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DYNAMIC PROGRAMMING ALGORITHM FOR OPTIMUM MANPOWER RECRUITMENT POLICY

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ABSTRACT

The specific use of dynamic programming models for manpower planning has been an extension of the general manpower planning problem. One of such models has been the Rao's manpower recruitment model which is developed to determine minimum total recruitment cost based on the policy of recruiting at an earlier period to meet the requirements of future periods thereby incurring overstaffing cost. The new model developed in this paper differs from Rao's model in the sense that it advocates for backward recursive approach in solving the manpower planning problem using dynamic programming technique instead of the forward recursive method. The numerical results obtained by using our algorithm have lower minimum suboptimal costs for the intermediate stages compared to the results obtained using Rao's algorithm.

KEYWORDS: Dynamic Programming optimal manpower, recruitment policy.

1. INTRODUCTION

Dynamic programming (DP) is a mathematical technique in which a given problem is decomposed into a number of sub-problems called stages, Taha (2002). The objective in such problems is to find a combination of decisions that will optimize some appropriate measure of effectiveness, (Wagner 2001). The manpower planning models deal with how changes take place in a manpower planning system under various operations and policy constraints (Rao 1990). Several models have been developed for various constraints and operating policies under which the system operates. For example, the earlier work or Sterman (2000), Aidman et al (2002) and Galanis (2002), were centred on modeling civil and military manpower planning using Markov chains in the determination of optimal workforce size. Ekoko (2006), has applied Markov Chain models to manpower planning with consideration of promotion and recruitment factors without incorporating the cost elements.

According to Hillier and Lieberman (2001) dynamic programming is applicable to many types of practical problems in which a series of interrelated decisions are required. As contained in Taha (2002), hiring and firing are exercised to maintain a labourforce that meets the needs of the project at hand in any organization.

Mehlmann (1980) developed optimal recruitment and transition strategies for manpower systems using dynamic programming techniques and shown that these strategies are linear functions of the present state and of present and future goals. Edward (1983) reviewed the various manpower planning models which have been developed, with emphasis on their assumptions and applications and concluded that good presentation of results and ease of use are more important than theoretical sophistication. And in line with this suggested direction of research contained in Edward (1983), we formulate a dynamic programming model for the determination of optimal solution of manpower recruitment problem. According to Rao (1990), the relevant costs in a manpower system consist of the following: recruitment costs, overstaffing costs, understaffing costs, firing/retirement cost and retention cost.

2. THE MANPOWER RECRUITMENT PROBLEM AND MODELS

Manpower Recruitment Problem:

In an establishment, the required number of staff (R), the fixed recruitment cost (k) and the overstaffing cost (i) are said to be known for each period of n-period horizon. These known values are given in Table 1.

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Table 1.

Year/Period (n)	No of Staff required (n)	Fixed recruitment cost (k)	Overstaffing cost (i)
1	R_1	k_1	
2	R_2	K_2	i_2
3	R_3	K_3	i_3
.	.	.	.
.	.	.	.
N	R_n	k_n	i_n

Based on the known values in Table 1, the problem on hand is how to determine which periods should recruitment be carried out and how many staff should be recruited at such periods in such a way so as to minimize total recruitment costs.

Rao's Manpower Planning Model and Algorithm for the Manpower Recruitment Problem.

Rao (1990) developed a manpower planning model to solve the manpower recruitment policy problem of the type given above in this section. Rao (1990) model is based on the following assumptions:

(a) The recruitment size is known and fixed – this can be the case when the available facilities and needed manpower are constant or can be increased in a regular proportion to facilitate good estimation.
(b) Recruitment at a particular grade is considered.
(c) Recruitment and overstaffing costs are known and fixed
(d) Understaffing is not allowed.

Rao's Manpower Planning Dynamic Programming Algorithm

The dynamic programming algorithm involves solving the earlier stated manpower recruitment problem by starting from the first stage to the last stage. Thus, in a ten–year manpower planning–horizon problem, we are to start from stage 1 and end in stage 10.

The major steps adopted by this approach for computing the minimum recruitment cost at each stage of the manpower planning model are as follows:

For period n^*, $n^* = 1, 2, \ldots, T$

i. Consider the policies of recruiting at period n^{**}, $n^{**} = 1, 2, \ldots, n^*$ by this order.
ii. Determine the total cost of the n^* different policies of period n^* by adding the fixed recruitment costs and overstaffing costs associated with the recruitment at period n^{**} and the cost of acting optimally for periods 1 to $n^{**} - 1$, consider by themselves. The later cost has been determined previously in the computation for period $n = 1, 2, \ldots, n^* - 1$
iii. From these n^* alternatives, select the minimum cost policy for periods 1 to n^* considered independently.
iv. Proceed to period $n^* + 1$ (or stop if $n^* = T$)

The algorithm is based on forward recursive method of solving dynamic programming problems.

Our Dynamic Programming Model for Manpower Planning Problem

Our model also incorporates the assumptions (a) – (d) in Rao (1990) model but does not use the manpower Planning Horizon Theorem. In this section, a dynamic programming model for manpower planning is developed with the objective of minimizing total recruitment cost. The algorithm associated with the model is also presented and applied to a numerical example.

Let n be the number of stages or periods in which recruitment is planned for a given establishment. As explained earlier, the manpower recruitment requirements and fixed recruitment costs vary from period to period, the overstaffing cost per recruitee per period (which is denoted by) also varies from period to period. In each period several decisions denoted by d_1, d_2, d_3, ... are to be made and we seek to obtain the suboptimal decision which corresponds to the minimum cost for the period.

Let n denote the last stage. We define stage $(n - r)$ as the general stage when there are r more stages ahead of it. There are $(r + 1)$ decisions to be made in stage $(n - r)$ and they are denoted by

$d_{j,n-r} = \{d_{1,n-r}, d_{2,n-r}, \cdots, d_{r+1,n-r}\}$. Their corresponding costs are

$c_{j,n-r} = \{c_{1,n-r}, c_{2,n-r}, c_{3,n-r}, \cdots, c_{r+1,n-r}\}$. But the suboptimal decision of stage $(n-r)$ is $d_{j^*,n-r}$ which has its corresponding suboptimal cost to be

$$c_{j^*,n-r} = \min_{1 \le j \le r+1} \{c_{1,n-r}, c_{2,n-r}, \cdots, c_{r+1,n-r}\} \quad (1)$$

For $r = 0,1,2,\ldots,(n-1)$

Note that $d_{j,n-r}$ simply means the decision j to recruit manpower for the first j periods at period $(n-r)$ and its corresponding cost is $c_{j,n-r}$.

$$c_{j,n-r} = k_{n-r} + R_{n-r+1}(i_{n-r}) + R_{n-r+2}\,2(i_{n-r}) + \cdots + R_{n-r+j-1}(j-1)(i_{n-r}) + c_{j^*,n-r+j} \quad \ldots\ldots\ldots(2)$$

While implementing equation (2), it should be noted that

$$c_{j^*,p} = 0, \ \forall\ p > n \quad \ldots\ldots\ldots(3)$$

As in other dynamic programming models, we shall use the recursive approach by starting from the nth stage which is the last stage of the problem. At the nth stage the decision is denoted by

$$d_{j,n} = d_{1,n} = d_{1^*,n} \quad \ldots\ldots\ldots(4)$$

The corresponding cost is

$$c_{1^*,n} = k_n + c_{j^*,n+1} = k_n \quad \ldots\ldots\ldots(5)$$

Note that at the nth stage $r = 0$, hence $c_{j^*,n+1} = 0$ from equation (3)

Dynamic Programming Algorithm

Step 1: nth stage computation

Since this is the last stage, the optimal recruitment policy cost can be determined from equation (5) (Note that at the nth stage, $r = 0$)

$c_{j^*,n} = k_n + c_{j^*,n+1} = k_n$ and the corresponding optimal decision is $d_{j,n} = d_{1,n} = d_{1^*,n}$ from equation (4)

Step 2: $(n-r)th$ stage computations

For each stage $(n-r)$ computation, we apply equation (2) to determine $C_{j,n-r}$ for $j = 1,2,\ldots,(r+1)$.

From equation (1) the suboptimal cost is $c_{j^*,n-r} = \min_{1 \le j \le r+1}\{c_{1,n-r}, c_{2,n-r}, \cdots, c_{r+1,n-r}\}$ while the corresponding suboptimal decision is $d_{j^*,n-r}$.

Note: Stage $(n-r)$ has $(r+1)$ decisions to make and they are:

$$c_{1,n-r} = k_{n-r} + c_{j^*,n-r+1}$$

$$c_{2,n-r} = k_{n-r} + R_{n-r+1}\, i_{n-r} + c_{j^*,n-r+2}$$

$$c_{3,n-r} = k_{n-r} + R_{n-r+1}\, i_{n-r} + R_{n-r+2}\, 2(i_{n-r}) + c_{j^*,n-r+3}$$

$$c_{r,n-r} = k_{n-r} + R_{n-r+1}\, i_{n-r} + R_{n-r+2}\, 2(i_{n-r}) + \cdots + R_{n-1}(r-1)(i_{n-r}) + c_{j^*,n}$$

....

$$c_{r+1,n-r} = k_{n-r} + R_{n-r+1}\, i_{n-r} + (R_{n-r+2})2(i_{n-r}) + \cdots + R_{n-1}(r-1)(i_{n-r}) + R_n(r)(i_{n-r}) + c_{j^*,n+1}$$

Step 3: Determination of Overall Policy

By a recursive process, the corresponding suboptimal recruitment costs are used to obtain the overall optimal recruitment policy that will give the minimum total recruitment cost

3. NUMERICAL EXAMPLE

In an establishment, the required number of staff (R), the fixed recruitment cost (k) and the overstaffing cost (i) are said to be known for each period of n-period horizon. These known values are given in Table 2.

Table 2:

Year n	No. of Staff required R	Fixed Recruitment Cost k (in ₦100s)	Overstaffing cost i (in ₦100s)
1	74	718	13
2	35	707	11
3	47	688	14
4	62	716	15
5	20	698	14
6	90	741	16
7	51	685	13
8	30	706	10
9	43	679	11
10	35	714	15

Based on the known values in Table 2, we are to determine which periods should recruitment be carried out and how many staff should be recruited at such periods in such a way that the total recruitment costs is minimized.

Solution

We now apply our algorithm to the data for the above 10-year planning period of a manpower system. The numerical example has 10 stages or periods i.e. $n = 10$

At stage 10 when $r = 0$

$$c_{1,10} = k_{10} + c_{1,11} = 714 \ (where\ c_{1,11} = 0)$$

The suboptimal decision is $d_{1,10}$ which means recruit 35 for period 10.

Stage 9 when $r = 1$

$$c_{1,9} = k_9 + c_{1,10} = 679 + 714 = ₦1393$$

$$c_{2,9} = k_9 + R_{10} i_9 + c_{1,11} = 679 + 35 \times 11 = 678 + 385 = ₦1064$$

The suboptimal cost at stage 9 is ₦1064* and the corresponding suboptimal decision $d_{2,9}$ which means to recruit for the first two periods starting from period 9 i.e. periods 9 and 10.

Stage 8 when $r = 2$

$$c_{1,8} = k_8 + c_{2,9} = 706 + 1064 = ₦1770$$

$$c_{2,8} = k_8 + R_9 i_8 + c_{1,10} = 706 + 43 \times 10 + 714 = ₦18[illegible]$$

$$c_{3,8} = k_8 + R_9 i_8 + R_{10} 2(i_8) + c_{1,11} = [illegible]$$

suboptimal cost is $c_{1,8} = 1770$, suboptimal decision = $d_{1,8}$

Stage 7, when $r = 3$

$$c_{1,7} = k_7 + c_{1,8} = 685 + 1770 = ₦2455$$

$$c_{2,7} = k_7 + R_8 i_7 + c_{2,9} = 685 + 30 \times 13 + 1064 = ₦2139$$

$$c_{3,7} = k_7 + R_8 i_7 + R_9 2(i_7) + c_{1,10} = 685 + 30 \times 13 + 43 \times 2 \times 13 + 714 = ₦2907$$

$c_{4,7} = k_7 + R_8 i_7 + R_9 2(i_7) + R_{10} 3(i_7) + c_{1,11} = 685 + 390 + 1118 + 35 \times 3 \times 13 = ₦3558$ suboptimal cost for stage 7 is $c_{2,7} = 2139$ while the suboptimal decision is $d_{2,7}$.

Similarly the remaining stages have been computed for using the same procedure and results are tabulated in Table 3 and the discussion of the procedure follows in section 4

Table 3: Summary of Results using the New Algorithm for the Numerical Example.

YEAR n	10	9	8	7	6	5	4	3	2	1
Compt S/N	1	2	3	4	5	6	7	8	9	10
K	714	679	706	685	741	698	716	688	707	718
I	15	11	10	13	16	14	15	14	11	13
R	35	43	30	51	90	20	62	17	35	74
1	714*	1393	1770*	2455	2880*	3578*	4294	4584	5291	5838
2		1064*	1850	2139*	3327	4097	3896*	5134	5120*	5757*
3			1836	2907	3581	5156	5855	4996	6166	6291
4				3558	5295	5710	7781	8035	6128	8391
5					6821	7768	8875	10522	9347	8733
6						9504	11750	11916	11784	13842
7							14186	15178	13057	17451
8								17894	16018	19475
9									18384	23597
10										26978
Minimum cost	714	1064	1770	2139	2880	3578*	3896	4584	5120	5757
Optimum policy	$d_{1^*,10}$	$d_{2^*,9}$	$d_{1^*,8}$	$d_{2^*,7}$	$d_{1^*,6}$	$d_{1^*,5}$	$d_{2^*,4}$	$d_{1^*,3}$	$d_{2^*,2}$	$d_{2^*,1}$

Compt. S/N means computational serial number, which refers to the order in which the computations were carried out

4. DISCUSSION OF COMPUTATIONAL PROCEDURE

Using the new dynamic programming algorithm, by recursive process, we start the computation with nth stage (i.e. stage 10). The suboptimal policy for period 10 is to recruit 35 employees thereby incurring a fixed recruitment cost of 714. In period 9, two possibilities are to be evaluated: recruit in period 9 and use the suboptimal policy of period 10 which cost 1393 (679+714). Or recruit for periods 9 and 10 and use the fixed recruitment cost and the overstaffing cost of period 9. The policy cost is 1064 (679+385). Thus the suboptimal decision in period 9 is d_{2^*9} with a suboptimal total cost of 1064. (i.e. $C_{2^*,9}$=1064). In period 8 (i.e, n-r = 8), there are three alternatives to be evaluated. The cost of the three alternatives are $C_{1^*,8}$ = 1770, $C_{2,8}$ = 1850 and $C_{3,8}$ = 1836. The suboptimal cost is $C_{1^*,8}$ = 1770 and the suboptimal decision is $d_{1^*,8}$. In period 7 alone, the fixed recruitment cost is 685; also there are four possibilities or alternatives to be evaluated: Recruit in period 7 and again in period 8, which cost 2455 (685+1770) or use the overstaffing and recruitment costs of period 7 resulting in 2139, 2907 and 3558 as suboptimal costs of the remaining three alternatives. Thus the suboptimal cost is $C_{2^*,7}$ =2139 and the corresponding optimal decision is $d_{2^*,7}$. In a similar way the suboptimal costs for the remaining stages (i.e. for n=6, n=5, n=4, n=3 n=2 and n=1) are computed using the dynamic programming algorithm contained in this work. The results are summarized in Table 2. The suboptimal policies of the given numerical examples are summarized as follows:

a) In **period 1**, recruit for the next two years (i.e. R_1+ R_2=74 + 35 =109).
b) Recruit in **period 3** alone, R_3 =47 and use the suboptimal policy decision from period 1 and 2.
c) In **period 4,** we are to recruit for the next two years. That is, (R_4 + R_5 = 62+20) for periods 4 and 5 and use the suboptimal decision for period 1 and 3.
d) In **period 6,** recruit for period 6 alone, R_6 = 90 and use the suboptimal policy for period 1 to 5 .
e) In **period 7** recruit for the next two periods (i.e, for periods 7 and 8). R_7 + R_8 = 51 + 30 = 81) and use the suboptimal policy of period 1 to 6.
f) Recruit in **period 9** for the next two periods, i.e, recruit for periods 9 and 10. Hence R_9 + R_{10} = 43 + 35 = 78 and stop. This shows that recruitment should be carried out in periods 1,3,4,6,7, and 9 if the total recruitment cost is to be minimized. The minimum total recruitment cost is ₦ 575,700.

Table 4: Summary of Results using The New Algorithm for the numerical example

Period n	10	9	8	7	6	5	4	3	2	1
Compt S/N	1	2	3	4	5	6	7	8	9	10
K	714	679	706	685	741	698	716	688	707	718
R	35	43	30	51	90	20	62	47	35	74
I	15	11	10	13	16	14	15	14	11	13
Minimum costs (N100s)	**714**	**1064**	**1770**	**2139**	**2880**	**3578**	**3896**	**4584**	**5120**	**5757**

Table 5: Summary of result using Rao's Algorithm for numerical example

Year n	1	2	3	4	5	6	7	8	9	10
Compt S/N	1	2	3	4	5	6	7	8	9	10
K R I	718 74 13	707 35 11	688 47 14	716 62 15	698 20 14	741 90 16	685 51 13	706 30 10	679 43 11	714 35 15
	718*	1425 1173*	1861* 1943 2395	2577* 2729	3275 2877*	3618* 4535 5577	4303* 4434	5009 4693*	5372* 5439 5811	6006 5757*
Minimum Cost	718	1173	1861	2577	2877	3618	4303	4693	5372	5757

Compt. S/N means computational serial number

Note that by Rao's algorithm both the year n and the computational serial number are the same. This is because the implementation of the Rao's algorithm starts from year 1.

Table 5: Comparison of Suboptimal Costs from Rao and The New Algorithms for the Numerical Example

Compt. S/N	1	2	3	4	5	6	7	8	9	10
Suboptimal Costs using Rao's Algorithm (A) (xN100)	718	1173	1861	2577	2897	3618	4303	4693	5372	5757
Suboptimal Costs using The New Algorithm (B) (xN100)	714	1064	1770	2139	2880	3578	3896	4584	5120	5757

5. Graphical Representation of Results of Numerical Example from both Algorithms.

Compt. S/N	1	2	3	4	5	6	7	8	9	10
A ($\times 10^4$)	7.14	11.73	18.61	25.77	28.97	36.18	43.03	46.93	53.72	57.57
B (xN100)	7.18	10.64	17.70	21.39	28.80	35.78	38.96	45.84	51.20	57.57

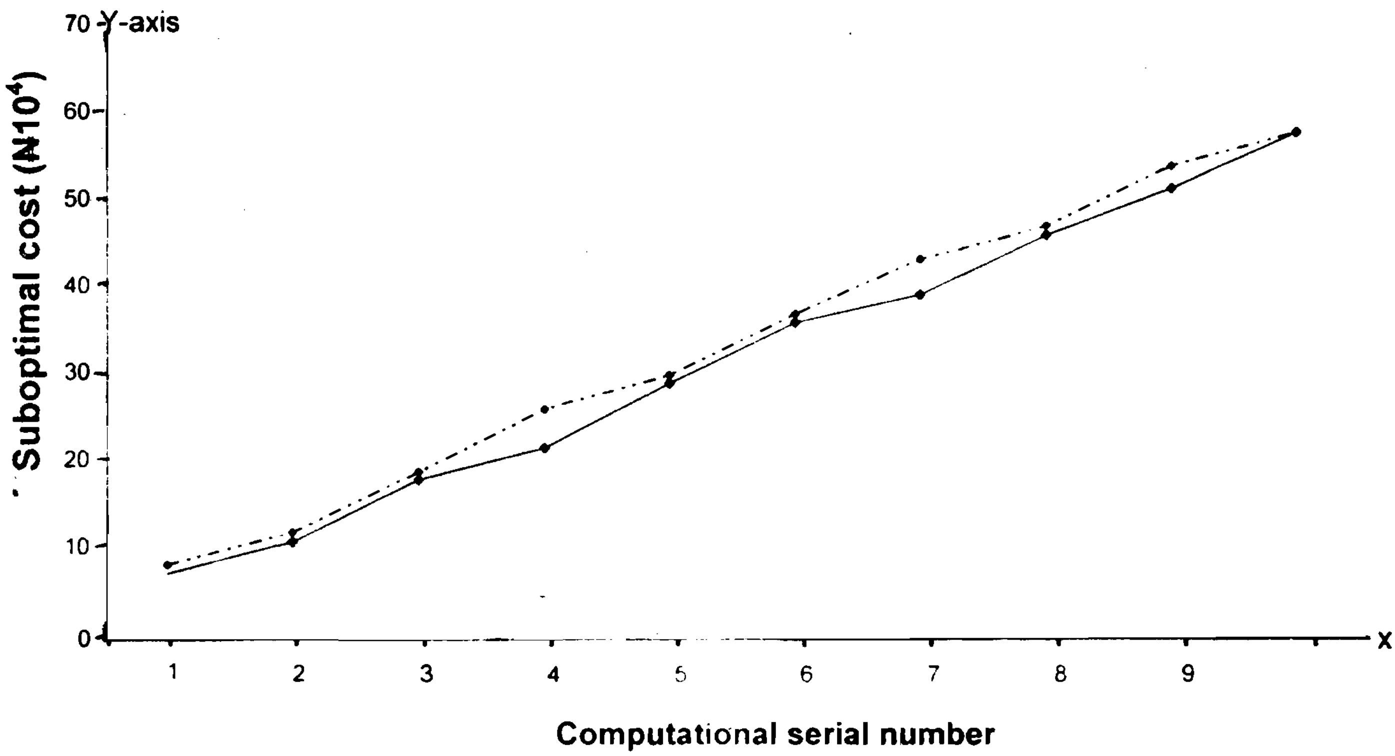

Fig 1: Graph of Suboptimal costs for Numerical Example

— · — · — · Results from Rao's Algorithm (A)

———— Results from the proposed Algorithm (B)

6. ANALYSIS OF RESULTS OBTAINED FROM THE RAO (1990) ALGORITHM AND THE NEW ALGORITHM

In comparing the results from the two numerical illustrations using the two algorithms, we observe that each of the results obtained from the proposed algorithm is lower than the corresponding ones obtained from Rao's algorithm. The minimum total recruitment costs obtained from the proposed algorithm and the Rao's algorithm are equal in each of the numerical example. The minimum total costs are ₦575,700. This shows that our algorithm is more efficient than Rao's algorithm.

Judging from the tables of results of numerical example presented in this paper, the suboptimal costs of all the periods or stages revealed that the corresponding suboptimal decisions are for batch recruitment for only one or two years. This agrees with some practical situations when an employee is recruited prior to when his services are actually needed. Instances when these situations occur include:

(1) During orientation of new employees
(2) When a staff is granted leave of absence
(3) Time of probation
(4) Study leave etc.

All these instances are not often given approval for too many years. For example, during orientation, a newly employed staff is given a training which prepares him for the job. During leave of absence, another staff could be employed to do his job thereby causing overstaffing to the establishment.

Another situation that can result in overstaffing is probation. Many newly employed staff are given a period of probation before their appointments are confirmed. Study-leave period is another identified situation which causes overstaffing cost to any establishment. Usually, in some establishments, employees are given study leave which ranges from three months to two years or or even more. In most cases, other workers are employed to fill their positions. The staff sent on Study Leave constitutes overstaffing cost because the staff on study leave will still be receiving his salary. The above mentioned instances of overstaffing attract overstaffing costs against any establishment. They are therefore, in reality, not often granted for too many years because of their high cost against the establishment, and this is what is reflected in the dynamic programming solution of manpower recruitment problem.

CONCLUSION

This research work is focused on the use of dynamic programming models and their algorithms for solving manpower planning problems. The specific use of dynamic programming models for manpower planning has been another extension of the general manpower planning problem. Two cost oriented methods have been adopted in this paper for solving the manpower planning problem using dynamic programming techniques. These are (a) The Forward Recursive method in Rao (1990), (b) The Backward Recursive method developed and incorporated in our algorithm for computing minimum total recruitment cost.

The backward recursive method was adopted in implementing our new algorithm for the manpower planning system without using the Manpower Planning-Horizon Theorem. In the formulation of the manpower planning model and algorithm the assumptions of fixed recruitment size and fixed overstaffing costs are taken into consideration. By our new algorithm, we can determine the manpower planning system. Our model and algorithms are found to be more computationally efficient since the suboptimal costs from our algorithm are lower than the corresponding results obtained from Rao's algorithm.

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