
6. $K_4 = 1, 2, \ldots, 6$ for fixed values of K_3 at 6, K_2 at 3 and K_1 at 2;

Table 4 TC for some possible combination of K_i

K_4	K_3	K_2	K_1 1	2	3	4	5	6
3	1	1	546.03	670.54	797.83	919.98	1037.57	1151.34
		2	546.62	651.93	766.17	876.54	983.08	1068.33
		3	579.39	679.82	791.43	899.66	1004.33	1105.91
		4	614.05	710.96	820.06	927.71	1031.26	1131.87
		5	648.58	742.33	850.70	956.43	1059.03	1158.82
		6	682.68	773.50	880.49	985.16	1086.87	1185.90
	2	1	542.45	625.24	725.73	823.85	918.87	1011.13
		2	479.83	490.47	555.11	623.05	689.92	755.20
		3	494.91	485.49	542.21	604.42	666.21	726.73
		4	517.71	498.90	552.70	613.01	673.19	732.59
		5	541.71	515.98	568.00	627.23	686.55	744.87
		6	565.81	533.98	584.58	643.00	701.68	759.45
	3	1	599.64	670.82	765.86	859.53	950.63	1039.29
		2	511.89	480.91	529.26	585.77	642.63	698.57
		3	521.28	453.10	484.68	530.57	578.55	626.34
		4	541.22	450.97	481.87	524.00	509.05	614.23
		5	562.22	468.24	489.74	530.09	573.84	617.91
		6	585.06	481.74	500.87	540.12	583.09	626.51
	4	1	661.05	724.62	816.60	907.93	997.02	1083.92
		2	556.47	503.35	544.67	596.92	650.28	703.06
		3	562.74	462.64	480.98	591.52	561.86	604.69
		4	581.29	461.12	469.21	502.11	540.34	579.62
		5	602.04	469.59	472.48	502.13	539.12	578.88
		6	623.2	481.11	480.68	509.38	544.76	581.70
	5	1	721.91	779.03	868.59	958.08	045.63	1131.18
		2	602.73	533.45	570.58	620.55	672.13	723.37
		3	606.55	484.42	495.01	529.68	569.29	609.81
		4	624.10	479.64	477.61	505.41	540.17	576.60
		5	644.15	486.07	478.07	502.54	535.09	569.74
		6	664.75	496.97	484.59	507.16	538.51	572.22
	6	1	781.59	832.83	920.28	10081.18	1094.38	1178.75
		2	648.59	565.53	599.50	647.84	698.26	748.48
		3	650.40	510.03	515.21	547.43	585.41	624.61
		4	667.10	502.90	493.95	598.54	551.27	586.10
		5	585.56	508.65	492.54	515.58	543.67	576.56
		6	706.67	518.44	497.78	516.71	545.69	677.57

The associated TC of these K-combinations have been presented in table 8. a study of figure 2 and table 8 gives two important revelation about the total cost TC namely:-

(1) The total cost function is convex with respect to each for fixed values of other K's
(2) The total cost generally increased with higher combination of K's

This revelation lead to the conclusion that there exists a global minimum value of TC for all possible K-combinations which can be arrived at by minimizing equation 14.

For the exhaustive enumeration solution procedure, a computer programme has been written to carry out the search for the minimum TC after the generation of all the possible K-combinations and computations of the associated TC [see Offiong, 1996]. For the four photocopying machines presently under consideration, the global minimum value of TC is given by plot 3 and its value is ₦453.10. this value is encircled in **Table 4** and it corresponds to K-combination [K_1 = 2, K_2 = 3, K_3 = 3, K_4 = 3]. Thus for the four photocopying machines, the best frequency of minor maintenance is 2 for machine 1, 3 for machine 2, 3 for machine 3 and 3 for machine 4.

DETERMINATION OF THE OPTIMUM CYCLE TIME:

After the selection of the K-combination which gives the minimum TC, the next and final stage in the numerical analysis will be the determination of the optimum interval of major preventive maintenance Tm. The optimum cycle time or the best value of the interval of major preventive maintenance Tm can be calculated using the optimum K-combination or the optimum frequencies of minor maintenance and equation 12. It has been shown that for the four photocopying machines, the optimum K-combination is [K_1 = 2, K_2 = 3, K_3 = 3, K_4 = 3]. Making use of this optimum K-combination the values in table 8 and equation 22, the best value of the interval of major preventive maintenance Tm has been calculated for the four photocopying machines and its value is 5.85 Weeks or approximately 6 weeks. A computer programme has been written to carry out this computation [see Offiong, 1996].

ENUMERATION ALGORITHM

To compute the best value of the interval of major preventive maintenance for a group of machines and

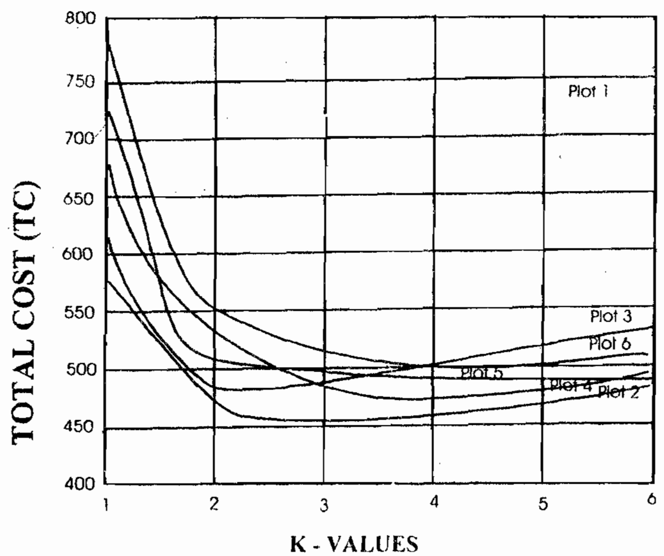

FIGURE 2: Plots of the Total Cost against the values 1, 2, ... ,6 at K_4 for fixed values of other K's.

select the optimum frequency of minor maintenance for each machine within the group in order to minimize the total cost of repair and production in this cycle, using exhaustive enumeration procedure, the following steps are required:

Step 1. Assuming that no machine in the group will have a minor preventive maintenance frequency of more than a reasonable number P, generate all the possible K-combination. Note that where there are N machines in a group there will be P^N possible K-combination.

Step 2. Using each of the generated K-combination calculate TC from equation 14
Step 3. Select from all the calculated TCs the minimum
Step 4. Using the K-combination associated with the minimum TC calculate Tm from equation 12.
Step 5. List Tm, Minimum TC and the K-combination associated with the minimum TC.
This exercise will require a very powerful computer because of the large number of K-combinations needed for the exhaustive examination of TC. For a system with N = 10 and P = 10, we will be talking about 10^{10} [that is 10,000,000,000] K-combinations.

INDUSTRIAL APPLICATION OF THE MODEL

In the case study, there are six expellers involved in the crushing of Kernel in a continuous production system. Preventive maintenance of this expeller under the existing group maintenance was done every three months. Analysis carried out by Offiong (1993) showed that with time, some machines within a group depreciation more than others hence requiring more frequent preventive maintenance. This implies that the existing practice of fixing preventive maintenance interval for all the expellers in a group was unreasonable. Under this existing practice the total cost production/repair for this group of expellers was ₦11400 per annum which amounted to ₦3.800 per month

The production cost data for the expellers in the group are given in **table 5**, while the repair cost data are

Table 5,	Production cost data and parameter for group 3 machines at FOM					
Time in fourth nights	Production Cost Data					
	Ex. 13	Ex. 14	Ex. 15	Ex. 16	Ex. 17	Ex. 18
0	25	38	32	28	30	
1	41	70	50	50	68	27
2	65	110	75	75	120	45
3	90	155	102	106	170	70
4	115	210	130	140	230	95
5	140	260	165	172	260	120
6	-	-	-		-	150
Parameter	Production Cost Parameter Value					
	Ex. 13	Ex. 14	Ex. 15	Ex. 16	Ex. 17	Ex. 18
A	25	38	32	28	30	27
B	17	32	19	21	38	18
N	1.2					

Table 6,	Repair cost parameter for group 3 machines at FOM					
Parameter	Ex. 13	Ex. 14	Ex. 15	Ex. 16	Ex. 17	Ex. 18
C	900	700	500	600	900	1000
λ	.04	.02	.03	.05	.04	.06
P	380	380	380	380	380	380
B	.36	.30	.40	.32	.38	.34
B	.25	.20	.30	.20	.30	.25
B	0	0	0	0	0	0
S	200	200	200	200	200	200
S	150	150	150	150	150	150
S	0	0	0	0	0	0
C	3800					

Table 7	Solution for group 3 machines at FOM					
Iterations	Number of repairs on machines					
	Ex. 13 (k_1)	Ex. 14 (k_2)	Ex. 15 (k_3)	Ex. 16 (k_4)	Ex. 17 (k_5)	Ex. 18 (k_6)	Total cost
1	1	1	1	1	1	1	1494.65
2	2	3	3	3	3	2	1456.45
3	3	5	4	4	3	4	1458.82
Optimum cycle time 17.00: optimum total cost = 1456.45 Optimum K-combination is $K_1 = 2, K_2 = 3, K_3 = 3, K_4 = 3, K_5 = 3, K_6 = 2$							

given in **Table 6**. Application of model yields **Table 7**. **Table 7** indicates that for the expellers in this group, simultaneous preventive maintenance should be performed on the entire 6 expellers in the group every 9 months (17.00 fortnights) and in between these 9 months individual preventive maintenance should be performed on

Expeller 1, (2 – 1) or 1 time; expeller 2, (3 – 1) or 2 times;
Expeller 3, (3 – 1) or 2 times; expeller 4, (3 – 1) or 2 times;
Expeller 5, (3 – 1) or 2 times; and expeller 6, (2 – 1) times.

From table 11 it can be seen that the total cost of production/repair per every fortnight (two weeks) is ₦1,456.45 which implies a cost of ₦2,912.90 per month. This amount when compared with the cost associated with the existing policy reveals a saving of (₦3,800.00 – ₦2,912.90) = ₦887.10 per month. For more details on the implementation of this model see Offiong (1996).

CONCLUSION

It is recommended that a work study should precede any design of a maintenance system. In this work, the method of carrying out maintenance tasks were not examined to see if they are the best possible methods. All scheduling were done assuming that the existing method of task performance is the best. This might not be so for most cases. It is expected that if a method study was carried out before this application of the model was carried out, some of the costs would have been reduced due to savings in labour, time, materials and spares. Furthermore work should take care of this. Generally this work has shown how management science can be used to improve the effectiveness of a maintenance programme.

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