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ESTIMATION OF PURE AUTOREGRESSIVE VECTOR MODELS FOR REVENUE SERIES

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ABSTRACT

This paper aims at applying multivariate approach to Box and Jenkins univariate time series modeling to three vector series. General Autoregressive Vector Models with time varying coefficients are estimated. The first vector is a response vector, while others are predictor vectors. By matrix expansion each vector, whether response or predictor is a linear combination of other time varying vectors and itself. The models are linear and parameters are estimated by least squares method. The estimates obtained prove reality of the models estimated.

KEYWORDS: Autoregressive Vector Models, Response and predictor vectors, Feedforward and feedback parameters, Vector series and white noise.

1. INTRODUCTION

According to Dufour (2006), an m-dimensional vector process $[X_t : t \in Z]$ follows an Autoregressive (p) model[or VAR(p) model] if it satisfies an equation of the form:

$X_t = \mu + \sum \Phi_k X_{t-k} + a_t$ for every t,

Where, $\Phi_1, \Phi_2, \ldots, \Phi_p$ are m×m fixed matrices and $[a_t : t \in Z]$ is a white noise process.

Multivariate Autoregressive analysis is an analysis multiple time series whose vector of current values of all variables is modelled as a linear sum of previous activities (Harrison and Penny, 2003). Consider d time series generated from d variables (brains regions) with a system such as a functional network in the brain and where p is the order of the model. Here the scalar p denotes order, however, later we will use p(α) to mean the probability of α. A Multivariate Autoregressive (p) model predicts the next value in a d-dimensional time series, y_n as a linear combination of the p previous values

$$y_n = \sum_{i=1}^{d} y_{n-i} A(i) + e_n$$

where $y_n = [y_{n(1)}, y_{n(2)}, \ldots, y_{n(d)}]$ is the n^{th} sample of a d-dimensional time series, each $A(i)$ is a d-by-d matrix of coefficients (weights) and $e_n = [e_{n(1)}, e_{n(2)}, \ldots, e_{n(d)}]$ is additive Gaussian noise with zero mean and covariance R.

Harrison and Penny (2003) assumed that the data mean has been subtracted from the time series. The model can be written in the standard form of a multivariate linear regression model as follows,

$$y_n = x_n W + e_n$$

where $x_n = [y_{n-1}, y_{n-2}, \ldots, y_{n-p}]$ are the p previous multivariate time series samples and W is a (p×d) - by- d matrix of MAR coefficients (weights). There are therefore a total of k = p×d×d MAR coefficients. If the n rows of Y, X and E are y_n, x_n and e_n respectively and there are n = 1,...,N samples then we can write

$$Y = XW + E$$

Where Y is an (N – p)-by-(p×d) matrix and E is an (N-p)-by-d matrix. The number of rows N-p (rather than N) arises as samples at time points before p do not have sufficient preceding samples to allow prediction.

Sims (1996) carried out multivariate time series modelling of consumption and gross national products of United State America. He considered alternative approaches to modelling the joint behavior of consumption and Gross National Products. The approaches included ordinary least squares (OLS) and maximum likelihood methods. Including two lags of each variable, in logs, and a constant, the VAR linear regression model by OLS was obtained as

$$C = (.9542L + 0456L^2)C + (.1427L - .1432L^2)Y + .01258 + \varepsilon_c$$
$$Y = (.2991L - .2300L^2)C + (1.2139L - .2908L^2)Y + .09799 + \varepsilon_y$$

Apart from OLS method of obtaining the bivariate models, the same system was estimated maximum likelihood method. The estimated system is

$$C = (.9443L - .0364L^2)C + (.1571L - 1409L^2)Y + .01385 + \varepsilon_c$$
$$Y = (.2877L - .2228L^2)C + (1.2045L - .2773L^2)Y + 09635 + \varepsilon_y$$

The coefficients of this system are closed to the system estimated by OLS.

This paper considers modeling of revenue series, which consists of three vector series, a response (X_{1t}) and two predictor vectors (X_{2t}) and (X_{3t}). The distributions of autocorrelation and partial autocorrelation functions show that the greater proportion of the observations assumed by the vector series is accounted for by a pure autoregressive process.

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2. LINEAR VECTOR MODELS

The general Vector Autoregressive, VAR(p) process is given as

$$\begin{pmatrix} X_{1t} \\ X_{2t} \\ X_{3t} \\ \vdots \\ X_{nt} \end{pmatrix} = \begin{pmatrix} \gamma_{1.11} & \gamma_{1.12} & \gamma_{1.13} & \cdots & \gamma_{1.1k} \\ \gamma_{1.21} & \gamma_{1.22} & \gamma_{1.23} & \cdots & \gamma_{1.2k} \\ \gamma_{1.31} & \gamma_{1.32} & \gamma_{1.33} & \cdots & \gamma_{1.3k} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ \gamma_{1.r1} & \gamma_{1.r2} & \gamma_{1.r3} & \cdots & \gamma_{1.rk} \end{pmatrix} \begin{pmatrix} X_{1t-1} \\ X_{2t-1} \\ X_{3t-1} \\ \vdots \\ X_{nt-1} \end{pmatrix} + \begin{pmatrix} \gamma_{2.11} & \gamma_{2.12} & \gamma_{2.13} & \cdots & \gamma_{2.1k} \\ \gamma_{2.21} & \gamma_{2.22} & \gamma_{2.23} & \cdots & \gamma_{2.2k} \\ \gamma_{2.31} & \gamma_{2.32} & \gamma_{2.33} & \cdots & \gamma_{2.3k} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ \gamma_{2.r1} & \gamma_{2.r2} & \gamma_{2.r3} & \cdots & \gamma_{2.rk} \end{pmatrix} \begin{pmatrix} X_{1t-2} \\ X_{2t-2} \\ X_{3t-2} \\ \vdots \\ X_{nt-2} \end{pmatrix}$$

$$+ \begin{pmatrix} \gamma_{3.11} & \gamma_{3.12} & \gamma_{3.13} & \cdots & \gamma_{3.1k} \\ \gamma_{3.21} & \gamma_{3.22} & \gamma_{3.23} & \cdots & \gamma_{3.2k} \\ \gamma_{3.31} & \gamma_{3.32} & \gamma_{3.33} & \cdots & \gamma_{3.3k} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ \gamma_{3.r1} & \gamma_{3.r2} & \gamma_{3.r3} & \cdots & \gamma_{3.rk} \end{pmatrix} \begin{pmatrix} X_{1t-3} \\ X_{2t-3} \\ X_{3t-3} \\ \vdots \\ X_{nt-3} \end{pmatrix} + \ldots + \begin{pmatrix} \gamma_{p.11} & \gamma_{p.12} & \gamma_{p.13} & \cdots & \gamma_{p.1k} \\ \gamma_{p.21} & \gamma_{p.22} & \gamma_{p.23} & \cdots & \gamma_{p.2k} \\ \gamma_{p.31} & \gamma_{p.32} & \gamma_{p.33} & \cdots & \gamma_{p.3k} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ \gamma_{p.r1} & \gamma_{p.r2} & \gamma_{p.r3} & \cdots & \gamma_{p.rk} \end{pmatrix} \begin{pmatrix} X_{1t-p} \\ X_{2t-p} \\ X_{3t-p} \\ \vdots \\ X_{nt-p} \end{pmatrix}$$

$$+ \begin{pmatrix} U_{1t} \\ U_{2t} \\ U_{3t} \\ \vdots \\ U_{nt} \end{pmatrix}$$

The above matrices give the following models:

$$X_{1t} = \sum_{a=1}^{p} \sum_{i=1}^{n} \sum_{f=1}^{k} \gamma_{a.1f} X_{it-a} \quad \ldots \quad (2.1)$$

$$X_{2t} = \sum_{a=1}^{p} \sum_{i=1}^{n} \sum_{f=1}^{k} \gamma_{a.2f} X_{it-a} \quad \ldots \quad (2.2)$$

$$X_{3t} = \sum_{a=1}^{p} \sum_{i=1}^{n} \sum_{f=1}^{k} \gamma_{a.3f} X_{it-a} \quad \ldots \quad (2.3)$$

$$\vdots$$

$$X_{nt} = \sum_{a=1}^{p} \sum_{i=1}^{n} \sum_{f=1}^{k} \gamma_{a.rf} X_{it-a} \quad \ldots \quad (2.4)$$

Therefore, models '2.1', '2.2','2.3' and '2.4' express linear autoregressive relationships of X_{1t}, X_{2t}, $X_{3t}, \ldots X_{nt}$ vectors with distributed lags. $\gamma_{a.1f}$, $\gamma_{a.2f}$, $\gamma_{a.3f}, \ldots, \gamma_{a.pf}$ are the matrices of coefficients of the autoregressive vector series.

3.1 VECTOR AUTOREGRESSIVE LNEAR MODELS

The distribution of autocorrelation and partial autocorrelation functions

As shown in Figure 3.1. 3.2 and 3.3 of non-stationary vector series suggested pure autoregressive process of order three for X_{1t}, order two for X_{2t} and order one for X_{3t}. On the basis of the order for each vector series, the vector models are:

$$\begin{pmatrix} X_{1t} \\ X_{2t} \\ X_{3t} \end{pmatrix} = \begin{pmatrix} \gamma_{1.11} & \gamma_{1.12} & \gamma_{1.13} \\ \gamma_{1.21} & \gamma_{1.22} & \gamma_{1.23} \\ \gamma_{1.31} & \gamma_{1.32} & \gamma_{1.33} \end{pmatrix} \begin{pmatrix} X_{1t-1} \\ X_{2t-1} \\ X_{3t-1} \end{pmatrix} + \begin{pmatrix} \gamma_{2.11} & \gamma_{2.12} & 0 \\ \gamma_{2.21} & \gamma_{2.22} & 0 \\ \gamma_{2.31} & \gamma_{2.32} & 0 \end{pmatrix} \begin{pmatrix} X_{1t-2} \\ X_{2t-2} \\ X_{3t-2} \end{pmatrix}$$

$$+ \begin{pmatrix} \gamma_{3.11} & 0 & 0 \\ \gamma_{3.21} & 0 & 0 \\ \gamma_{3.31} & 0 & 0 \end{pmatrix} \begin{pmatrix} X_{1t-3} \\ X_{2t-3} \\ X_{3t-3} \end{pmatrix} + \begin{pmatrix} U_{1t} \\ U_{2t} \\ U_{3t} \end{pmatrix}$$

The expansion of the matrices produces the following models:

$$X_{1t} = \gamma_{1.11}X_{1t-1}+\gamma_{1.12}X_{2t-1}+\gamma_{1.13}X_{3t-1}+\gamma_{2.11}X_{1t-2}+\gamma_{2.12}X_{2t-2}+\gamma_{3.11}X_{1t-3}+U_{1t} \quad \ldots\ldots (3.1)$$
$$X_{2t} = \gamma_{1.21}X_{1t-1}+\gamma_{1.22}X_{2t-1}+\gamma_{1.23}X_{3t-1}+\gamma_{2.21}X_{1t-2}+\gamma_{2.22}X_{2t-2}+\gamma_{3.21}X_{1t-3}+U_{2t} \quad \ldots\ldots (3.2)$$
$$X_{3t} = \gamma_{1.31}X_{1t-1}+\gamma_{1.32}X_{2t-1}+\gamma_{1.33}X_{3t-1}+\gamma_{2.31}X_{1t-2}+\gamma_{2.32}X_{2t-2}+\gamma_{3.31}X_{1t-3}+U_{3t} \quad \ldots\ldots (3.3)$$

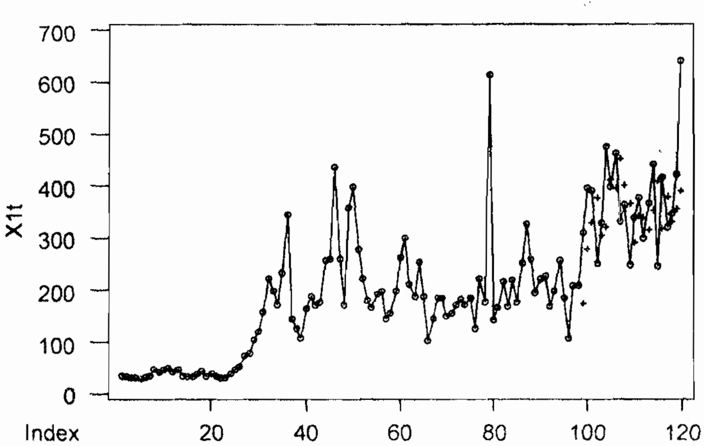
The above models are models of linear relationship between each vector and the distributed lag of the vector itself and other vectors. The models are similar to multiple regression models, and application of ordinary least squares method provides estimates for the parameters. The values shown in appendix (1) are values of the predictor lagged vectors whose regression estimates give the following results:

$$X_{1t} = 0.720X_{1t-1} - 0.271X_{2t-1} + 0.052X_{1t-2} + 0.197X_{2t-2} + 0.288X_{1t-3} \quad \ldots\ldots (3.4)$$
$$X_{2t} = 0.264X_{1t-1} + 0.069X_{2t-1} + 0.143X_{1t-2} + 0.108X_{2t-2} + 0.196X_{1t-3} \quad \ldots\ldots (3.5)$$
$$X_{3t} = 0.456X_{1t-1} - 0.340X_{2t-1} - 0.0916X_{1t-2} + 0.088X_{2t-2} + 0.0915X_{1t-3} \quad \ldots\ldots (3.6)$$

The above estimated models are used to obtain the estimate of the non-stationary revenue series as shown in appendix '2' Appendix '1' contains actual values of the non-stationary revenue series. Moreover, estimates obtained for the vectors, X_{1t}, X_{2t} and X_{3t} are graphically shown with the original revenue series in figures '3.1', '3.2' and '3.3'.

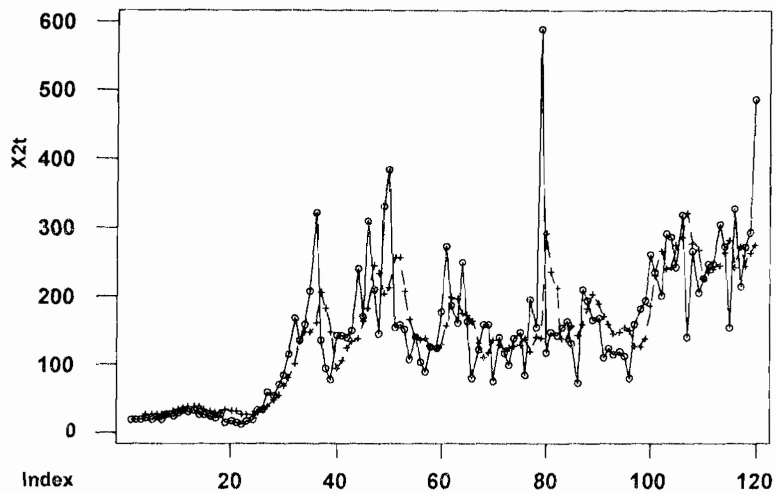
CONCLUSION

The distribution of autocorrelation and partial autocorrelation functions suggested the order for each vector with which the trivariate models were obtained. The least squares regression estimates of the models show the contribution of each feedforward and feedback parameter to each response vector in the autoregressive vector models. It is established that in multivariate time series modeling, every vector, be it response or predictor, could as well be modeled with time varying coefficients.



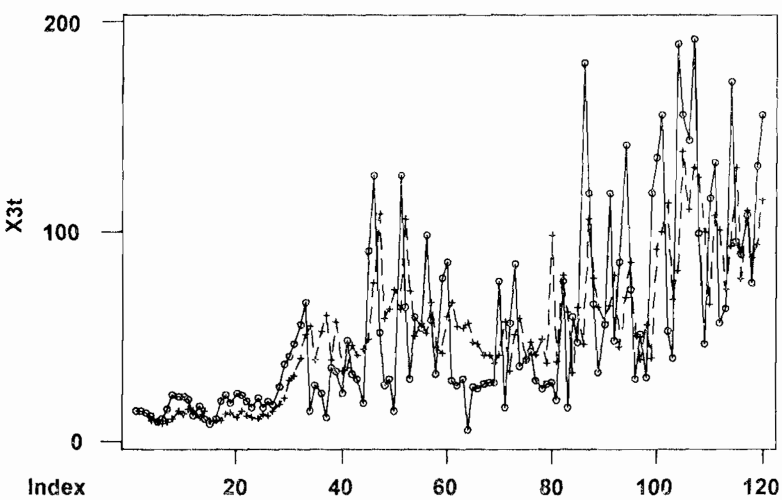
ACTUAL PLOTS WITH BLACK DOTS
ESTIMATES PLOTS WITH BLACK PLUS

Figure 3.1: Plots of actual and Estimates of X1t



ACTUAL PLOTS WITH BLACK DOTS
ESTIMATES PLOTS WITH BLACK PLUS

Fiqure 3.2: Plots of actual and estimates of X2t



ACTUAL PLOTS WITH BLACK DOTS
ESTIMATES PLOTS WITH BLACK PLUS

Figure 3.3: Plots of actual and estimates of X3t

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APPENDIX 1: ACTUAL INTERNALLY GENERATED REVENUE SERIES REPRESENTED BY THREE VECTORS.

s/n	X_{1t}	X_{2t}	X_{3t}	s/n	X_{1t}	X_{2t}	X_{3t}	s/n	X_{1t}	X_{2t}	X_{3t}
1	30.87	17.01	13.86	41	186.82	139.41	47.41	81	164.91	145.21	19.70
2	31.26	17.31	13.95	42	169.89	137.98	31.91	82	215.65	139.52	76.13
3	29.35	16.10	13.25	43	176.91	147.73	29.18	83	167.03	151.33	15.70
4	30.05	18.68	11.37	44	256.21	238.38	17.83	84	219.36	160.19	59.17
5	25.96	17.46	8.50	45	260.00	169.12	90.88	85	176.06	129.01	47.05
6	30.31	20.55	9.76	46	434.75	308.15	126.60	86	251.51	70.66	180.85
7	31.54	17.04	14.50	47	258.23	207.11	51.12	87	325.11	207.01	118.10
8	45.20	23.85	21.35	48	169.79	143.58	26.21	88	257.86	192.54	65.32
9	41.07	20.57	20.50	49	358.15	328.97	29.18	89	195.03	162.92	32.11
10	45.46	24.86	20.60	50	397.26	383.01	14.25	90	220.52	165.52	55.00
11	48.68	29.65	19.03	51	279.01	152.71	126.30	91	225.77	107.42	118.35
12	40.17	28.67	11.50	52	220.75	157.39	63.36	92	167.89	120.52	47.37
13	45.79	29.76	16.03	53	178.99	149.68	29.31	93	198.30	112.85	85.45
14	32.76	22.89	9.87	54	164.50	105.69	58.81	94	257.08	115.70	141.38
15	30.77	23.25	7.52	55	192.33	138.53	53.80	95	183.01	110.86	72.15
16	32.07	21.97	10.10	56	198.54	100.29	98.25	96	106.12	76.75	29.37
17	37.83	19.64	18.19	57	143.54	86.21	57.33	97	207.17	156.60	50.57
18	43.85	22.60	21.25	58	155.90	124.20	31.70	98	209.36	179.21	30.15
19	30.77	12.60	18.17	59	198.51	120.68	77.83	99	309.66	191.79	117.87
20	37.06	14.53	22.53	60	260.93	175.79	85.14	100	394.27	258.99	135.28
21	31.96	10.61	21.35	61	299.44	270.84	28.60	101	388.93	232.97	155.96
22	29.00	10.30	18.70	62	211.02	185.08	25.94	102	250.32	198.14	52.18
23	30.36	15.04	15.32	63	188.06	158.68	29.38	103	328.70	289.35	39.15
24	36.63	16.90	19.73	64	252.71	247.66	5.05	104	475.41	285.73	189.68
25	45.77	30.45	15.32	65	185.72	160.47	25.25	105	396.98	241.31	155.67
26	50.00	31.50	18.50	66	101.75	77.25	24.50	106	461.13	317.68	143.45
27	72.50	55.20	17.30	67	145.56	118.56	27.00	107	331.10	138.69	192.41
28	77.18	51.73	25.45	68	184.41	156.59	27.83	108	363.17	263.85	99.32
29	104.08	67.58	36.50	69	184.41	156.59	27.82	109	248.50	202.20	46.30
30	120.70	80.90	39.80	70	149.33	73.20	76.13	110	339.98	224.38	115.60
31	157.34	111.47	45.87	71	153.39	138.19	15.70	111	377.75	245.45	132.30
32	220.45	164.79	55.66	72	171.38	115.46	55.92	112	300.42	244.67	55.75
33	198.76	132.35	66.41	73	180.48	96.33	84.15	113	366.28	303.08	63.20
34	171.03	156.70	14.33	74	170.13	135.03	35.10	114	441.37	270.02	171.35
35	231.97	205.76	26.21	75	184.16	145.96	38.20	115	246.69	151.69	95.00
36	343.58	321.12	22.46	76	124.36	81.71	42.65	116	416.48	327.73	88.75
37	143.73	132.88	10.85	77	222.96	194.45	28.51	117	320.97	213.59	107.35
38	126.16	91.21	34.95	78	175.75	151.25	24.50	118	347.35	272.14	75.21
39	107.93	74.75	33.18	79	614.93	587.93	27.00	119	422.91	291.66	131.25
40	162.04	139.41	22.63	80	142.32	114.50	27.82	120	641.23	485.56	155.67

APPENDIX 2: ESTIMATES FROM PURE AUTOREGRESSIVE MODELS FOR THREE VECTOR SERIES:

s/n	X_{1t}	X_{2t}	X_{3t}	s/n	X_{1t}	X_{2t}	X_{3t}	s/n	X_{1t}	X_{2t}	X_{3t}
1				41	135.41	100.67	34.74	81	269.42	231.69	37.72
2				42	163.52	118.38	45.14	82	286.12	206.97	79.15
3				43	168.57	127.99	40.58	83	195.43	133.77	61.66
4	30.66	21.26	9.40	44	176.95	132.79	44.16	84	165.23	132.86	32.37
5	30.24	21.30	8.95	45	206.86	158.71	48.15	85	214.90	151.54	63.37
6	27.62	20.14	7.49	46	252.33	177.47	74.86	86	182.65	136.88	45.77
7	29.67	20.92	8.75	47	349.79	241.75	108.04	87	259.47	153.42	106.06
8	31.16	21.15	10.01	48	287.61	229.07	58.54	88	255.43	178.25	77.18
9	39.77	25.88	13.90	49	262.45	199.43	63.02	89	263.28	199.64	63.65
10	40.09	27.50	12.59	50	279.89	207.71	72.18	90	240.94	184.27	56.67
11	45.15	30.68	14.47	51	314.15	251.46	62.69	91	230.16	165.76	64.40
12	46.06	32.15	13.91	52	358.35	252.78	105.57	92	233.45	154.73	78.73
13	42.57	31.68	10.89	53	274.97	203.54	71.43	93	184.41	139.84	44.57
14	46.61	32.54	14.07	54	210.89	160.96	49.94	94	209.48	141.48	68.00
15	37.15	27.88	9.27	55	191.96	135.84	56.11	95	234.43	149.34	85.09
16	35.21	25.88	9.34	56	181.66	130.41	51.25	96	194.77	144.18	50.59
17	32.72	23.33	9.39	57	200.24	134.10	66.13	97	160.81	121.95	38.85
18	36.74	24.34	12.49	58	165.27	120.84	44.42	98	179.89	124.87	55.02
19	40.48	26.96	13.52	59	160.03	118.55	41.49	99	174.14	135.05	39.10
20	36.33	25.14	11.19	60	183.95	124.62	59.32	100	276.56	184.94	91.62
21	39.42	25.15	14.27	61	219.00	153.04	65.96	101	327.53	228.05	99.48
22	33.76	22.08	11.68	62	247.27	193.04	54.23	102	377.20	263.93	113.27
23	32.48	21.36	11.13	63	245.53	191.86	53.67	103	305.82	288.01	67.81
24	30.50	20.58	9.91	64	225.80	169.58	56.22	104	321.89	240.31	81.57
25	34.66	22.49	12.16	65	216.38	169.28	47.10	105	410.69	272.64	138.05
26	38.64	27.21	11.43	66	206.06	159.98	46.07	106	395.66	284.88	110.78
27	46.34	32.40	13.94	67	166.17	125.74	40.43	107	450.54	319.82	130.72
28	59.17	42.48	16.68	68	146.51	105.96	40.56	108	401.27	275.23	126.04
29	70.52	50.10	20.42	69	150.39	113.10	37.29	109	366.92	266.92	100.00
30	91.61	62.99	28.62	70	172.49	131.38	41.11	110	289.97	225.07	64.91
31	105.82	74.79	31.04	71	181.03	123.99	57.05	111	340.98	233.90	107.08
32	135.12	95.66	39.46	72	148.47	115.63	32.83	112	338.55	238.32	100.24
33	178.78	127.80	50.97	73	170.15	119.48	50.68	113	315.52	243.53	71.99
34	196.26	141.85	54.41	74	179.64	121.49	58.15	114	353.80	261.18	92.62
35	180.35	141.99	38.36	75	163.43	124.10	39.32	115	409.46	279.26	130.20
36	208.02	155.85	52.17	76	180.26	133.05	47.20	116	317.75	239.87	77.88
37	261.89	201.86	60.04	77	154.54	114.07	40.53	117	380.48	270.83	109.65
38	215.10	176.60	38.50	78	183.24	135.02	48.21	118	330.07	242.94	87.13
39	198.50	141.97	56.53	79	171.05	134.19	36.85	119	354.65	261.22	93.40
40	123.23	89.78	33.46	80	386.16	288.07	98.09	120	389.14	273.86	115.28