GLOBAL JOURNAL OF MATHEMATICAL SCIENNCES VOL. 5, NO. 2, 2006: 161-166

DUAL LP MODEL SOLUTION OF AN INVENTORY PROBLEM USING DYNAMIC PROGRAMMING TECHNIQUE.

P. O. EKOKO

(Received 8 September, 2006; Revision Accepted 20 October, 2006)

ABSTRACT

In this paper an inventory problem that seeks to determine the quantity of an item to be ordered into a warehouse and quantity to be sold (in each period) that will maximize total profit in *N* periods has been identified. The dual linear programming (LP) model of the inventory problem is presented and subsequently modified to an alternative method of solution to the inventory problem by dynamic programming approach. The results of a numerical example obtained by dynamic programming technique are exactly the optimal solution of the inventory problem when the primal LP is solved. This alternative method is a short cut procedure, which has the advantage of being computationally less cumbersome when compared to the primal LP approach.

KEYWORDS: Inventory problem, dynamic programming, linear programming.

1.0 INTRODUCTION

Over the years dynamic programming has been applied to solve many real life problems which before now could not be considered for the applicability of the technique. For example, Clarke (1988) applied dynamic programming to shed some light on possible batting tactics for optimal scoring rates in the game of cricket while Wood (1978) formulated and solved the problem of loom box sequence planning in wool textile industry using dynamic programming approach. The number of applications of dynamic programming has also recently increased in areas that are more susceptible to the technique. Mehlmann (1980) stated that in the past two decades a body of literature on dynamic programming has developed to focus on manpower planning. And in applying dynamic programming to control British Local Government finance, Smith (1989) expressed further that more research work is being carried out in the area of dynamic programming.

Though many applications of dynamic programming abound in the literature, Smith (1989) observed that many authors on dynamic programming still complain about the lack of enough practical applications of the technique. The increasingly powerful computing facilities now available mean that the solution of many hitherto intractable problems is becoming a reality. However, there remain a problem in encouraging students and practitioners to adopt a dynamic programming approach in solving relevant practical problems. This paper takes the reader on a girded tour of an example of inventory problem used to illustrate an alternative solution method for solving such problems. It is hoped that it will encourage more applications of the dynamic programming technique.

A major problem in inventory theory is how to strike a balance between having too many quantities of an item on hand in a warehouse and running out of stock, Agbadudu (1996) and Lucey (1996). The study of inventory enables us to formulate an optimal inventory policy which specifies: the optimal ordering or manufacturing quantity, the lead time and the minimum total inventory cost, Ekoko (1999). For most companies the expenses associated with financing and maintaining inventories are a substantial part of the cost of doing business. A lot of research work on inventory theory including Taha (2002), and Hillier and Lieberman (2001) have been centred on finding the Economic Order Quantity (EOQ) and minimum total inventory cost for inventory models. Our concern in this paper is the formulation of an inventory problem in dual linear programming (LP) model and subsequent solution using dynamic programming approach. The dynamic approach of the model in this paper which is formulated in section 2.0 seeks to maximize the total profit made in *N* periods when $x_j(t)$ quantity of an item is ordered in time t into a warehouse and $y_j(t)$ quantity is sold in time t of period j.

2.0 DUAL LP MODEL OF AN INVENTORY PROBLEM.

Let D be the capacity of a warehouse having d items initially. Let x_j be the quantity ordered at the beginning of period j which are delivered at the end of the same period at a unit cost of c_j. Let y_j be the quantity sold at period j at a unit selling price of s_j. The total profit in this inventory problem of *N* periods is $\sum_{j=1}^{N}(s_j y_j - c_j x_j)$.

Therefore the objective function is

P. O. Ekoko, Department of Mathematics, University of Benin, Benin City, Edo State Nigeria

of the objective function (16), it will never pay to let R_1 or Q_1, deviate from their smallest possible values. Therefore, the minimization of $(D-d)\ R_1 + dQ_1$ is equivalent, in our case, to requiring that R_1 *and* Q_1 be as small as possible.

Suppose that we have determined the optimal solution of (17) and find that the inventory at the end of period 1 is d'. Let us now set up a new problem where everything is as before except that the initial inventory is d' rather than d and that we wish to optimize the operation over the remaining $(N-1)$ periods. By analogy with (17) the system for this problem is given by

$$\left.\begin{array}{l} R'_N \geq -c_N \\ R'_{k-1} \geq Q'_k - c_{k-1}\ ,\ k = 3(1)N \\ R_{k-1} \geq R'_k\ ,\ k - 3(1)N \\ Q'_k \geq R'_k + s_k\ ,\ k = 2(1)N \\ Q'_{k-1} \geq Q'_k\ ,\ k = 3(1)N \\ (D - d')R'_2 + d'Q'_2 = min \\ R'_k \geq 0,\ Q'_k \geq 0 \end{array}\right\} \qquad (20)$$

where the primes denote new values of R_k *and* Q_k. Repeating the previous argument, we see that R'_2 *and* Q'_2 must be as small as possible for the optimal solution of (20).

Continuing this procedure, we see that the optimal solution of (17) has the property that all R_k and all Q_k are as small as possible. This fact is the key to the computational procedure, and the solution can now be performed practically by inspection.

If R_N is to be as small as possible consistent with (13) i.e. $R_N \geq -c_N$ and nonnegativity requirement i.e. $R_N \geq 0$, then

$$R_N = max\,(-c_N, 0) \qquad (21)$$

Similarly, from (15) where Q_N is bounded from below by $R_N + s_N$ and nonnegativity requirement $Q_N \geq 0$,

$$Q_N = max(R_N + s_N, 0) \qquad (22)$$

if Q_N is to be as small as possible.

Considering (14), (11) and $R_{N-1} \geq 0$, we have

$$R_{N-1} = max\{Q_N - c_{N-1}, R_N, 0\} \qquad (23)$$

if R_{N-1} is to be as small as possible.

Also from (12), $Q_{N-1} \geq R_{N-1} + s_{N-1}$ *and* $Q_{N-1} \geq 0$, we have

$$Q_{N-1} = max\{R_{N-1} + s_{N-1}, Q_N, 0\} \qquad (24)$$

if Q_{N-1} is to be as small as possible. The variables R_{N-2} *and* Q_{N-2} are determined in a similar way, and so on.

Thus computing R_N *and* Q_N using (21) and (22) respectively, the following recurrence relations can be used to determine R_{k-1} *and* Q_{k-1}.

$$\left.\begin{array}{l} R_{k-1} = max(Q_k - c_{k-1}, R_k, 0) \\ Q_{k-1} = max[R_{k-1} + s_{k-1}, Q_k, 0] \end{array}\right\} k = 2(1)N \qquad \begin{array}{r}(25)\\(26)\end{array}$$

(15) and (16) are repeated each time until the process terminates at R_1 *and* Q_1. R_1 *and* Q_1 are then substituted into (16) to yield the minimum value of the objective function of the dual LP problem which by duality theorem is equal to the maximum value of the objective function of the primal LP problem.

Determination of the Original Primal Variables.

So far the optimal value of the objective function can be found using (16) (i.e. $(D-d)\ R_1 + dQ_1$) which is not yet in terms of x_j *and* y_j. If we had solved the original dual in (5) - (8) by the simplex method, then the optimal solution of the primal (in terms of x_j *and* y_j) world have been obtained using the optimal simplex multipliers of the dual. It is not possible doing so now since we modified the procedure to a dynamic programming approach for the dual

LP problem. But since from (21), (22), (25) and (26), R_k *and* Q_k are partial sums of c_k *and* s_k then it is possible to simplify the objective function (16) i.e. $(D-d)\ R_1 + dQ_1$ to the form:

$$-\alpha_1 c_1 - \alpha_2 c_2 - \cdots - \alpha_N c_N + \beta_1 s_1 + \beta_2 s_2 + \cdots + \beta_N s_N$$

Which can further be equated to (1), the original primal objective function. That is

$$\sum_{j=1}^{N}\left(s_j y_j - c_j x_j\right) = -\alpha_1 c_1 - \alpha_2 c_2 - \cdots - \alpha_N c_N + \beta_1 s_1 + \beta_2 s_2 + \cdots + \beta_N s_N \qquad (27)$$

If ties occur in evaluating (21), (22), (25) and (26), then multiple optimal solutions exist, because the ties mean that the R_k *and* Q_k can be differently expressed in terms of the partial sums over the c_k *and* s_k. These different expressions will lead to different optimal policies in terms of the x_j *and* y_j, but they will, of course, all be equally profitable.

4.0 NUMERICAL ILLUSTRATION OF THE DYNAMIC APPROACH.

Example

The unit cost prices and unit selling prices of a commodity for five time periods are tabulated as follows:

Period j	1	2	3	4	5
c_j (₦)	25	25	25	35	45
s_j (₦)	20	35	30	25	50

If initially there are 50 units of the commodity in a warehouse which has capacity for 200 units, determine by dynamic programming approach the quantities to be ordered and sold that will maximize the total profit from all the periods.

Solution

We have 5 periods in this example, $N = 5$ and we proceed as follows:

$R_5 = max\ (-c_5, 0) = 0$ $\qquad Q_5 = max\ (R_5 + s_5, 0) = s_5$

$R_4 = max\ (Q_5 - c_4, R_5, 0) = Q_5 - c_4$ $\qquad Q_4 = max\ (R_4 + s_4, Q_5, 0) = Q_5$

$R_3 = max\ (Q_4 - c_3, R_4, 0) = Q_4 - c_3$ $\qquad Q_3 = max\ (R_3 + s_3, Q_4, 0) = R_3 + s_3$

$R_2 = max\ (Q_3 - c_2, R_3, 0) = Q_3 - c_2$ $\qquad Q_2 = max\ (R_2 + s_2, Q_3, 0) = R_2 + s_2$

$R_1 = max\ (Q_2 - c_1, R_2, 0) = 40 = Q_2 - c_1$ $\qquad Q_1 = max\ (R_1 + s_1, Q_2, 0) = 65 = Q_2$

We need the values of R_1 *and* Q_1 because the objective function in (17) is in terms of R_1 *and* Q_1. To obtain for example R_1, we do continued substitution for Q_2 *and* R_2 in terms of only c_k *and* s_k as shown in (28). c_k *and* s_k are given in the question. $Q_2 - c_1 = 40,\ R_2 = 30. \quad \therefore R_1 = 40.$

$\therefore$ Maximum profit $w = (D-d)R_1 + dQ_1 = 150 \times 40 + 50 \times 65$

= ₦9,250.00

Expressing the R_k *and* Q_k in terms of the c_k *and* s_k, we obtain

$$\left.\begin{aligned}
R_5 &= 0 & Q_5 &= s_5\\
R_4 &= s_5 - c_4 & Q_4 &= s_5\\
R_3 &= s_5 - c_3 & Q_3 &= s_5 - c_3 + s_3\\
R_2 &= s_5 - c_3 + s_3 - c_2 & Q_2 &= s_5 - c_3 + s_3 - c_2 + s_2\\
R_1 &= s_5 - c_3 + s_3 - c_2 + s_1 - c_2 & Q_1 &= s_5 - c_3 + s_3 - c_2 + s_2
\end{aligned}\right\} \qquad (28)$$

$$\therefore \qquad (D-d)R_1 + dQ_1 = -(D-d)c_1 - Dc_2 - Dc_3 + Ds_2 + Ds_3 + Ds_5$$

$$\sum_{j=1}^{5}\left(s_j y_j - c_j x_j\right) = -(D-d)c_1 - Dc_2 - Dc_3 + Ds_2 + Ds_3 + Ds_5 \qquad (29)$$

By comparing coefficients of like terms in equation (29), x_j *and* y_j are obtained as follows:

$$\Rightarrow x_1 = D - d = 150 \qquad y_1 = 0$$
$$x_2 = D = 200 \qquad y_2 = D = 200$$
$$x_3 = D = 200 \qquad y_3 = D = 200$$
$$x_4 = 0 \qquad y_4 = 0$$
$$x_5 = 0 \qquad y_5 = D = 200$$

5.0 CONCLUSION

The type of inventory problem treated in this paper is different from the common type of inventory problems that seek to determine EOQ and minimum total inventory cost. We have found that the quantity of an item ordered into a warehouse and the quantity sold at each period which will maximize total profit from N periods can be obtained through dual LP model formulation and subsequent solution by dynamic programming. From the results of the numerical example presented in this paper, it can be explained that in the first three periods which have lower ordering cost, ordering should be done to full warehouse capacity, while nothing should be ordered in the last two periods because of high ordering cost of those periods. On sales, nothing should be sold during the first and fourth periods when unit selling prices are low, while every item in the warehouse should be sold during the second, third and fifth periods when the selling prices are high. Though these results of the numerical example obtained by dynamic programming are exactly the same as the optimal solution of the inventory problem when the primal LP model is solved, the dynamic programming approach is computationally less cumbersome.

ACKNOWLEDGEMENT

The author hereby acknowledge the comments of the reviewers which have led to this final piece of work.

REFERENCES

Agbadudu, A. B., 1996. Elementary Operations Research, Benin City: Uniben Press.

Clarke, S. R., 1988. Dynamic Programming in One-Day Cricket-Optimal Scoring Rates, J. Opl. Res. Soc., Vol. 39, No. 4, pp. 331-337.

Ekoko, P. O., 1999. Basic Operations Research, Benin City: United City Press.

Hillier, F. S. and Lieberman, G. J., 2001. Introduction to Operations Research, London: Holden-Day, Inc.

Lucey, T., 1996. Quantitative Techniques, London: ELBS.

Mehlmann, A., 1980. An Approach to Optimal Recruitment and Transition Strategies for Manpower Systems Using Dynamic Programming, J. Opl. Res. Soc, Vol. 31, pp. 1009-1015.

Smith, P., 1989. Dynamic Programming in Action, J. Opl. Res. Soc. Vol. 40, No. 9 pp. 779-787.

Taha, H. A., 2002. Operations Research, An Introduction, New York: Mcmillan Publishing Co., Inc.

Wood, D. M., 1978. A Dynamic Programming Solution to a Problem of Loom Box Sequence Planning, J. Opl. Res. Soc., Vol. 29, No. 12, pp. 1159-1165.