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A QUADRATIC PROGRAMMING MODEL FOR CROP COMBINATION IN INTERCROPPING

I. A. ETUKUDO, and M. U. UMOREN

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ABSTRACT

One particularly known problem that confronts the farmer is that of determining optimal crop combinations for an intercropping scheme. This work therefore sets out to develop a model which determines optimally a crop combination that will yield maximum profit when interactive effects are present. A quadratic programming model is developed herein for selecting such an optimal crop combination for the intercropping scheme. A numerical application of the model shows that the model is adequate for such purposes.

Keywords: Quadratic Programming, Intercropping, Optimal Selection

INTRODUCTION

As a result of the rapidly growing need for increased food production arising form the high population growth rate, both small and large scale farmers are faced with the problem of optimal intercropping scheme that will yield maximum profit. Intercropping scheme is defined as a delibrate practice of cultivating two or more crops simultaneously on the same field (Onwueme and Sinha, 1991, Igbozurike, 1977).

Various researches including Meed and Riley (1981), and Wanki, Fawusi and Nanju (1982) have shown that intercropping is more popular in the economically under developed nations and occupies about ninety percent of cropped area in most countries, particularly in the Tropical Rain Forest and Semi Arid Tropics (Wanki, Fawusi and Nanju, 1982). Onwueme and Sinha (1991) have observed that there is a yield advantage in growing crops together rather than growing each one separately because of the fact that crops complement one another in their use of field time. In addition, they further stated that the spread of disease and pests is considerably less rapid in intercropping than in sole cropping.

In order to solve the problem of this optimal intercropping scheme faced by farmers, a quadratic programming model is developed herein for selecting an optimal crop combination for the intercropping scheme. A numerical example is given to illustrate the application of the model.

DEFINITION OF VARIABLES

In order to develop an appropriate model for the intercropping scheme, we define variables that are involved in the model.

t = total number of crops available for intercropping scheme.

n = total number of crops taken from t to be combined in the intercropping scheme.

l_j = cost of preparation of land per hectare with respect to crop j, j =1, 2, ...n

s_j = cost of seeds/seedlings per hectare of crop j, j = 1,2, ... n

f_j = cost of fertilizer needed per hectare of crop j, j, = 1, 2 ... n

I. A. ETUKUDO, Dept. of Mathematics/Statistics and Computer Science, University of Calabar, Calabar, Nigeria.
M. U. UMOREN, Dept. of Mathematics, Statistics and Computer Science, University of Uyo, Uyo, Nigeria.

p_j = cost of planting per hectare with respect to crop j, j =1, 2, ...,n
m_j = cost of farm management from planting to harvesting with respect to crop j, j = 1, 2,..., n
v_j = cost of harvesting per hectare with respect to crop j, j = 1, 2, ...,n
g_j = cost of insurance cover per hectare of crop j, j = 1, 2,..,n
h_j = cost of post harvest handling per hectare of crop j, j =1, 2, ...,n

q_{jk} = the per –hectare interaction effect of crop j and crop k (j,k =1, 2, .., n)
c_j = expected profit per hectare if crop j (j =1, 2, .. , n) is used on the same piece of land for a mono cropping scheme.
A = number of hectares of farmland available for intercropping scheme.
L = total funds available for preparing farmland for intercropping scheme.
S= total funds available for purchase of seedling with respect to all crops in the intercropping scheme.
F = total fund available for procurement of fertilizer for all the crops in the intercropping scheme
P = total funds available for planting all the crops in the intercropping scheme
M = total funds available for management of the intercropping scheme from planting to harvest time.
V = total funds available for harvesting of all crops in the intercropping scheme.
G = the total amount of money available for obtaining insurance cover for all the crops in the intercropping scheme.
H = total funds available for post harvest handling of the intercropping scheme.
x_j = the decision variable representing the hectares of land allocated to crop j, j = 1, 2, ...,n
x_k= the decision variable representing the hectares of land allocated to crop k, k=1, 2, ...,n.

DEVELOPMENT OF THE QUADRATIC PROGRAMMING MODEL

Let the profit realizable by planting crop j on x_j hectares be $c_j x_j$. Then the total profit that will be realized by planting each one of a set of n crops singly on the farmland is $\sum_{j=1}^{n} c_j x_j$ 1

If x_j and x_k hectares are allocated to crops j and k respectively for intercropping on the farmland, the interaction effect in terms of profit is
$q_{jk} x_j x_k$. The total effect of interaction for intercropping n crops on the farmland is

$$\sum_{j=1}^{n} \sum_{k=1}^{n} q_{jk} x_j x_k \quad \text{.......................................2}$$

Therefore, the objective function for the intercropping scheme is

$$\text{Maximize } f(x) = \sum_{j=1}^{n} c_j x_j + \frac{1}{2} \sum_{j=1}^{n} \sum_{k=1}^{n} q_{jk} x_j x_k \quad \text{.......................................3.1}$$

The cost of preparing land for x_j hectares of land allocated to crop j is $l_j x_j$. Therefore, for the entire farmland, the cost of preparing land for a set of n crops for planting under the intercropping scheme is $\sum_{j=1}^{n} l_j x_j$. Now, since the total funds available for land preparation is L, the first constraint is

$$\sum_{j=1}^{n} l_j x_j \leq L \quad \text{.......................................3.2}$$

The second constraint is

$$\sum_{j=1}^{n} s_j x_j \leq S \quad \text{.......... 3.3}$$

since the cost of seeds procurement for x_j hectares of land allocated to crop j is $s_j x_j$ and the total cost of procuring seeds for a set of n crops for cultivation under the scheme is $\sum_{j=1}^{n} s_j x_j$, S is the total funds available for procuring seeds for intercropping.

The cost of fertilizer procurement for x_j hectares of land allocated to crop j is $f_j x_j$. Therefore, the total cost of procuring fertilizer with respect to all n crops is $\sum_{j=1}^{n} f_j x_j$. Since the total funds available for the procurement of fertilizer is F, the third constraint is

$$\sum_{j=1}^{n} f_j x_j \leq F \quad \text{.......... 3.4}$$

The planting cost of x_j hectares of land allocated to crop j is $p_j x_j$. Therefore, the total planting cost with respect to all n crops is $\sum_{j=1}^{n} p_j x_j$. Since the total funds allocated for planting is P, the fourth constraint is

$$\sum_{j=1}^{n} p_j x_j \leq P \quad \text{.......... 3.5}$$

Cost of managing x_j hectares of land allocated to crop j is $m_j x_j$. therefore, the total cost of farm management with respect to all n crops planted is $\sum_{j=1}^{n} m_j x_j$. Since the total funds available for management of the intercropping scheme is M, then the fifth constraint is

$$\sum_{j=1}^{n} m_j x_j \leq M \quad \text{.......... 3.6}$$

Let $v_j x_j$ be the cost of harvesting crop j from x_j hectares of land. The total cost of harvesting all the n crops selected for the intercropping scheme is $\sum_{j=1}^{n} v_j x_j$. Since the total funds available for this operation is V, then the sixth constraint is

$$\sum_{j=1}^{n} v_j x_j \leq V \quad \text{.......... 3.7}$$

Furthermore, let $h_j x_j$ be the cost of post harvest handling of crop j from x_j hectares of land. The total post-harvest handling cost of all n crops is $\sum_{j=1}^{n} h_j x_j$. Since the total funds available for post-harvest handling is H, then the seventh constraint is

$$\sum_{j=1}^{n} h_j x_j \leq H \quad \ldots\ldots 3.8$$

The cost of obtaining insurance cover for j crops from x_j hectares of land is $g_j x_j$. Therefore, the total cost of buying insurance for all n crops cultivated is $\sum_{j=1}^{n} g_j x_j$. Since only G naira can be spent on insurance, the eighth constraint is

$$\sum_{j=1}^{n} g_j x_j \leq G \quad \ldots\ldots 3.9$$

Let A hectares be the size of the farmland available for intercropping scheme. Since the number of hectares of land allocated to crop j is x_j which cannot be larger than A, then the ninth constraint is

$$x_j \leq A,\ j = 1,2,\ \ldots\ ,n \quad \ldots\ldots 3.10$$

Combining equations 3.1 to 3.10, the quadratic programming (QP) model for selecting optimal crop combinations for intercropping scheme is:

$$\text{Maximize } f(x) = \sum_{j=1}^{n} c_j x_j + \frac{1}{2} \sum_{j=1}^{n} \sum_{k=1}^{n} q_{jk} x_j x_k$$

subject to:

$$\sum_{j=1}^{n} l_j x_j \leq L$$

$$\sum_{j=1}^{n} s_j x_j \leq S$$

$$\sum_{j=1}^{n} f_j x_j \leq F$$

$$\sum_{j=1}^{n} p_j x_j \leq P$$

$$\sum_{j=1}^{n} m_j x_j \leq M$$

$$\sum_{j=1}^{n} v_j x_j \leq V$$

$$\sum_{j=1}^{n} h_j x_j \leq H$$

$$\sum_{j=1}^{n} g_j x_j \quad \leq \quad G$$

$$x_j \leq A, \; j = 1, 2, \ldots, n$$
$$x_j \geq 0, \; j = 1, 2, \ldots, n$$

APPLICATION OF THE MODEL

The solution procedure adopted in obtaining optimal solution is the modified simplex algorithm with restricted entry rule, Hillier and Lieberman (1974).

To use the model, we assume that the soil analysis of the farmland to be used for the intercropping scheme has been carried out with respect to all the crops to be considered for the scheme and that all the crops can thrive very well on the farmland based on the analysis. Our interest is to select the crop combination that optimizes the objective function by the application of the model.

Since the Hessian matrix (Etukudo, 2002) from the data obtained is not symmetric which negates the assumption that for the quadratic programming model to hold the Hessian matrix must be symmetrical, a mathematical logic is here invoked to force it to be symmetrical. That is, since $q_{jk} \neq q_{kj}$ an optimal coefficient matrix used in this model to obtain an optimizer is here determined by taking the average of each of the q_{jk} and q_{kj} (Etukudo, 2002).

OPTIMIZATION WITH RESPECT TO EVERY POSSIBLE GROUP OF CROPS.

Let n be a fixed integer such that $2 \leq n \leq t$. Then there are t different groups of crops, each one consisting of n crops that can be formed from the crops for the inter-cropping scheme. Since n crops can be selected from the t crops in ${}^tC_n$ (n =2, 3, 4, ..., t) ways for the scheme, then there are $\sum_{n=2}^{t} {}^tC_n$ possible groups of crops in all from which selection can be made. This approach requires us to solve $\sum_{n=2}^{t} {}^tC_n$ different quadratic programming problems from which an optimal combination can be selected. This involves feeding the parameters of each combination into the model and solving the resulting problems one by one to obtain the optimal solution. The combination of groups of crops with the maximum objective function value gives the optimal solution.

A NUMERICAL EXAMPLE

We shall here illustrate the application of the model with a numerical example. In this example, the number of crops considered for the intercropping scheme is restricted to four, namely maize, yam, pepper and okro denoted respectively as crops 1, 2, 3, and 4, assuming however, that soil analysis favours the four crops on that piece of land. The data for the numerical example is given in Tables 1- 5.

Since there are four crops involved in the intercropping scheme, we have eleven possible crop combinations and the data associated with the crops in each of the eleven groups are fed into our model one after the other and the resulting eleven QP problems solved (Etukudo, 2002). The optimal solutions of the eleven different quadratic programming problems are summarized on Table 6. From Table 6, it can be seen that an intercropping scheme consisting of crops 1, 2 and 3 yields the highest profit of N57, 900.00

Table 1: Cost of Each Farming Operation per Crop per Hectare

Cost (N'00)	Crops (j)			
	1	2	3	4
l_j	8	9	7	6
s_j	4	3	6	5
f_j	4	2	3	5
p_j	1	4	4	3
m_j	3	2	4	1
v_j	1	4	3	2
h_j	5	3	1	2
g_j	3	4	1	2

Table 2: The per Hectare Effects of one crop and the other (in monetary terms) when planted together.

Crop j	1	2	3	4
1	0	-1	-4	-3
2	1	0	-2	-1
3	-3	2	0	-4
4	3	2	-5	0

Table 3: Optimal Coefficient Symmetric matrix of per Hectare Effects of one Cropand the other

Crop j	1	2	3	4
1	0	0	-3.5	0
2	0	0	0	0.5
3	-3.5	0	0	-4.5
4	0	0.5	-4.5	0

Table 4: Value of Resource (monetary) Constraints

Resource	Value (N'000)
L	20
S	5
F	10
P	6
M	18
V	30
H	15
G	22

Table 5: Expected Profit per Crop per Hectare

Crop j	1	2	3	4
Profit (N'00)	60	70	60	40

A = 600 Hectares

Table 6: Optimal Solution to the QP Problems

Model	Crop Combination	Value of Decision variables	Objective Function Value	Number of Iterations
QP 1	1,2	$x_1 = 0, x_2 = 0$	0	0
QP 2	1,3	$x_1 = 350, x_3 = 600$	N18,716.67	4
QP 3	1,4	$x_1 = 0, x_4 = 0$	0	0
QP 4	2,3	$x_2 = 0, x_3 = 0$	0	0
QP 5	2,4	$x_2 = 0, x_4 = 0$	0	0
QP 6	3,4	$x_3 = 333.33, x_4 = 600$	N11,100.00	4
QP 7	1,2,3	$x_1 = 0, x_2 = 466.67$ $x_3 = 600$	N57,900.00	5
QP 8	1,2,4	$x_1 = 0, x_2 = 0$ $x_4 = 0$	0	0
QP 9	1,3,4	$x_1 = 500, x_4 = 0$ $x_4 = 600$	N35,550.00	5
QP 10	2,3,4	$x_2 = 600, x_3 = 33.33$ $x_4 = 600$	N9,120.00	6
QP 11	1,2,3,4	$x_1 = 600, x_2 = 0$ $x_3 = 0, x_4 = 520$	N53,820.00	7

followed by the scheme consisting of crops 1, 2, 3, and 4 with the profit of N53, 820.00. Therefore, to have a maximum profit from his intercropping scheme, the farmer should select the crop combination consisting of crops 1, 2, and 3 and cultivate zero hectares of maize, 466. 67 hectares of yam and 600 hectares of pepper on the same farmland.

DISCUSSION AND CONCLUSION

A model has been developed for optimal selection of crops in intercropping scheme for the agricultural sector. The model developed herein prevents sub-optimality obtained by farmers which is caused by intercropping crops based on tradition or custom.

Since the application of the model encourages detailed planning of all activities involved in farming operations, thorough estimation and analyses of costs, risks, resources and profit, its adoption will eliminate waste and minimize incidents of failures in crop farming which will in turn enhance success in farming business.

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