

# LP MODEL FOR PERIODIC RECRUITMENT AND RETRENCHMENT OF MANPOWER IN A NEW ORGANIZATION.

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## ABSTRACT

In this paper we have examined the manpower planning problem in which the maximum total number of employees to be recruited when the organization is in full operation is known. The manpower system has no initial employees at period  $n=0$ . The system also allows a periodic recruitment and retrenchment for a finite time interval. In addition to the usual constraints in linear programming (LP) formulation, we included the periodic overstaffing employees in the LP formulation, which by a theorem reduced the LP problem to a sparser LP problem. The resulting sparse LP problem has the advantage of less computational time when solving either manually or by computer.

**KEYWORDS:** Manpower, Recruitment, Employees, Overstaffing, Disengagement.

## INTRODUCTION

Manpower (human resource) are people in their various roles as contributors to the production of goods and services, (Urhoma 2009). Thomason (1988) defines manpower planning as a process whereby course of actions are determined in advance and continually updated with the aim of ensuring that: (a) the organization's manpower demand to meet its projected needs is as accurately predicted as the adoption of modern forecasting techniques allow and (b) the supply of labour (manpower) to the organization is maintained by deliberate and systematic action to mobilize it in reasonable balance with these demands.

Manpower planning which is also known as Human Resource planning consists of putting right number of people into the right kind of place at the right time, doing the right things for which they are suited for the achievement of the organizations goals, Cole (2005). Manpower planning involves analyzing the current manpower inventory, making future manpower forecasts, developing employment programmes and designing training programme for employees. According to Cole (2005) manpower planning involves; (a) the recruitments of sufficient and suitable staff for an organization. (b) the retention in the organization (c) the optimum utilization of staff and (d) the improvement of staff performance through training and retrenchment or disengagement of staff as necessary. Cole (2005) identified three major categories of staff that are important in manpower planning: (i) existing staff, (ii) new recruits and (iii) leavers. Each of these categories requires different decision to be made by the managers concerned, and these are stated below:

Existing staff: performance appraisal, productivity, development, equal opportunities, training, remuneration etc.

New recruits: recruitment methods, selection procedures, terms of contract.

Leavers: Dismissal for poor performance, retirement, etc.

Bowey (1974), remarked that manpower planning is not a clearly defined practice; rather it is a statistical technique in which rates of wastage are incorporated into a computerized model, and predictions made about required rates of recruitment. According to Armstrong (2004), the aim of manpower planning is to anticipate the problems of potential surplus (overstaffing) or deficits of people (understaffing) in order to plan a recruitment schedule for the efficient operation of the organization.

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Many research works have been carried out by different researchers on manpower planning problem. The approach has always been either by using goal programming, dynamic programming or Markov chain, Ogumeyo and Ekoko (2008). There has been no specific reference to linear programming approach with respect to manpower planning models. For example in Taha (2002), Mehlmann (1980), Ogumeyo and Ekoko (2008) manpower planning models were based on dynamic programming approach.

On the other hand, Sterman (2000), Aidman et al (2002) and Galanis (2002) worked on civil and military manpower planning models which are based on Markov chain models. Raghavendra (1991) and Ekoko (2006) discussed manpower planning models with respect to recruitment and promotion factors using Markov chain while Price and Piskor (1972) developed military manpower planning model, Giannikos and Darzi, (1995) treated a model for office allocation for members of staff in a higher institution using goal programming techniques.

This paper examines a manpower planning model in which linear programming is used to determine the level of periodic recruitment and retrenchment (disengagement) of staff in a new organization. The model is based on periodic hiring and firing (recruitment and retrenchment or disengagement) due to seasonal fluctuations. Nwachukwu (2007) remarked that many organizations make use of analysis of labour turnover, labour stability and similar ratios to determine the extent to which new recruits are disengaged or retrenched with respect to measure of performance. The labour stability index which links the disengagement or retrenchment rate with length of service according to Nwachukwu (2007) is defined as:

$$\frac{\text{No of staff retrenched with more than one year service}}{\text{No of staff recruited one year ago}}$$

Planning manpower levels requires that an assessment of present and future manpower needs be compared to present and future resources. The penalties for not being correctly staffed are costly. These penalties hinge on two factors: understaffing and overstaffing. Rao (1990) and Ogumeyo and Ekoko (2008) manpower planning model assumptions allowed only overstaffing while the manpower planning model in this paper allows either overstaffing or understaffing based on the previous performance and productivity during periodic recruitment.

According to Mehlmann (1980), one of the principal purpose of manpower planning is to analyze the influence of recruitment and transition behaviour on the size and structure of an organization. Mehlmann (1980) further remarked that the personnel flow in organizations having various grades of employees can be subdivided into recruitment stream, the transition between grades and the disengagement or retrenchment from the system. Furthermore the model assumes that there is no initial staff in the organization at period  $n = 0$ . The inclusion of these assumption resulted in a more sparse linear programming problem.

Thus, the manpower problem presented in this paper is based on surplus (overstaffing) if the previous period has shortage of manpower and deficit (understaffing) if the previous period has excess manpower. Organization's executives are often faced with the problem of how many employees are to be recruited and how many are to be fired in such a way that the cost of operating the manpower planning system is minimized as a result of seasonal fluctuation, Hillier and Lieberman (2001). This problem is formulated as a linear programming problem in the next section.

## 2. LP Model Formulation.

The following are the assumptions of the model.

- (a) We can recruit any number of employees at any period subject to vacant positions in the manpower system.
- (b) During each finite time interval the recruitment cost and the retrenchment or disengagement costs are constant and may be different from those of other periods.
- (c) Either understaffing or overstaffing is allowed at each period subject to the result from previous period.
- (d) The manpower planning system is to be examined for a given length of time up to  $n$  finite time periods.

**Model Notations**

Let

$e_n =$  the number of staff that are employed in period  $n$  at a recruitment cost of  $c_n$ .

$d_n =$  the number of staff that are retrenched in period  $n$  at a retrenchment cost of  $c'_n$

$H =$  the total number of staff in cadre when an organization is working in full capacity.

$h =$  initial number of staff in the organization at period  $n = 0$ .

The problem of the manpower planning is to minimize the sum.

$$\text{Minimize } z = c_n e_n - c'_n d_n \tag{1}$$

There are two staffing constraints (2n) and two non-negativity constraints (2n) in this problem:

(a) The overstaffing constraints  $(e_n - d_n) > 0$  which states that the total overstaffing for the first  $i$  periods should not exceed  $H - h$  i.e.

$$\sum_{j=1}^i (e_j - d_j) = \sum_{j=1}^i e_j - \sum_{j=1}^i d_j \leq H - h \tag{2}$$

The LHS of equation (2) can also be called the net increase in manpower in the first periods.

(b) The understaffing constraints  $(d_j - e_j)$ : it states that the number of staff by which the organization is understaffed for the first  $(i - 1)$  periods + understaffing at period  $i$  should not exceed  $h$ , which is the number of staff originally in the organization at period  $n = 0$ . If it does, it means the organization has only material resources which is not real as existence of an organization is based on the contribution of human and material resources.

Mathematically, this is expressed as:

$$\sum_{j=1}^{i-1} (d_j - e_j) + d_i = - \sum_{j=1}^{i-1} e_j + \sum_{j=1}^i d_j \leq h, \quad i = 1(1)N \tag{3}$$

The left hand side of equation (3) can also be called the net increase in manpower subtracted from those retrenched in the first  $(i - 1)$  periods + the retrenched manpower in period  $i$ .

$$(c) \quad e_j, d_j \geq 0, \quad j = 1(1)N \tag{4}$$

Note that the first summation in (3) does not exist for  $i = 1$ . Moreover fractional values are approximated to be integers. This implies that no part staff is allowed.

Equation (1) stated above constitutes the total manpower planning cost from all the  $n$  periods while equation (1) - (4) constitute a LP problem which is stated thus:



$$\begin{array}{llll}
 \text{Min } z = c_1e_1 + c_2e_2 + \dots + c_n e_n - c'_1d_1 - c'_2d_2 - \dots - c'_n d_n & 7(a) & & \\
 \text{s.t.} & & & \\
 e_1 & -d_1 & \leq 0 & 7(b_1) \\
 e_1 + e_2 & -d_1 - d_2 & \leq 0 & 7(b_2) \\
 e_1 + e_2 + e_3 & -d_1 - d_2 - d_3 & \leq 0 & 7(b_3) \\
 \dots & & & \\
 e_1 + e_2 + \dots + e_n - d_1 - d_2 - \dots - d_n & \leq 0 & & 7(b_n) \\
 & + d_1 & \leq H & 7(d_1) \\
 -e_1 & + d_1 + d_2 & \leq H & 7(d_2) \\
 -e_1 - e_2 & + d_1 + d_2 + d_3 & \leq H & 7(d_3) \\
 \dots & & & \\
 -e_1 - e_2 - \dots - e_{n-1} + d_1 + d_2 + d_3 + \dots + d_n & \leq H & & 7(d_n) \\
 e_1 & -d_1 & \geq 0 & 7(f_1) \\
 & e_2 & -d_2 & \geq 0 & 7(f_2) \\
 & & e_3 & -d_3 & \geq 0 & 7(f_3) \\
 \dots & & & & & \\
 & & & e_n & -d_n & \geq 0 & 7(f_n) \\
 e_1, e_2, \dots, e_n, d_1, d_2, \dots, d_n & \geq 0 & & & & & 7(g)
 \end{array} \quad (7)$$

Equation 7a is called the objective function, which is the measure of effectiveness. Equation 7(b<sub>1</sub>) – 7(b<sub>n</sub>) constitute the set of linear constraints while equation 7(g) is the set of nonnegativity constraints. Specifically equations 7(f<sub>1</sub>) - 7(f<sub>n</sub>) constitute the n periodic nonnegative excess recruitment. The manpower problem in LP form which is expressed in 7(a) to 7(g) is compared to the former LP in (6) as follows: The objective functions and nonnegativity constraints are the same in both LP problems. The initial manpower quantity *h* is specified to be zero in the latter LP problem. By the addition of *n* extra non-negativity excess employees constraints to the later problem, the later LP problem has a total of 3n linear constraints (as against 2n in the former) in 2n variables. Hence we state and prove a proposition which is based on the later LP problem in 7(a) – 7(g).

Proposition: Given that the manpower system has no initial employees (*i.e.*  $h = 0$ ) and that the periodic excess employees is nonnegative ( $e_j - d_j \geq 0$ ), then

- (i)  $e_1 = d_1$
- (ii)  $0 \leq d_1 \leq H$  and  $0 \leq e_1 \leq H$  and
- (iii) The LP problem in 7(a) – 7(g) can be reduced to have only n-variables,  $d_i$  which are the number of disengaged or retrenched employees in period *i*.

**Proof:** The proof of the proposition is as follows:

From equations 7(b<sub>1</sub>) to 7(f<sub>1</sub>) we have

$$e_1 - d_1 = 0 \quad 8(a_1)$$

Substituting equation 8(a<sub>1</sub>) into equation 7(b<sub>2</sub>) and considering equation 7(f<sub>2</sub>), we have

$$e_2 - d_2 = 0 \quad 8(a_2)$$

i.e.  $e_2 = d_2$

Similarly,

$$e_n - d_n = 0 \quad 8(a_n)$$

i.e.  $e_n = d_n$

From equation 7(d<sub>1</sub>)

$$d_1 \leq H \quad 9(a_1)$$

By substituting equation 8(a<sub>1</sub>) into equation 7(d<sub>2</sub>), we have

$$d_2 \leq H \quad 9(a_2)$$

Similarly, substituting 8(a<sub>1</sub>) and 8(a<sub>2</sub>) into 7(d<sub>1</sub>), we have

$$d_3 \leq H \quad 9(a_3)$$

Similarly by substituting equations 8(a<sub>1</sub>) – 8(a<sub>n</sub>) in equation 7(d<sub>n</sub>), we have

$$d_n \leq H \quad 9(a_n)$$

Since  $e_i = d_i$  for every  $i = 1, 2, \dots, n$ , the objective function can be expressed only in terms of the  $d_i$  variables and the LP problem in 7(a) – 7(g) is now reduced to:

$$\left. \begin{array}{l} \text{Min } z = (c_1 - c'_1)e_1 + (c_2 - c'_2)e_2 + \dots + (c_n - c'_n)e_n \\ \text{s.t.} \\ \quad d_1 \leq H \\ \quad d_2 \leq H \\ \quad \vdots \\ \quad d_n \leq H \\ \quad d_1, d_2, d_3, \dots, d_n \geq 0 \end{array} \right\} \quad (10)$$

This completes the required proof.

## CONCLUSION

The model presented in this paper focused on a manpower planning system in which periodic recruitment and retrenchment are carried out in a newly established organization. The model consists of two constraints which are overstaffing and understaffing constraints. The manpower planning model has been formulated as a linear programming problem with the inclusion of periodic overstaffing to maintain adequate supply of labour in a reasonable balance with organizations' manpower requirement. While all the other models mentioned so far in this paper seek to address manpower planning problem using either dynamic programming, goal programming or Markov chain model approach, the model in this paper used linear programming (LP) approach. We have also stated and proved a theorem which reduced the formulated LP problem to a more sparse LP problem which has the advantage of less computational time to solve.

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