PURE DIAGONAL BILINEAR MOVING AVERAGE VECTOR MODELS

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ABSTRACT

The purpose of this work was to develop special models from the general Bilinear Moving Average Vector (BMAV) Models. These models were established with only pure diagonal coefficients of the vectors of nonlinear components. The autocorrelation and partial autocorrelation functions for the vector series obtained suggested a pure moving average process for the vector models. A Pure Diagonal Bilinear Moving Average Vector (PDBMAV) Models were obtained. This is a special case of the general Bilinear Moving Average (BMAV) Models. This is so because of the associative products of vectors and their respective white noise forming the principal diagonal of the vector matrices. The estimates obtained validate PDBMAV models established. These are shown in figures ‘1’, ‘2’, and ‘3’.

KEY WORDS: Pure Diagonal Vectors, Bilinear Vector Models, Moving Average Vector Models, Bilinear Moving Average Vector Models.

INTRODUCTION

Most time series analysts assume linearity and stationarity, for technical convenience, when analyzing macroeconomic and financial time series data, (Franses, 1998). However, some of the microeconomic and financial data are not linear, due to its dynamic behaviour. Classical linear models are not appropriate for modelling such nonlinear series, (Subba Rao and Gabr, 1984). In most cases, nonlinear forecast is superior to linear forecast. Maravall (1983) used a bilinear model to forecast Spanish monetary data and reported a near 10% improvement in one-step ahead mean square forecast errors over several ARMA alternatives. There is no-gainsaying the fact that most of the economic or financial data assume fluctuations due to certain factors. That is why the use of nonlinear models in forecast gives higher precision than linear models.

Let \( e_i \) be a sequence of independently and identically distributed random variables defined on a probability space \((\Omega, B, P)\) with \( E(e_i) = 0 \) and \( E(e_i^2) = \sigma^2 < \infty\). The general superdiagonal bilinear model \( X_t \) with respect to \( e_t \) is

\[
X_t = \sum_{i=1}^{r} a_i X_{t-i} + \sum_{j=1}^{h} b_{ij} e_{t-j} + \sum_{i=1}^{m} \sum_{j=1}^{s} c_{ij} X_{t-i} e_{t-j}
\]

where \( a_i, b_{ij}, c_{ij} \) are fixed time independent parameters, (Akamanam, Bhaskara Rao and Subramanyam, 1986).

Oyet (1991) defined a process \((X_t)_{t \in \mathbb{Z}}\) on a probability space \((\Omega, \mathcal{F}, P)\) as a time varying bilinear process of order \((p,q,p,q)\) and denoted by \(BL(p,q;p,q)\), if it satisfies the following stochastic difference equation:

\[
P \quad q \quad P \quad Q
X_t = \sum_{i=1}^{P} a_i(t) X_{t-i} + \sum_{j=1}^{Q} c_{ij} e_{t-j} + \sum_{j=1}^{Q} \sum_{i=1}^{P} b_{ij} X_{t-i} e_{t-j} + \varepsilon_t
\]

where \((a_i(t))_{t \in \mathbb{Z}}\), \((c_{ij})_{t \in \mathbb{Z}, j \in \mathbb{Z}}\), \((b_{ij})_{t \in \mathbb{Z}, j \in \mathbb{Z}}\) are time-varying coefficients which depend on finite dimensional unknown parameter vectors \(a_i\), \(c_{ij}\) and \(b_{ij}\) respectively. The sequence \((\varepsilon_t)_{t \in \mathbb{Z}}\) is a heteroscedastic white noise process. That is, \((\varepsilon_t)_{t \in \mathbb{Z}}\) is a sequence of independent random variables, not necessarily identically distributed, with mean zero and variance \(\sigma^2\). Moreover \(\varepsilon_t\) is independent of past \(X_t\). The initial values \(X_t, t < 1\), and \(\varepsilon_t, t \leq 1\) are assumed to be equal to zero.

Boonchai and Eivind (2005) stated the general form of a multivariate bilinear time series models as

\[
X_t = \sum A X_{t-i} + \sum M e_{t-i} + \sum \sum B_{ij} X_{t-i} e_{t-j} + \varepsilon_t
\]

Here the state \(X_t\) and noise \(e_t\) are n-vectors and the coefficients \(A, M,\) and \(B_{ij}\) are all zero, and we have the class of well-known vector ARMA models. The bilinear models include additional product terms \(B_{ij} X_{t-i} e_{t-j}\) as the
theoretical point of view, it is therefore natural to consider bilinear models in the process of extending linear theory to non-linear cases. According to Boonchai and Eivind (2005), a particular reason for introducing bilinear time series in population dynamics is that they are suitable for modelling environmental noise. One may start with a deterministic system with (constant) parameters that describe conditions that depend on a fluctuating environment. Boonchai and Eivind (2005) made extension first to univariate and then to multivariate bilinear models. The main results give conditions for stationarity, ergodicity, invertibility, and consistency of least square estimates.

Usoro and Omekara (2007) established Bilinear Moving Average Vector (BMAV) Models. Their models involved interactive products of vectors and their white noise. In their work, the parameters of the non-linear parts were the coefficients of the interactive products. Their models were used for estimation and forecast of values of each vector.

In this paper, the interest is in the pure diagonal parameters forming the vector matrices. This calls for a special case of the General BMAV models. The parameters of the pure diagonal matrices are the coefficients of only the associative products of the vectors and their respective white noise.

2. **ESTIMATION OF PARAMETERS OF THE VECTOR MODEL**

A. **LINEAR MOVING AVERAGE VECTOR MODELS**

The general Vector Moving Average, VMA(q) process is given

\[
\begin{align*}
X_{1t} &= C_{1t} + \lambda_{11} C_{1t-1} + \lambda_{12} C_{2t} + \sum_{h=1}^{m} \lambda_{1h} \cdot C_{1t-h} \\
X_{2t} &= C_{2t} + \lambda_{21} C_{1t-1} + \lambda_{22} C_{2t-1} + \sum_{h=1}^{m} \lambda_{2h} \cdot C_{2t-h} \\
\vdots & \vdots \\
X_{vt} &= C_{vt} + \lambda_{vt} C_{vt-1} + \sum_{h=1}^{m} \lambda_{vh} \cdot C_{vt-h}
\end{align*}
\]

The expansion of the above matrices gives the following models,

\[
X_{1t} = C_{1t} + \sum_{h=1}^{q} \sum_{j=1}^{v} \sum_{n=1}^{m} \lambda_{hnj} \cdot C_{n} \\
X_{2t} = C_{2t} + \sum_{h=1}^{q} \sum_{j=1}^{v} \sum_{n=1}^{m} \lambda_{hnj} \cdot C_{n}
\]

(2.1)
\[ X_{3t} = C_{3t} + \sum_{b=1}^{q} \sum_{j=1}^{v} \sum_{h=1}^{m} \lambda_{b, j, h} e_{j-b} \] 

\[ X_{mt} = C_{mt} + \sum_{b=1}^{q} \sum_{j=1}^{v} \sum_{h=1}^{m} \lambda_{b, v, h} e_{j-b} \]

Therefore, models '2.1', '2.2', '2.3' and '2.4' express linear moving average relationships of \( X_{1t}, X_{2t}, X_{3t}, \ldots X_{nt} \) vectors with distributed lags of \( C_{1t}, C_{2t}, C_{3t}, \ldots C_{mt}, \lambda_{0, 1t}, \lambda_{0, 2t}, \lambda_{0, 3t}, \ldots, \lambda_{0, nt} \) are the matrices of coefficients of the moving average vector series. The above models can further be written as

\[ X_{yt} = C_{yt} + \sum_{b=1}^{q} \sum_{j=1}^{v} \sum_{h=1}^{m} \lambda_{b, v, h} e_{j-b} \]

where, \( X_{yt} \) is vector series, \( \lambda_{0, 1}, \lambda_{0, 2}, \lambda_{0, 3}, \ldots \lambda_{0, q} \) are the matrices of coefficients of the moving average vector series.

\section{B. NONLINEAR PURE DIAGONAL MOVING AVERAGE VECTOR MODELS}

The nonlinear vector models involving pure diagonal coefficients is given as,

\[
\begin{bmatrix}
X_{1t} \\
X_{2t} \\
X_{3t} \\
\vdots \\
X_{nt}
\end{bmatrix}
= \begin{bmatrix}
\beta_{01, 11} & 0 & 0 & \ldots & 0 \\
0 & \beta_{01, 22} & 0 & \ldots & 0 \\
0 & 0 & \beta_{01, 33} & \ldots & 0 \\
0 & \ldots & \ldots & \ldots & \ldots \\
0 & 0 & 0 & \ldots & \beta_{01, vq}
\end{bmatrix}
\begin{bmatrix}
C_{1t-1} \\
C_{2t-1} \\
C_{3t-1} \\
\vdots \\
C_{vt-1}
\end{bmatrix}
+ \begin{bmatrix}
(X_{1t-0}, X_{2t-0}, X_{3t-0}, \ldots, X_{nt-0})
\end{bmatrix}
\]

\[
\begin{bmatrix}
\beta_{02, 11} & 0 & 0 & \ldots & 0 \\
0 & \beta_{02, 22} & 0 & \ldots & 0 \\
0 & 0 & \beta_{02, 33} & \ldots & 0 \\
0 & \ldots & \ldots & \ldots & \ldots \\
0 & 0 & 0 & \ldots & \beta_{02, vq}
\end{bmatrix}
\begin{bmatrix}
C_{1t-2} \\
C_{2t-2} \\
C_{3t-2} \\
\vdots \\
C_{vt-2}
\end{bmatrix}
+ \begin{bmatrix}
(X_{1t-0}, X_{2t-0}, X_{3t-0}, \ldots, X_{nt-0})
\end{bmatrix}
\]

\[
\begin{bmatrix}
\beta_{03, 11} & 0 & 0 & \ldots & 0 \\
0 & \beta_{03, 22} & 0 & \ldots & 0 \\
0 & 0 & \beta_{03, 33} & \ldots & 0 \\
0 & \ldots & \ldots & \ldots & \ldots \\
0 & 0 & 0 & \ldots & \beta_{03, vq}
\end{bmatrix}
\begin{bmatrix}
C_{1t-3} \\
C_{2t-3} \\
C_{3t-3} \\
\vdots \\
C_{vt-3}
\end{bmatrix}
+ \begin{bmatrix}
(X_{1t-0}, X_{2t-0}, X_{3t-0}, \ldots, X_{nt-0})
\end{bmatrix}
\]
The above matrices remain principal diagonal for \( i=j \) and \( h=1 \). The model becomes,

\[
X_d = \sum_{i=j}^{n} \sum_{a=b}^{v} \sum_{p,q} \beta_{0a,0b} X_d e_{ij-b}
\]

\[X_{ij} = e_{ij} + \sum_{b=1}^{q} \sum_{h=1}^{m} \lambda_{h,0i} e_{ij-b} + \sum_{i=j}^{n} \sum_{b=1}^{v} \beta_{0b,i} X_d e_{ij-b}
\]

3 ESTIMATION OF PARAMETERS OF THE VECTOR MODELS

Expansion of the matrices provides the following models.

\[
X_1 = e_{11} + \lambda_{11} e_{11} + \lambda_{12} e_{21} + \lambda_{13} e_{31} + \lambda_{2,13} e_{11} + \lambda_{23} e_{21} + \lambda_{23} e_{31} + \beta_{01,11} X_{11} + \beta_{01,22} X_{21} + \beta_{02,33} X_{31}
\]

\[
X_2 = e_{21} + \lambda_{21} e_{11} + \lambda_{22} e_{21} + \lambda_{23} e_{31} + \lambda_{23} e_{31} + \lambda_{23} e_{31} + \beta_{01,11} X_{11} + \beta_{01,22} X_{21} + \beta_{02,33} X_{31}
\]

\[
X_3 = e_{31} + \lambda_{31} e_{11} + \lambda_{23} e_{31} + \lambda_{23} e_{31} + \lambda_{23} e_{31} + \lambda_{23} e_{31} + \beta_{01,11} X_{11} + \beta_{01,22} X_{21} + \beta_{02,33} X_{31}
\]

The above models are called Pure Bilinear Moving Average Vector Diagonal (PDBMAV) models.
According to Lewis and Stevens (1991), the multiple partitions and predictor vector interactions calls for multivariate adaptive regression method for the estimation of the above models. $X_{11}$, $X_{21}$ and $X_{31}$ are functions of linear vectors on one part and nonlinear interactions of vectors and their respective white noise on the other part. The least squares regression, using 'minitab' give the following estimated models for $X_{11}$, $X_{21}$ and $X_{31}$:

\[
X_{11} = \beta_{11} - 0.496\beta_{11,1} + 0.178\beta_{21,1} - 0.375\beta_{31,1} + 0.00227X_{11,0}\beta_{11,1} \\
- 0.00044b_{01,22}X_{21,0}\beta_{21,1} + 0.00111X_{31,0}\beta_{31,1} - 0.00208b_{02,33}X_{31,0}\beta_{31,2} \quad 3.4
\]

\[
X_{21} = \beta_{21} + 0.004\beta_{11,1} - 0.407\beta_{21,1} + 0.056\beta_{31,2} + 0.00070X_{11,0}\beta_{11,1} \\
+ 0.00091X_{21,0}\beta_{21,1} + 0.00178X_{31,0}\beta_{31,2} - 0.00174b_{02,33}X_{31,0}\beta_{31,2} \quad 3.5
\]

\[
X_{31} = \beta_{31} - 500\beta_{11,1} + 0.585\beta_{21,1} - 0.431\beta_{31,2} + 0.00157X_{11,0}\beta_{11,1} \\
- 0.00135X_{21,0}\beta_{21,1} - 0.00067b_{01,33}X_{31,0}\beta_{31,1} - 0.00034b_{02,33}X_{31,0}\beta_{31,2} \quad 3.6
\]

CONCLUSION

A Pure Diagonal Bilinear Moving Average (PDBMAV) model obtained for each vector series made the model a special case of Bilinear Moving Average Vector (BMAV) models. This is because the coefficients of the nonlinear part were restricted to only the principal diagonal of the coefficient matrices. This has significantly reduced the number of multiple coefficients in the General BMAV models to smaller number of coefficients, whose estimates are slightly better than that of the BMAV models. Appendices '1' and '2' are the original and estimated values of the vector series. The values in appendix '2' show slight deviations from actual values in appendix '1'. This further proves fitness of the models established.

**PLOTS OF ACTUAL AND ESTIMATES OF X1t**

![Plot of Actual and Estimates of X1t](image)

**FIGURE 1: Plots of Actual and Estimates of X1t**
PLOTS OF ACTUAL AND ESTIMATES OF X2t

ACTUAL PLOTS WITH BLACK DOTS
ESTIMATES WITH BLACK PLUS

FIGURE 2: Plots Actual and Estimates of X2t

PLOTS OF ACTUAL AND ESTIMATES OF X3t

ACTUAL PLOTS WITH BLACK DOTS
ESTIMATES WITH BLACK PLUS

FIGURE 3: Plots of Actual and Estimates of X3t
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