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FORECASTING IN ONE-DIMENSIONAL AND GENERALIZED INTEGRATED AUTOREGRESSIVE BILINEAR TIME SERIES MODELS

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ABSTRACT

In this paper, forecast of one-dimensional integrated autoregressive bilinear is compared with forecast of generalized integrated autoregressive bilinear model. We describe the method for estimation of these models and the forecast. It is also pointed out that for this class of non-linear time series models; it is possible to obtain optimal forecast. The estimation technique is illustrated with respect to a time series, and the optimal forecast of these time series are calculated. A comparison of these forecasts is made using the two models under study. The mean square error for forecast in generalized integrated autoregressive bilinear model is smaller than the mean square error for forecast in one-dimensional integrated autoregressive bilinear model. Though the two models are adequate for forecast when compared with the real series but forecast with generalized integrated autoregressive bilinear model is more adequate.

KEY WORDS: Optimal Forecast, Non-Linear Time Series Models, Bilinear Models, Estimation Technique, Mean Square Error.

INTRODUCTION

The bilinear time series models have attracted considerable attention during the last years. An overview of models and their application can be found in Subba Rao (1981), Pham and Tran (1981), Gabr and Subba Rao (1981), Rao et al. (1983), Liu (1992), Cathy (1997), Gonclaves et al. (2000), Shangodoyin and Ojo (2003), Wang and Wei (2004), Boonaick et al. (2005), Doukhan et al. (2006), Drost et al. (2007), Usoro and Omekara (2008) and Ojo(2009). The bilinear modes studied by the above researchers could not achieve stationary for all nonlinear series. Ojo (2011) proposed one-dimensional integrated autoregressive bilinear time series model that could achieve stationary for all non linear time series. Also, Ojo and Shangodoyin (2010) proposed generalized integrated autoregressive bilinear time series model that could achieve stationary for all non linear time series.

Forecasting connote an attempt to see into the future. There are two words, which are used to denote numerical forecasting methods namely forecasting, and prediction. Forecasting is the process of estimation in unknown situations. Prediction is a similar, but more general term, and usually refers to estimation of time series, cross-sectional or longitudinal data. Forecasting is commonly used in discussion of time-series data. Therefore forecasting is a powerful useful instrument in planning and making a wise decision about future. As a result of feature of stationary for all non linear series in one dimensional and generalized integrated bilinear model we shall attempt to study optimal forecast using these two models and see the one that perform better.

Generalized and One-Dimensional Integrated Autoregressive Bilinear Time Series Models

We define generalized integrated autoregressive bilinear time series model as follows:

$\psi(B)X_t = \phi(B)\nabla^d X_t + \sum_{k=1}^{r}\sum_{l=1}^{s} b_{kl} X_{t-k} e_{t-l} + e_t$, denoted as GBL (p, d, 0, r, s)

where $\phi(B) = 1 - \phi_1 B - \phi_2 B^2 .... - \phi_p B^p$ and

$$X_t = \psi_1 X_{t-1} + ... + \psi_{p+d} X_{t-p-d} + b_{11} X_{t-1} e_{t-1} + ... + b_{rs} X_{t-r} e_{t-s} + e_t \qquad (1)$$

$\phi_1, ..., \phi_p$ are the parameters of the autoregressive component; $b_{11}, ..., b_{rs}$ are the parameters of the non-linear component; and *d* is the degree of consecutive differencing required to achieve stationary.

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We define one-dimensional integrated autoregressive bilinear time series models as follows:

$$\psi(B)X_t = \phi(B)\nabla^d X_t + \left(\sum_{k=1}^{r} b_{k1} X_{t-k}\right) e_{t-1} + e_t$$, denoted as BL (p, d, 0, r, 1),

where $\phi(B) = 1 - \phi_1 B - \phi_2 B^2 .... - \phi_p B^p$ and

$$X_t = \psi_1 X_{t-1} + ... + \psi_{p+d} X_{t-p-d} + \left(\sum_{k=1}^{r} b_{k1} X_{t-k}\right) e_{t-1} + e_t . \quad (2)$$

$\phi_1, ...\phi_p$ are the parameters of the autoregressive component; $b_{11}, ..., b_{r1}$ are the parameters of the nonlinear component and *d* is the degree of consecutive differencing required to achieve stationary.

Model Estimation

The estimation of the models are similar, we shall report the estimation of generalized type since $m_i = 1,2,3...., s$ for the generalized case include $m_i = 1$ the one dimensional case. Suppose that X_t are generated by equation (1), the sequence of random deviates $\{e_t\}$ could be determined from the relation

$$e_t = X_t - \psi_1 X_{t-1} - .... - \psi_{p+d} X_{t-p-d} - \sum_{k=1}^{r}\sum_{l=1}^{s} b_{kl} X_{t-k} e_{t-l} \quad (3)$$

To estimate the unknown parameters in equation (3), we make the following assumptions:

(i) The errors $\{e_t\}$ are independent and identically distributed with mean zero and variance σ^2 with finite kurtosis.

(ii) The values of $|\psi' s| < 1$ and $|b_{kl}' s| < 1$ ensure that invertibility condition required of the bilinear process is satisfied. For details see (Ojo and Shangodoyin, 2010).

Thus maximizing the likelihood function is equivalent to minimizing the function $Q(G)$, which is as follows:

$$Q(G) = \sum_{i=m}^{n} e_t^2 , \quad (4)$$

with respect to the parameter $G' = (\psi_1, ..., \psi_p ; B_{11}, ..., B_{rs})$

Then the partial derivatives of Q(G) are given by

$$\frac{dQ(G)}{dG_i} = 2\sum_{t=m}^{n} e_t \frac{de_t}{dG_i} \qquad \text{(i = 1, 2,é ..,R)} \quad (5)$$

$$\frac{d^2Q(G)}{dG_i dG_j} = 2\left(\sum_{t=m}^{n} e_t \frac{de_t}{dG_i}\frac{de_t}{dG_j} + \sum_{t=m}^{n} e_t \frac{d^2 e_t}{dG_i dG_j}\right)$$

where these partial derivatives of e_t satisfy the recursive equations

$$\frac{de_t}{d\psi_i} + \sum_{j=1}^{s} W_t(t)\frac{de_{t-j}}{d\psi_i} = X_{t-i} \text{ if i = 1, 2, é , p} \quad (6)$$

$$\frac{de_t}{dB_{kmi}} + \sum_{j=1}^{s} W_j(t)\frac{de_{t-j}}{dB_{kmi}} = -X_{t-k} e_{t-m} \quad \text{(k=1,2,é ,r ; } m_i \text{ =1,2,é ,s)} \quad (7)$$

$$\frac{d^2 e_t}{d\psi_i d\psi_i^{'}} + \sum_{j=1}^{s} W_j(t) \frac{d^2 e_{t-j}}{d\psi_i d\psi_i^{'}} = 0 \text{ (i, i' = 0, 1, 2, é , p)} \quad (8)$$

$$\frac{d^2 e_t}{d\psi_i dB_{kmi}} + \sum_{j=1}^{s} W_j(t) \frac{d^2 e_{t-j}}{dB_{kmi} d\phi_i} + X_{t-k} \frac{d^2 e_{t-mi}}{d\psi_i} = 0$$

(i=0,1,2,é ,p ; k_i =1,2,é ,r; m_i=1,2,é ,s) (9)

$$\frac{d^2 e_t}{dB_{kmi} dB_{kmi}^{'}} + \sum_{j=1}^{s} W_j(t) \frac{d^2 e_{t-j}}{dB_{kmi} dB_{kmi}^{'}} + X_{t-k}^{'} \frac{d^2 e_{t-mi}}{dB_{kmi}} = -X_{t-k} \frac{de_{t-m}}{dB_{kmi}^{'}}$$

(k, k' =1,2,é ,r ; m_i m_i' = 1,2,é ,s) (10)

$W_j(t) = \sum_{j=1}^{s} B_{ij} X_{t-j}$. We assume e_t = 0 (t = 1, 2, é , m-1) and also

$$\frac{de_t}{dG_i} = 0, \frac{d^2 e_t}{dG_i dG_j} = 0, \text{ (i, j = 1, 2, é , R; t = 1, 2, é , m-1)}$$

From e_t=0 (t= 1, 2, é , m-1), $\frac{de_t}{dG_i} = 0, \frac{d^2 e_t}{dG_i dG_j} = 0,$, and $\frac{de_t}{dB_{kmi}} + \sum_{j=1}^{s} W_j(t) \frac{de_{t-j}}{dB_{kmi}} = -X_{t-k} e_{t-m}$ (k=1,2,é ,r ; m_i =1,2,é ,s), it follows that the second order derivatives with respect to ψ_i (i = 0, 1, 2, é , p) and θ_i (i = 0, 1, 2, é , q) are zero. For a given set of values $\{\psi_i\}$ and {B_{ij} } one can evaluate the first and second order derivatives using the recursive equations 6, 7 and 10.

Let $V(G) = \frac{dQ(G)}{dG_1}, \frac{dQ(G)}{dG_2}, \ldots\ldots, \frac{dQ(G)}{dG_R}$

and let $H(G) = [d^2 Q(G) / dG_i dG_j]$ be a matrix of second partial derivatives. Expanding *V*(**G**), near $G = \hat{G}$ in a Taylor series, we obtain $[V(G)]_{\hat{G}=G} = 0 = V(G) + H(G)(\hat{G} - G)$. Rewriting this equation, we have $\hat{G} - G = -H^{-1}(G)V(G),$ thereby obtaining an iterative equation given by $G^{(k+1)} = G^{(k)} - H^{-1}(G^{(k)})V(G^{(k)})$,

where $G^{(k)}$ is the set of estimates obtained at the k^{th} stage of iteration. The estimates obtained by the above iterative equations usually converge. For starting the iteration, we need to have good sets of initial values of the parameters. This is done by fitting the best subset of the linear part of the bilinear model.

Predictive Performance of the Models

In order to compare the performance of the bilinear models, it is necessary that we should obtain the forecasts and these are obtained as follows:

Suppose {X_t} is a discrete time series and we wish to predict X_{t_0+h} given the semi-infinite realization $(X_s, s \le t_0)$. Let the predictor be $\tilde{X}_{t_0}(h)$. Then it is well known that $E\left[X_{t_0+h} - \tilde{X}_{t_0}(h)\right]^2$ is minimum if and only if $\tilde{X}_{t_0}(h) = E(X_{t_0+h} / X_s, s \le t_0)$. The evaluation of $\tilde{X}_{t_0}(h)$ from the model depends on the unknown parameters.

Typically, we substitute the least squares estimates of these parameters, and then calculate the predictors. The predictors thus obtained are denoted by $\tilde{X}_{t_0}(h)$, (h = 1, 2, …) and the error by $\tilde{e}_{t_0}(h) = X_{t_0+h} - \tilde{X}_{t_0}(h)$ and the mean sum of squares of the errors of the predictors for the period (t_o+h, t_o+ h + 1, … , t_o + h + M) is $\tilde{\sigma}^2_{\tilde{e}}(h) = \frac{1}{M}\sum_{j=1}^{M} \tilde{e}^2_{t_0+j}(h)$

RESULTS AND DISCUSSION

To present the application of the model and its forecast, we will use a real time series dataset, the Wolfer sunspot. The scientists track solar cycle by counting sunspots – cool planet-sized areas on the Sun where intense magnetic loops poke through the star's visible surface. We have used annual sunspot numbers for the years 1730-1879, giving 150 observations.

Generalized Integrated Autoregressive Model

Fitted Model at t=150

$X_t = 0.412820X_{t-1} - 0.271125X_{t-2} - 0.270908X_{t-3} - 0.339150X_{t-5} - 0.293320X_{t-7} + 0.000325X_{t-1}e_{t-1} - 0.020870X_{t-1}e_{t-2} - 0.002425X_{t-1}e_{t-3} + 0.018075X_{t-2}e_{t-1} + 0.009283X_{t-2}e_{t-2} - 0.008691X_{t-2}e_{t-3} - 0.019234X_{t-3}e_{t-1} - 0.007737X_{t-3}e_{t-2} + e_t$

One-Dimensional Integrated Autoregressive Bilinear Time Series Model

Fitted Model at t=150

$X_t = 0.412820X_{t-1} - 0.271125X_{t-2} - 0.270908X_{t-3} - 0.339150X_{t-5} - 0.293320X_{t-7} - 0.002709X_{t-1}e_{t-1} - 0.006085X_{t-2}e_{t-1} - 0.002411X_{t-3}e_{t-1} - 0.009225X_{t-4}e_{t-1} - 0.006196X_{t-5}e_{t-1} + 0.002575X_{t-6}e_{t-1} - 0.021601X_{t-7}e_{t-1} + 0.010533X_{t-8}e_{t-1} + e_t$

Table 1: Residual variance and mean squares error for forecast (sunspot data)

MODEL	One-Dimensional Integrated Autoregressive Bilinear Model	Generalized Integrated Autoregressive Bilinear Model
σ_e^2	207.50	193.20
MSEF	15.55	14.28

From Table 1, it is clear that the generalized integrated autoregressive bilinear time series model has smaller residual variance when compared with one-dimensional integrated autoregressive models. Also generalized integrated autoregressive bilinear time series model has the smaller mean squares error for the forecast when compared with one-dimensional integrated autoregressive time series models. And as a result the performance of generalized integrated autoregressive bilinear time series models is better when it is used for forecasting.

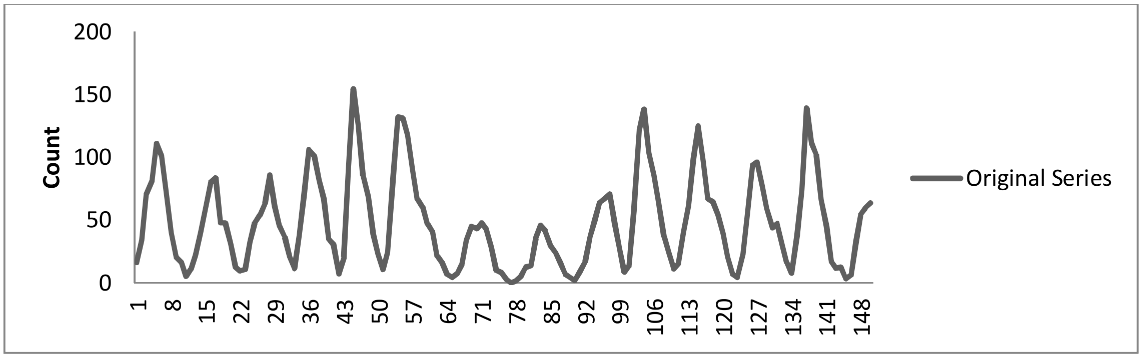

Figure 1: Time Plot of Sunspot Data (Original Series)

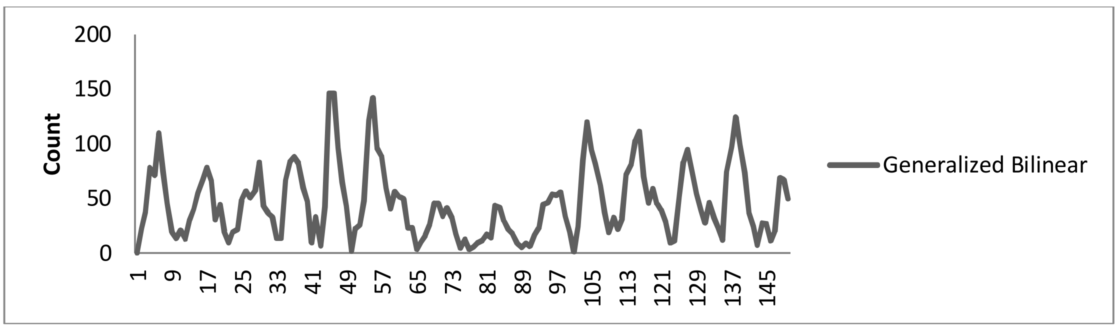

Figure 2: Time Plot of Forecast using Generalized Integrated Autoregressive Bilinear Model

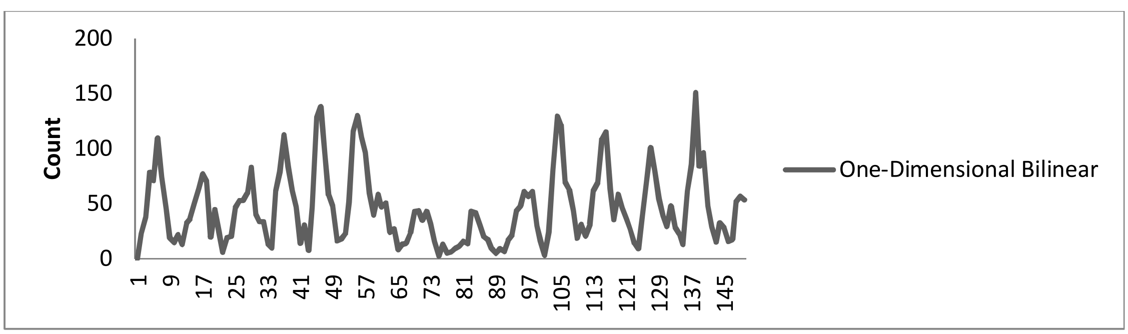

Figure 3: Time Plot of Forecast using One-Dimensional Integrated Autoregressive Bilinear Model

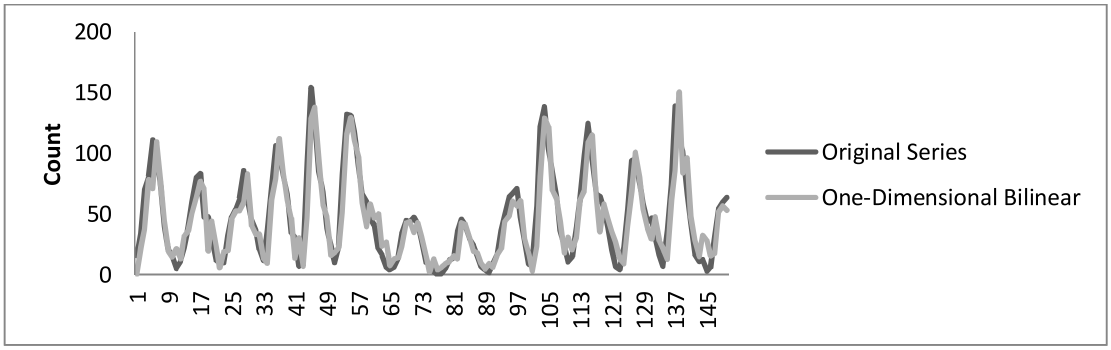

Figure 4: Time Plot of Forecast using Original Series and One-Dimensional Integrated Autoregressive Bilinear Model

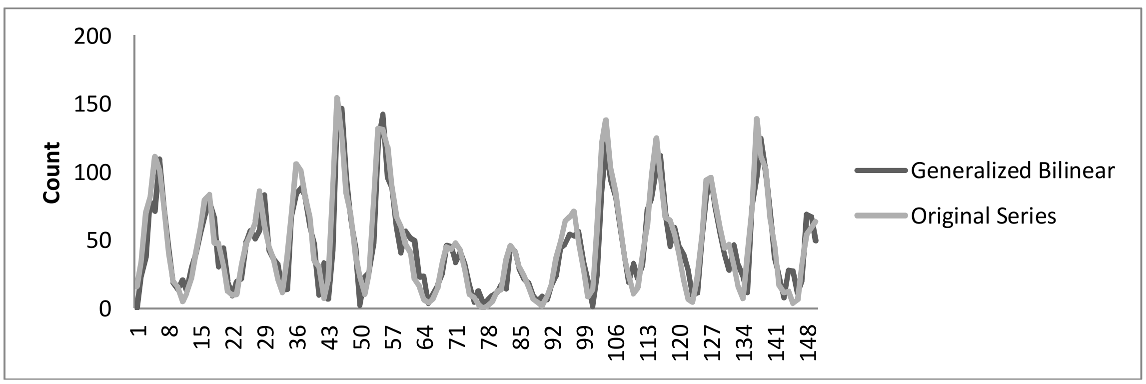

Figure 5: Time Plot of Forecast using Original Series and Generalized Integrated Autoregressive Bilinear Model

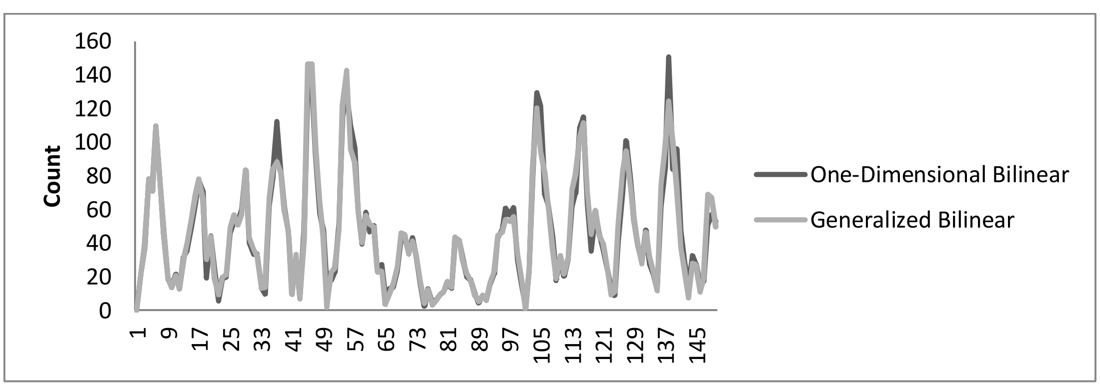

Figure 6: Time Plot of Forecast using Generalized and One-Dimensional Integrated Autoregressive Bilinear Models

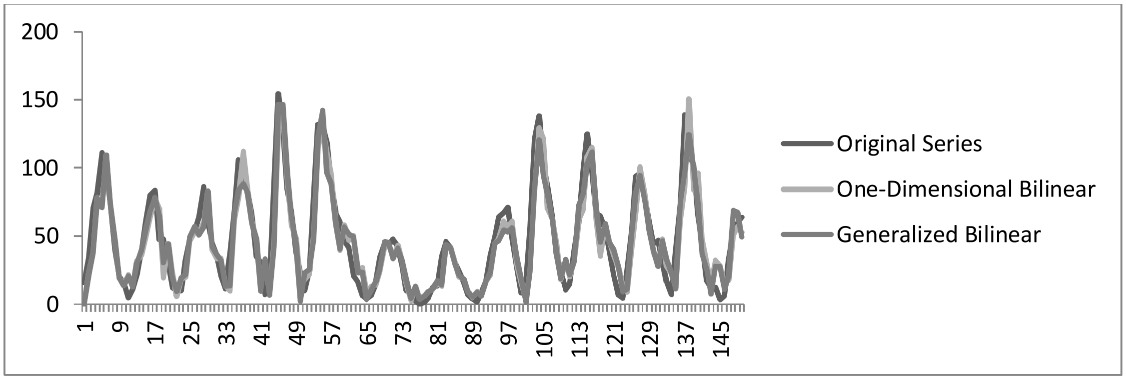

Figure 7: Time Plot of Original Series and Forecast using Generalized and One-Dimensional Integrated Autoregressive Bilinear Models

Figure 1 shows the graph of original series. Figure 2 shows graph of forecasts of generalized model while Figure 3 shows graph of forecast of one-dimensional model. Figures 4 and 5 compare graph of original series with graph of forecast of one-dimensional and generalized models while figure 5 compares graph of one-dimensional and generalized models. Figure 7 compares graph of original series and graph of forecast of one-dimensional and generalized models together.

CONCLUSION

Two bilinear time series models that were capable of achieving stationary for all non linear series were considered. These two models were used to forecast the future value having estimated their parameters. Generalized integrated autoregressive model outperformed one dimensional integrated autoregressive model after we have studied the residual variance attached to the two models. The mean square error for forecast for the models were studied and we found out that the mean square error attached to generalized bilinear model was smaller than one dimensional model. The two models were used to forecast. On the basis of the forecasting performance, generalized integrated autoregressive bilinear time series model formed a useful class of non-linear model for forecasting.

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