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MAINTAINABILITY OF MANPOWER SYSTEM WITH RESTRICTED RECRUITMENT

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ABSTRACT

The maintainability of a manpower system is studied under a Markov framework. The classical method of controlling only one factor of flow is extended to highlight the case in which two factors are under control simultaneously. One special case of this extension, where recruitment of units faces partial embargo, is given, which led to other important results.

KEY WORDS: Manpower structure, Maintainability, Factors of flow, Markov model Probability.

INTRODUCTION

Maintainability is an aspect of manpower control that has to do with making a desired manpower structure to remain the same at the mth future accounting periods, Batholomew (1982). It is regarded as one-step maintainability if m = 1 or r–step maintainability if m = r. Manpower structures are maintained by management control over the factors of control, usually promotion flow and recruitment flow. Hence, in the literature, it is regarded as promotion control when the promotion flow is controlled to induce the required change in manpower structure, or recruitment control when the recruitment flow is used, Batholomew, et al (1991). The third type of flow, which is wastage, does not enjoy the same wide acceptance as a tool for control in manpower planning due to the problem of undesirability, Bartholomew (1982). Ossai (2008) however demonstrated the use of concepts related to wastage in controlling departmentalized manpower systems.

What is common in the above types of statistical manpower control is that only one factor of flow is controlled. In practical situation the management has the free hand to exercise control in more than one factor of flow at a time. This paper is, therefore, set out to statistically study a case where both the recruitment flow and the promotion flow are under control at the same time. It is a kind of promotion control with restricted recruitment or embargo on recruitment. The manpower system is considered under Markov Framework.

Maintainability in Markov Manpower Framework

Let *n*(*t*), a 1 × k row vector of number of unit stocks, be the manpower structure at time *t* and *p* the homogenous k × k probability transition matrix of the manpower system; *r* and *w* be 1 × k row vector of recruitment and wastage respectively. We consider a manpower system that follows a first order Markov process, Kijima (1997). The manpower structure at time *t* + 1 is expressed as

$$n(t+1) = n(t)P + n(t)w'r \qquad (2.1)$$

In Uche and Ossai (2008), *P* is redefined to give the model in (2.1) as

$$n(t+1) = b(t) + n(t)P' + n(t)w'r \qquad (2.2)$$

where, $b(t) = n(t)(A - A'), A = P - B$ and $B = diag\{P_{ii}\}, i = 1, 2, \cdots, k;$ that is, *B* is a k × k diagonal matrix whose diagonal entries correspond to the diagonal entries of *P*.

Now, the current structure, *n*(*t*), is maintainable into the *t* + 1 time horizon if $n(t) = n(t+1) = n$, Georgiou and Tsantas (2002). Hence, for maintainability, (2.2) translates to

$$n = b^* + nP' + nw'r \qquad (2.3)$$

where $b^* = n(A - A')$

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Result 1

If a transition matrix *P* exists, which maintains the structure *n*(*t*), then such matrix *P* must satisfy the condition that

$$b^* \le n(I - w'r) \qquad (2.4)$$

where *I* is the identify matrix.

The above result is true from (2.3) if we note that

$nP - b^* = nP'$ and since $nP' \le 0$ it follows that $nP - b^* \ge 0$, which implies that $n(I - w'r) - b^* \ge 0.$

Where 0 is the k × k zero matrix and the mathematical signs between vectors and matrices are evaluated at their corresponding entries.

3. Control by Embargo

As mentioned earlier, we discuss a situation of manpower control in which control is exercised on both the transition matrix P and the recruitment vector *r*. It is a type of restriction or embargo on recruitment in the case of *r*, and normal promotion control in the case of *P*. In the study of n-step maintainability, Davies (1981) suggests the use of total restriction on recruitment as a means of maintaining a manpower structure at the n^{th} step. No statistical demonstration is however carried out in line with this suggestion in his work. This type of discussion is presented in Ossai (2008), where two types of embargo are defined as total embargo and partial embargo.

In total embargo on recruitment, there is no unit recruited into the system in the time period under control. The implication of this on the maintainability of the manpower structure is seen from equation (2.3). The structure will be maintained under no recruitment plan when

$$b^* = n(I - P') \qquad (3.1)$$

For instance, it can be shown that under total embargo on recruitment, the manpower structure is maintained when *P* is an identity matrix since then the condition in (3.1) is satisfied. Other forms of *P* can be found which satisfies this maintainability condition.

Any other type of restriction on recruitment that is not total restriction is referred to as partial embargo on recruitment. For instance, ban on recruitment can just be placed on selected grades or levels of the manpower system for some special reasons. It can be on the top grades to avoid top-heavy structures, etc. In each case, the resultant recruitment vector can be represented which guides the way the structure will be maintained. Here we discuss one such partial embargo on recruitment, which seems to represent closely what obtains in many manpower systems. This is a situation where recruitment is made only to replace the exact number and type of units (or workers) that leave by wastage. That is, recruitment is only made to correspond to the number and type of workers who leave the system in each grade. Let the recruitment vector in this case be represented by $r_{(w)}$. Let also the i^{th} element of *n*(*t*) and *w* be $n_i(t)$ and w_i respectively. Since $n(t)w'$ represents the total number of units that leave through wastage, the recruitment vector that achieves the above partial embargo plan at time *t* is given by

$$r_{(w)} = \left[n_1(t)w_1, n_2(t)w_2, \cdots, n_k(t)w_k\right]\left\{n(t)w'\right\}^{-1}$$

It is easy to see that $r_{(W)}$ as above is a probability vector, which qualifies it as a suitable recruitment vector. The following results follow from the latter discussions.

Results 2

Any diagonal transition matrix P that satisfies the stochastic condition of a manpower structure, that is $P + diag(w_i) = I,\ i = 1, 2, ..., \text{k},$ will maintain the given structure, *n*, under partial embargo with $r_{(W)}$ as the recruitment vector, $r_{(w)} < \infty$. Where $diag(w_i)$ is a k x k diagonal matrix whose i^{th} diagonal entry is w_i.

Proof: We first prove the prerequisite (support) result that under partial embargo on recruitment any such P must satisfy the condition given in equation (2.4). Under Result 1, this is easily done because if *P* is diagonal, $b^* = 0$, so that according to the condition in (2.4), we need to show that $0 \le n(I - w'r).$ This follows since

$$n(I - w'r) = n - nw'\left(r_{(w)}\right) = n\left(I - diag(w_i)\right) = nP \ge 0.$$

We now turn to prove the full result. Recall that the steady-state equation to be satisfied for any maintainable structure, *n*, given in (2.3) is that

$$n = b^* + nP' + nw'r$$

If $r_{(w)} < \infty$, then

$$n = b^* + nP' + nw'r_{(w)}$$

$$= b^* + nP' + nw'\left[n_1 w_1, n_2 w_2, ..., n_k w_k\right]\left(nw'\right)^{-1}$$

$$= b^* + nP' + n\ diag\left(w_i\right),\ i = 1,2,...,k.$$

$$\Rightarrow n\left\{I - P' - diag\left(w_i\right)\right\} = b^* \qquad (3.2)$$

The condition in (3.2) is the maintainability condition for *n* in the case specified. Without loss of generality, let P be diagonal so that $P' = P$ and, hence, $b^* = 0.$ The condition in (3.2) becomes

$$n\left\{I - P - diag\left(w_i\right)\right\} = 0 \qquad (3.3)$$

The structure *n* is necessarily greater than zero. So, one possible solution to (3.3) is that

$$I - P - diag\left(w_i\right) = 0 \qquad (3.4)$$

With P diagonal, (3.4) is true if P satisfies the stochastic condition that

$$P + diag\left(w_i\right) = I,\ \ i = 1,2,\cdots,k,$$

which concludes the proof.

We note that for a given wastage vector *w* and manpower structure *n*, such a *P* in Result 2 is unique. This is because there can only be one diagonal P which will satisfy the stochastic condition with the given *w*.

However, interest is not just on a diagonal transition matrix that will maintain a manpower structure. Neither is it an advantage in manpower planning for the control tool to be unique. In fact, multiplicity of options in planning tools is an advantage in manpower planning, especially in the area of manpower control, Bartholomew, et al (1991). The uniqueness property of *P* in the above result becomes desirable if one is interested in quick example of a *P* that will maintain the manpower structure as is sometimes the case in theoretical studies. The following result addresses the issue further.

Result 3

There can be many non-diagonal transition matrix *P* that will maintain the manpower structure, with $r_{(w)}$ as the recruitment vector.

Proof

This follows because the condition in (3.3) turns out to be the general condition for maintainability of *n*, (since $nP' + b^* = nP$), and though *n* is necessarily greater than zero, it is possible that $n\left\{I - P - diag\left(w_i\right)\right\} = 0$ when $I - P - dig\left(w_i\right) \neq 0$ and *P* is not diagonal.

4. Illustrative Example

Let a simple three-grade manpower system have a structure given by *n* = [20, 10, 10] with a wastage vector *w* = [0.2, 0.1, 0.4]. If the control policy is partial embargo on recruitment with $r = r_{(w)}$, then by Result 2 it follows that the diagonal transition matrix that will maintain the structure is given by

$$P = \begin{bmatrix} 0.8 & 0 & 0 \\ 0 & 0.9 & 0 \\ 0 & 0 & 0.6 \end{bmatrix}$$

since it is the only one that satisfies the stochastic condition of the system with the given wastage vector *w*. Here $r_{(w)}$ is calculated to be $r_{(w)} = [0.4444, 0.1111, 0.4444]$ and justification for maintainability is the satisfaction of equation (2.3), which readily follows using MATLAB software.

According to Result 3, there are other probability transition matrices outside the set of diagonal matrices that will satisfy the stochastic condition and still maintain the same structure with $r = r_{(w)}$. An example is

$$P = \begin{bmatrix} 0.8 & 0 & 0 \\ 0 & 0.6 & 0.3 \\ 0 & 0.3 & 0.3 \end{bmatrix}$$

In this case, using MATLAB with *w* as given above, *P* satisfies $P\mathbf{1}' + w' = \mathbf{1}'$, which is the general stochastic condition for non-diagonal transition probability matrix, where $\mathbf{1}' = (1,1,...,1)'$, that is a column vector of one's. Also, using MATLAB with the foregoing definitions $b^* = [0,0,0]$, $r_{(w)} = [0.4444, 0.1111, 0.4444]$ and hence $b^* + nP' + nw'r = [20,10,10] = n$. This shows that the non-diagonal matrix *P* also maintains the structure *n* as equation (2.3) is then satisfied.

CONCLUSION

In this work the methods of maintaining manpower systems in a Markov model have been extended to highlight the case in which two factors of flow are under control simultaneously. In the particular case of partial embargo on recruitment, the resultant recruitment vector is defined, which led to two important results in the maintainability of the manpower structure.

It is noteworthy that other cases of partial embargo on recruitment are possible and can be studied in line with the procedure used in this work, by first finding their resultant recruitment vectors. Also, the situation in which the system is changing in size can readily be incorporated.

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