BAND MODELS APPROACH IN ATMOSPHERIC TRANSMISSION FUNCTION
CALCULATION - A REVIEW

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ABSTRACT

Attempts to proffer solutions to radiative transfer equations have always been
impeded due principally to the presence of numerous absorption lines whose strength, distribution, spacing
and half width within a particular absorption spectrum of a given substance is not fixed.

This review looks at the use of band models as solution to the problem of radiative transfer equation
of solar fluxes. Different band models like the regular (Elsasser), Goody (Statistical or Random), Random
Elsasser and the Quasi random are overtly examined with special emphasis on their respective operational
conditions, advantages and disadvantages.

INTRODUCTION

The passage of solar radiation through the atmosphere usually results in processes such as absorption,
transmission, scattering and reflection. About 31% of the incoming solar radiation is scattered at the upper
atmosphere while the remaining is transmitted, absorbed and scattered at the lower atmosphere (Liou, 1980;
LaRocca, 1978). An enormous portion of the solar radiation is finally absorbed by the earth surface while
emission from the earth's surface will send back part of it to the atmosphere to take care of the equilibrium
energy requirement of the earth (Liou, 1980).

The earth's atmosphere comprises solids and gases as shown in table 1. (United State Standard
Atmosphere, 1976). With the dust and water vapour components properly filtered out, the other permanent
constituents are fairly stable with 99% being nitrogen and oxygen (Berger et al., 1988; Liou, 1980; LaRocca,
1978; Lounsbury, 1973). The constituents vary as one ascends each of the strata of the atmosphere.

Solar radiation is generally absorbed in the atmosphere by molecules of oxygen, ozone, nitrogen,
carbon IV oxide and methane all of which are trace elements in a normal atmosphere (LaRocca, 1978: Liou,
1980; Harries, 1997; Berger et al., 1988). As discussed by Liou, (1980), Palttridge et al., (1976), Berger et al.,
(1988) and LaRocca (1978) within the infrared region of solar radiation, absorption is primarily due to triatomic
molecules of ozone, water vapour and carbon IV oxide. The absorption spectrum of these gases is largely due
to the vibrational and rotational transitions of their atoms. These spectra are characterised by the presence
of numerous absorption lines whose strength, distribution, spacing and halfwidths vary according to prevailing
conditions (imposed or real). The conditions are lines being assumed to be of equal intensity and/or lines having strength
with an exponential probability distribution. With these imposed conditions, the weak line, strong line and non
overlapping approximation are applied to obtain the transmittance. The absorption coefficient of these gases
depend on wave-number (Liou, 1980; LaRocca, 1978; Udo, 1997; Kondratyev, 1969).

The presence of these lines is mainly responsible for the inability of researchers to calculate
atmospheric transmittance which for a non scattering and non reflecting atmosphere, equals one minus the
atmospheric absorbtance. Also the dependence of absorption coefficient on physical conditions does not help
matters here. Temperature, pressure and absorber amount are not fixed in the atmosphere but rather varies
with altitude. These variations rendered the task of evaluating the halfwidths of the absorption lines very
difficult (Udo, 1997). Udo (1997) reported that the transmittance in certain vibrational bands is jointly
influenced by a combination of different absorbers, consequently, the lines will overlap. Transmittance in the
atmosphere in the vicinity of 15 and 9.6µm bands is co-influenced by carbon IV oxide, ozone and water vapour
(Udo, 1997). To simplify calculations, many researchers neglected this overlapping effect (Young, 1964;
Russel et al., 1973; Drayson, 1965; Scott, 1974; Mcmillin, 1976; Roberts et al., 1976; Fleming et al., 1970; Scott
et al., 1981; Chou, 1984).

A solution was obtained when the entire absorption spectrum of the absorbing substance was
partitioned into bands with defined spectral limits with the assumption that the distribution of lines, strength and

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<table>
<thead>
<tr>
<th>Permanent Constituents</th>
<th>Variable Constituents</th>
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<tbody>
<tr>
<td>Nitrogen (N\textsubscript{2})</td>
<td>Water Vapor (H\textsubscript{2}O)</td>
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<tr>
<td>Oxygen (O\textsubscript{2})</td>
<td>Ozone (O\textsubscript{3})</td>
</tr>
<tr>
<td>Argon (Ar)</td>
<td>Sulphur dioxide (SO\textsubscript{2})</td>
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<tr>
<td>Carbon dioxide CO\textsubscript{2}</td>
<td>Nitrogen dioxide (NO\textsubscript{2})</td>
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<tr>
<td>Neon (Ne)</td>
<td>Ammonia (NH\textsubscript{3})</td>
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<td>Helium (He)</td>
<td>Nitric Oxide (NO\textsubscript{2})</td>
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<tr>
<td>Krypton (Kr)</td>
<td>Hydrogen Sulphide (H\textsubscript{2}S)</td>
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<tr>
<td>Xenon (Xe)</td>
<td>Nitric acid Vapor (H\textsubscript{2}NO\textsubscript{3})</td>
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<td>Hydrogen (H)</td>
<td>Trace</td>
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<tr>
<td>Methane (CH\textsubscript{4})</td>
<td>0.5x10\textsuperscript{-4}</td>
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<tr>
<td>Nitrous Oxide (N\textsubscript{2}O)</td>
<td>1.5x10\textsuperscript{-4}</td>
</tr>
<tr>
<td>Carbon Monoxide (CO)</td>
<td>0.27x10\textsuperscript{-4}</td>
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*Concentration near the earth’s surface.


Spacing can be represented by a simple mathematical model (Liou, 1980; LaRocca, 1978; Udo, 1987; Coulson, 1976). By integrating over finite limits, the effective transmission function was uniquely obtained and subsequently the solar flux. It is this idea of partitioning that gave birth to the present day band models in which several contributions have been made towards eliciting viable solution to the problem of calculating atmospheric transmission function (Goody, 1952; Elsasser, 1938; Plass, 1958; Wyatt et al., 1962; Coulson, 1965; Udo, 1987; 1997). Band model is purely a mathematical technique with avenues for practical applications. It is reputed to be the most correct of all other techniques channeled toward realizing this same objective (LaRocca, 1978).

A band with an infinite array of Lorentz line shape which is characterised by having equal halfwidth, intensity and spacing is termed the regular Elsasser band model (Elsasser, 1938; LaRocca, 1978; Coulson, 1965; Liou, 1980; Udo, 1987). The Random Elsasser band model is obtained when the Lorentzian lines are randomly superposed on each other (Plass, 1958; Udo, 87). This regular repetition in the Lorentz line shape is very noticeable in the 16μm CO\textsubscript{2} band but will not work well with water vapour and ozone absorption bands.

The Random band model came as a back up for all the absorption bands where the regular line structures could not be observed (Goody, 1952; Mayer, 1947; LaRocca, 1978; Liou, 1980; Udo, 1987). The distribution of lines as well as their strengths can be expressed by probability functions which can be adjusted to favour the true line distribution (Drayson, 1967; Udo, 1987).

The Quasi-Random model is an apt generalization of the previous two models. It does not believe in regularity of the lines but rather believes that the lines are not disorderly arranged (LaRocca, 1978). It is reputed to be the most correct of all the forms of band models.

Band models do fail in many cases and it is these limitations that prompted researchers to develop alternative methods (Udo, 1987). Such methods are computer generated procedures that utilise direct numerical integration to work (Drayson, 1965; Drayson et al, 1966).

This paper discusses band models and their role in eliciting viable solution to the problem of calculating atmospheric transmission function entirely in its theoretical form. It is hoped that this review will help researchers and would-be researchers in the field to be fully aware of the various types of band models, their operational conditions and their limitations.

EQUATION OF RADIATIVE TRANSFER

A ray of solar radiation traversing a medium will be continuously attenuated through absorption as it
interacts with the medium. At the same time, the weakened beam of radiation will be reinforced by emission from the medium itself. If the medium is non-scattering, non-reflecting, non-refracting and isotropic at the wavelengths under consideration, the resultant intensity will then be given according to Liou, (1980) by

$$dI_\lambda = -K_\lambda \rho I_\lambda ds + j_\lambda \rho ds$$  

(1)

where $\rho = $ density of the absorbing medium, $K_\lambda = $ mass extinction cross section for radiation of wavelength $\lambda$, $j_\lambda = $ source function coefficient and $ds = $ thickness of the medium traversed by the radiation.

For a plane parallel atmosphere with an isotropic source function in which linear distances can be measured as shown in fig. 1, then equation (1) becomes

$$\frac{\mu dI_\lambda (z; \theta, \phi)}{de} = -I_\lambda (z; \theta, \phi) + J_\lambda (z; \theta, \phi)$$  

(2)

where $J_\lambda = j_\lambda/K_\lambda = $ source function, $\theta$ and $\phi$ are the inclinations to the upward normal and the azimuthal angle with reference to the x-axis respectively, $\mu = \cos \theta$ and $e$ is the optical depth and it is a function of altitude and is defined as

$$e = \int_{z}^{z_f} K_\lambda \rho dz'$$  

(3)

Equation (2) is the equation of radiative transfer for a plane parallel atmosphere.

Liou (1980), LaRocca (1978), Wayne et al. (1983) and Patridge et al. (1976) have solved equation (2) for the upward intensity that is independent of the azimuthal angle to obtain and the corresponding solution to the downward intensity is

$$I_\lambda (e; \mu) = I_\lambda (e; e, \mu) e^{(e_1 - e)/\mu} + \int_{e}^{e_1} \frac{J_\lambda (e', \mu)}{e} e^{(e' - e)/\mu}$$  

(4)$\mu > 0$

$$I_\lambda (e; -\mu) = I_\lambda (0; \mu) e^{e/\mu} + \int_{0}^{e} \frac{J_\lambda (e'; -\mu)}{e} e^{(e' - e)/\mu}$$  

(5)$\mu > 0$

where the primes denote optical depth of the emitting bodies, $\mu, \varepsilon_1 = $ thickness of the medium and $e^{\mu}$ is a multiplication factor.
Numerical form of equation (5) is given by LaRocca (1978), Harries (1997), Udo (1997) and Liou (1980) as

$$I_{\lambda} = I_{\lambda}(\varepsilon, \mu) z(\lambda) + \sum_{j=1}^{n} B_{j}(T(z)) \left( \frac{\partial I_{\lambda}}{\partial T} \right) \Delta \varepsilon_{j}$$  \hspace{1cm} (6)

where we have taken the emitting gas to be in local thermodynamic equilibrium (LTE). This condition is only met at altitudes not exceeding 70km as Ketepov et al. (1978) pointed out. With this condition, the source function $J_{\nu}$ and $I_{\nu}$ can be replaced by Planck's blackbody function $B_{\nu}(T_{s})$ in the infra-red region. $T_{s}$ is the surface temperature of the earth. 

Liou (1980) gave an expression to the solution for monochromatic upward flux density $F^{u}(\varepsilon)$ for the upper atmosphere where there is hardly any matter to emit radiation as

$$F^{u}(\varepsilon) = \int_{0}^{t} F_{e}^{u} \delta k + 2 \int_{0}^{t} n B_{e}(T(s)) E_{\nu}(\varepsilon_{\nu} - \varepsilon) \delta k$$

$$+ 2 \int_{t}^{t_{\nu}} \int_{0}^{t} n B_{e}(T(t')) E_{\nu}(\varepsilon_{\nu} - \varepsilon) \delta k \delta \varepsilon'$$ \hspace{1cm} (7)

and

$$F^{d}(\varepsilon) = \int_{0}^{t} F_{e}^{d} \delta k = \int_{0}^{t} \int_{0}^{n} B_{e}(T(t')) E_{\nu}(\varepsilon_{\nu} - \varepsilon) \delta k$$ \hspace{1cm} (8)

for the downward flux density where the integration is for all wavelengths in the whole infra-red spectrum. Note the change from wavelength domain to wave number, $K$ domain. Also note that in arriving at equations (7) and (8), he used

$$\frac{dE}{d\varepsilon} = \int_{1}^{\infty} \frac{\varepsilon^{x}}{X^{n-1}} dx = - E_{n-1}(\varepsilon)$$ \hspace{1cm} (9)

where $E_{n} = \int_{1}^{\infty} \frac{\varepsilon^{x}}{X^{n}} dx$ and $d\mu = dx/x^{2}$.

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**Fig. 2:** Absorption spectrum of water vapour rotational band and 15 um carbon dioxide band at high resolution (after McClatchey and Selby, 1972)
Mathematically, solutions to equation (7) and (8) are supposed to yield unique solutions to the total upward and downward thermal infrared fluxes in a clear atmosphere but for the existence of practical barriers. According to Liou (1980), and Udo (1997), the over dependence of absorption coefficient on wavelength especially in the rotational and vibrational spectrum is one of such impediments. This practical difficulty is so conspicuous that for us to obtain the fluxes at any height (say $\varepsilon$) in the atmosphere, we are faced with evaluating the double integration for very many absorption lines. A model absorption spectrum of water vapour and 15 $\mu$m Carbon IV Oxide are as shown in Fig. 2. This approach is not only time consuming but very impracticable even with our present generation of computers.

Another strong impediment that strongly barred researchers from actualising their aim is the dependence of absorption coefficient on halfwidth of the lines which in turn depends on physical conditions such as temperature, pressure and the amount of the absorbing substance in the atmosphere. Since these parameters vary as one ascends or descends the atmosphere, the absorbance will in turn be varying also.

The overlapping effect of the lines in the observed spectra caused by the presence of more than one absorber is another factor that frustrates the efforts of researchers (Udo, 1987, 1997). Though many researchers ignore this factor in their work (Young, 1974; Drayson, 1965; Russel et. al., 1973; Scott, 1974; McMillin, 1976; Roberts et. al., 1976; Fleming et. al., 1979; Scott et. al., 1981), concerted attempts have been made to properly solve this problem (Wang et al., 1983; Udo, 1997; Harries, 1997). Udo (1997) pointed out that there exists an appreciable overlapping effect on transmission function as the overlapping distance changes. Goody (1964) and Elsasser (1942) have shown that the effect of overlapping of gases on radiation calculation depends on both the strengths of the absorption lines and the amount of gases in the atmosphere.

The only viable way out of this problem is to partition the entire absorption spectrum into bands with well defined limits from where we can determine the effective transmission function. The band will not only be able to accommodate many absorption lines but will also enclose several wings from lines outside the interval that contributes to absorption within the interval.

CONCEPT OF TRANSMISSION FUNCTION

Imagine a spectral interval of width $\Delta v$ which is large enough so that it can accommodate many absorption lines but is small enough so that the mean value of the Planck's function $B_v(T)$ can be applied to it. Liou (1980) and LaRocca (1978) gave an expression for the transmission function $I(\varepsilon)$ as

$$\tau(\varepsilon) = \frac{1}{\Delta v} \int_{h\varepsilon} e \varepsilon d\varepsilon$$

where

$$\varepsilon = \varepsilon_1(v, q) = \int_{v}^{q} K_v dq$$

is the monochromatic optical depth, $q$ is the total normal path and $\varepsilon_1$ is the optical depth. $q_1$ and $\varepsilon_1$ are defined respectively as

$$q_1 = \int_{o}^{v} \rho dz \quad \text{and} \quad \varepsilon_1 = \int_{o}^{q_1} K_v dq$$

It should be noted here that if scattering, reflection, refraction are neglected in an isotropic medium then the transmission function is related to the absorption function for a particular wavelength by

$$\tau_1 + A_1 = 1$$

BAND MODELS FOR TRANSMISSION FUNCTIONS

The various types of band models in use are:

(i) The Regular or Elsasser band model
(ii) The Goody or Statistical or Random band model
(iii) The Random Elsasser model
(iv) The Quasi Random model
REGULAR BAND MODEL

The Regular or Elsasser band model has lines assumed to be of equal halfwidths, strengths and equal spacings (Elsasser, 1938, LaRocca, 1978; Coulson, 1965; Liou 1980). The transmission function is averaged over an equal interval to the spacing between the line centers.

Elsasser, (1938); Liou, (1980); LaRocca, (1978) and Coulson, (1965) gave an expression for the absorption coefficient at a wavenumber displacement from the centre of one particular line as

\[ K_v = \sum \frac{\omega \alpha}{(v - i\delta)^{i\alpha}} \]  \hspace{1cm} (13)

where \( \delta \) is the line spacing. By applying Mittag - Leffler theorem on equation (13) and substituting the result into equation (10), Liou (1980) arrived at:

\[ \frac{\partial^v}{\partial v} = -\sinh \beta \exp \{ -y \cosh \beta J_v (iy) \} \]  \hspace{1cm} (14)

where \( J_v (iy) \) is the Bessel function of imaginary argument, \( \beta = 2\pi \alpha \delta \) and \( y = 2\pi \nu \delta \). Equation (14) has solution under two conditions which are:

(i) Weak line approximation: Under high pressures, the spectral lines will overlap strongly in such a way that it will become unobservable. LaRocca (1978) and Liou (1980) have shown that by suitably approximating the parameters in equation (14) under such conditions, equation (14) will become

\[ A = 1 - e^{-\beta \nu} \]  \hspace{1cm} (15)

The absorption function here is independent of the position of the spectral lines within the band. This result is particularly useful in obtaining the values of the absorption function to small \( \nu \) and large values of pressure.

(ii) Strong line approximation: LaRocca (1978) pointed out that for long paths at high altitudes, the pressure is usually low while the amount of absorbing substances are large. This condition will make absorption to be completed at the center of the lines, the halfwidth to be small but narrow and the lines to be less overlapping. Equation (14) will be approximated to an error function provided \( \nu \) is large enough and \( \beta \) is sufficiently small as Liou (1980) and LaRocca (1978) have shown. Thus

\[ A = \text{erf} \left( \frac{\sqrt{2} \beta \nu}{2} \right) \]  \hspace{1cm} (16)

where

\[ \text{erf}(a) = \frac{2}{\sqrt{\pi}} \int_0^a e^{-t^2} \, dt \]

This approximation is known as strong line approximation to the Elsasser band model. Equation (16) is usually valid provided \( 0.1 \leq A \leq 10 \)

(iii) Non overlapped Approximation: For short paths at high altitudes, the pressure is usually high while the absorber amount is small. LaRocca (1978) and Liou (1980) pointed out that under such conditions, equation (14) becomes

\[ A = \beta \psi e^{-\beta \nu} [J_0 (\psi) + J_1 (\psi)] \]  \hspace{1cm} (17)

RANDOM BAND MODEL

Random or statistical or Goody band model works with the assumption that the position of lines together with their strength can be described by a probability function which can be adjusted to favour the true line distribution (Drayson, 1957; Udo, 1987; Goody, 1952; LaRocca, 1978; Liou, 1980). Consider a finite spectral interval comprising of \( n \) lines of mean spacing \( \delta \) so that \( \Delta \nu = n \delta \). Let \( P(S) \) be the probability that the \( i \)th line will have an intensity \( S_i \). Also let \( P(s) \) be the normalized probability that a line will have a strength between \( S \) and \( S+\Delta S \) so that we assumed that the lines have similar probability of being anywhere within the interval \( \Delta \nu \) and the average
transmission function was obtained by taking the mean over line positions and of intensity of lines. LaRocca (1978), Liou (1980) and Goody (1952) gave expression for transmission function as

$$
\tau(\nu) = \left[ 1 - \frac{1}{\Delta \nu} \int_{\nu}^{\nu + \Delta \nu} P(s) \left( 1 - e^{-ks} \right) ds \right]^n
$$

(19)

For large values of $n$, LaRocca (1978) and Liou (1980) rewrite equation (19) as

$$
\tau(\nu) = \exp \left\{ -\frac{1}{d} \int_{\nu}^{\nu + \Delta \nu} P(s) \left[ \int_{\nu}^{\nu + \Delta \nu} \left( 1 - e^{-ks} \right) dv \right] ds \right\}
$$

(20)

where

$$
\int_{\nu}^{\nu + \Delta \nu} \left( 1 - e^{-ks} \right) dv
$$

absorption by a single line integrated over the interval $\Delta \nu$. Equation (20) was solved under the following conditions:

(i) Equal intensity lines: The lines were assumed to be of equal intensity such that when the Dirac-Delta function was applied to equation (20), we had

$$
A(\nu) = 1 - \exp \left\{ -[\beta P e^{\psi} I_0(\psi) + I_1(\psi)] \right\}
$$

(21)

If the lines are weakly absorbing, $\psi$ will be small and equation (21) becomes according to LaRocca (1978) and Liou (1980)

$$
A(\nu) = 1 - \exp \left\{ -\beta \psi \right\}
$$

(22)

LaRocca (1978) and Liou (1980) have also shown that if the lines are strongly absorbing, equation (21) becomes

$$
A(\nu) = 1 - \exp \left\{ -2 \frac{S_{\alpha \chi}}{d} \right\}
$$

(23)

and finally when the non-overlapping approximation was applied to the equation we got

$$
A(\nu) = \frac{\beta P e^{\psi} I_0(\psi) + I_1(\psi)}{1 + 2\psi}
$$

(24)

A line strength with an exponential probability distribution: This situation occurs when both the lines and the probability distribution function are of different strengths (LaRocca (1978) and Liou (1980)). Imagine a Poisson distribution (exponential distribution) for the probability of their intensities to be of the form:

$$
P(s) = \frac{1}{S_o} e^{-s/s_o}
$$

(25)

$$
A(\nu) = 1 - \exp \left\{ -\frac{\beta \psi_o}{(1+2\psi_o)^{\nu}} \right\}
$$

(26)

where $S_o$ is the mean line strength. By applying a statistical regression analysis, the absorptance as obtained by Liou (1980) and LaRocca (1978) is where $\psi_o = S_{\alpha \chi}/2\pi a$. By considering $\psi_o << 1$, we obtained expression for the weak line absorption from equation (25) as

$$
A(\nu) = 1 - \exp \left\{ -\beta \psi_o \right\}
$$

(27)
When $\psi_0 >>> 1$, the strong line approximation becomes
\[ A(v) = 1 - \exp \left[ -\left( \frac{\beta \psi_0}{1+2\psi_0} \right)^2 \right] \] \hspace{1cm} (28)

When the exponent is small, we obtained the non overlapping approximation as
\[ A(v) = \frac{\beta \psi_0}{(1+2\psi_0)^{\frac{3}{2}}} \] \hspace{1cm} (29)

**RANDOM-ELSASSER BAND MODEL**

This model assumes that the spectral lines though regular are randomly placed for some gases and spectral regions. It is therefore a generalisation of the Elsasser and the Goody band models. Each line may have different line spacing, halfwidth and strength. Plass (1958) reported that the transmission function for $N$ randomly superposed Elsasser band with arbitrary strength, halfwidth and line spacing is given by
\[ A(v) = 1 - \prod_{i=1}^{N} \left[ 1 - A_{E_i}(\psi_i, \beta_i) \right] \] \hspace{1cm} (30)

where $\psi_i = S_i/(2\pi \sigma_i)$, $\beta_i = 2\pi \sigma_i d_i$, $S_i$ and $\sigma_i$ are the line strength and halfwidth for the $i$th Elsasser band respectively, $d_i$ is the line spacing of the $i$th band and $A_{E_i}$ is absorbance for the $i$th line.

**QUASI-RANDOM MODEL**

This model presupposes that the lines are neither regularly arranged nor randomly arranged but it still believes that there exists some order in its arrangement. The transmittance was calculated in bits here. First it was calculated for the frequency range that was very much less than the range of interest. This did not only act as a precaution against overlapping effects from other lines but also confined the lines to narrow intervals around their true positions. (LaRocca, 1978).

The transmittance for $N$ spectral lines within the interval was calculated independently and then by assuming a random positioning of the lines, the results were later combined. The mean results for at least two different arrangements of the mesh that divides the spectrum into frequency were determined. The procedure was equally extended to take care of absorption from the wings of lines in neighbouring intervals as well as the numerous weak spectral lines and their spacing. Finally, the transmittance values for each line in every sub-interval that forms part of the major interval were averaged to determine the final value for the transmission function. LaRocca (1978) gave an expression for the transmittance for this model as
\[ \tau = \frac{1}{n} \sum_{j=1}^{M} A_j \] \hspace{1cm} (31)

where $A_j$ is the absorbance of each of the $n$ smaller wavelength intervals of width $D$ in which the initial interval $\Delta v$ was subdivided. Wyatt et al (1962) pointed out that the absorbance was obtained from
\[ A_j = 1 - \prod_{i=1}^{N} \left[ 1 - A_{E_i}(\psi_{i,j}, \beta_{i,j}) \right] \] \hspace{1cm} (32)

where $A_{E_i} = $ single line absorbance of the $i$th line over $j$th finite interval, $D$ and $M =$ the number of lines in the $i$th frequency interval.

**LIMITATIONS OF BAND MODELS**

In spite of the numerous contributions made by band models, they still have some limitations. Udo (1987) listed these limitations as:

(i) The exact line distribution within the band can never be ascertained but can only be approximated by band models.

(ii) It does not account for the contribution from wings of lines lying outside the spectral region under
consideration.

(iii) The spectral resolution of the transmission is finite. For Elsasser model, it is a multiple of line spacings whereas the statistical model uses an averaging interval assumed to be sufficiently large so as to ensure that the true distribution of the lines is adequately simulated by the statistical distribution.

(iv) There exist a lot of complexity in the calculation even for a Lorentz line shape with a tremendous increase as we go over to other line shapes. It is the existence of these limitations that prompted researchers to develop some alternative approaches to the treatment of this problem. This alternative approach is a computer-generated method that uses the direct numerical solution. It is simpler to use than the band model. It also has many advantages than the band models. Inspite of these advantages, it is sad to note that their results are slightly less accurate than the results from band models (LaRocca, 1975) and it is time consuming.

CONCLUSION

Band models are very reliable tools which have played a very important role in enabling us to understand radiative transfer processes in the atmosphere. It is also a technique which transforms a problem with impossible complexity into one with manageable proportion.

Liou (1980) and LaRocca (1978) studied results from band models instead of the numerous assumptions and approximations and independently concluded that they have a whopping 90% accuracy. It is hoped that its the foremost position of the Band models will continue to remain undeterred since the transmission function is a function of a few measurable physical quantities.

REFERENCES


