ERROR CONTROL CODING SCHEMES IN DIGITAL SYSTEMS

B. J. KWAIKA, S. F. A. AKANDE and A. H. GBADAMOSI

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ABSTRACT

Error control coding is a scheme adopted in detection, correction and prevention of errors due to distortions and perturbations accruing from modern defects, regeneration defects, interferences, linear and non-linear distortions encountered in digital communication networks. Five major classes of codes namely block codes, Systematic codes, Cyclic codes, Convolutional codes and Independent error control codes were reviewed in this study. The Hamming code of the class of Block codes was designed and implemented using Exclusive OR (XOR) gates obtained as IC SN7486. Also, the Parity check code of the independent error control class was designed and implemented using the AND-OR-INVERT (AOI) gates obtained as IC SN 74LS280.

Results indicate that coded transmission yields a lower word error probability than uncoded transmission; the effect of which is remarkable in excess of average signal power $S_n$ per noise spectral density $\eta_n$ ($S_n/\eta_n)^8$.

Key words: Errors, Coding Schemes, Digital Systems

INTRODUCTION

The digital communication system is a network of wires and mechanical parts whose voltage and current positions convey coded information (Chiiorobo, 1997). A codeword can only take a finite set of discrete levels in accordance with the binary code 0 and 1.

The digital communication system is versatile due to the impact of the digital computer flexibility and compatibility of wide band channels provided by geostationary satellites and the ever increasing integrated circuit (IC) technology in a cost effective manner (Haykin, 1988).

There are various potential sources of distortions and perturbations encountered in digital communications. These accrue from:

i. the propagation within the physical medium (atmosphere, cables, space etc.),

ii. the electromagnetic environment created by signal bearing lines,

iii. the inverse operation that takes place within the receiver, and

iv. the Additive White Gaussian Noise (AWGN).

All these, among many others, contribute in various ways to make it almost impossible to produce an exact replica of a digitized analogue signal from its digital representation. Errors in digital systems are linked to impairments in modern equipment, noise, imperfect filtering, selective and deep fading, attenuation due to regeneration defects, non-linearities within amplifiers operated near saturation points (hard limits) and Interferences — viz — Inter symbol Interference (ISI), co-channel Interference (CCI), Adjacent channel Interference (ACI), and Inter Channel Interference (ICI) (Haykin, 1988; Akande and Chiiorobo, 1997).

An early approach to error control analyses assesses performances in bit error rate (BER) coding parameters such as the Q-function, the error probability function and the signal to noise ratio (S/N) (Gray, 1979). Error control coding involves the calculated use of redundancies using functional blocks such as the channel encoder and the channel decoder.

However, the redundancies, when introduced, reduce the effective data rate through the channel. In practice, the channel encoder systematically adds digits to the transmitted message bits while the channel decoder uses the added redundancies to detect and correct any errors in the information bearing digits (Posa, 1981).

Errors in digital systems can be classified into two-Random errors and Burst errors. Random errors are due to Gaussian noise sourced from thermal, shot and flicker types. Burst errors are due to impulse noise, which are characterized by long quiet intervals followed by high amplitude burst. Such errors are

B. J. KWAIKA. Department of Physics, University of Jos, Jos, Nigeria.

S. F. A. AKANDE. Department of Physics, University of Jos, Jos, Nigeria.

A. H. GBADAMOSI. Department of Physics, University of Jos, Jos, Nigeria.
reduced using two prominent methods – the Forward-acting error correction (FEC) method and the Automatic retransmission request (ARQ) method.

With FEC, errors are corrected at the receiver end while the ARQ accepts or discards a received message sequence depending on its authenticity and reply the transmitter through a feedback channel. These two are used separately but sometimes cascaded when necessary as in digital audio tracking system (Posa 1981).

The general features of any error control code include the code length which specifies the length of a codeword, coderate which states the rate at which codewords are transmitted, and the correction properties associated with the distance.

The thrust of this paper is to study error control coding schemes including the design and implementing techniques as applicable to a wide range of message formats in digital communication. The designed error control codes – Hamming code and Parity check code, were selected because of their flexibility and versatility in application with most digital systems.

ERROR CONTROL CODING SCHEMES

Error control codes are classified into five – Block codes, systematic codes, Cyclic codes, Convolutional codes and the Independent error control codes (Dupontiel et al. 1983; Haykin, 1988).

In Block codes, each k-block of message bits is encoded into a block of n > k bits by adding redundancies called check bits. Error control codes in this class include the Repetition code, Maximum length code and the Hamming code.

In Systematic codes, the stream, of transmitted data, which comprises the k information bits and the n-k checkbits, are grouped into blocks of 23. Thus, a Systematic code operates on multiple bits. One of such is the Reed-Solomon (R-S) code.

A Cyclic code is formed when a circular permutation of S-codewords is carried out to yield another codeword. Error control codes in this class include the Cyclic redundancy code, Bose-Chandhuri-Hocquenghen (BCH) code and the Expurgated code.

In convolutional codes, check bits are continuously interleaved with information bits. This implies that code digits generated by an encoder at a time depend not only on the k-message digits at that time but also on the preceding operation. Codes in this class include the Concatenated code, and the Trellis code.

The independent error control codes have their properties sparsely distributed in the four aforementioned classes. They have no restriction with respect to codelength and coderate. Codes in this class include the Constant ratio code, Parity check code and the Lattice code.

Shamugam (1979) reported that Shannon theory states that for coded or uncoded information, a block of k message digits is transmitted in a duration time Tw given as $Tw = k/r_0$ where $r_0$ is the message bit rate. The bit rate into the channel is given as $r_c = r_0(n/k)$. This implies that the transmitted bit duration in the coded case must be less than the bit duration in the uncoded case. If the transmitter power $S_T$ (also called the signal strength) is held constant, then the energy per bit is decreased by the use of coding and the probability of decoding a transmitted bit increases.

It is expedient to determine whether coding results in a significant reduction in the probability of incorrectly decoding a message bit. This is expressed as a probability of error $P_e$ of a binary systematical with a fairly constant average signal power, $S_{av}$, over a defined noise spectral density $\eta$. 

for both coded and uncoded case and is given as (Shanmugam 1979):

\[ P_c = Q \left( \frac{2S_n}{\eta r_b} \right)^{1/2} \]  \hspace{1cm} (1)

where \( r_b \) is the message bit rate and \( Q \) is a constant (\( \approx 0.0023 \)) (Shanmugam, 1979).

Where error control is employed, the probability of error is less than one. For arbitrary values of \( 10^{-1} \) to \( 10^{-4} \) and a constant ratio of signal power to noise power or range 2 to 10dB, plots of \( q_u, q_c, P_u \) and \( P_c \) are shown in Fig. 5.

\[ S_m / \eta_b \text{ (Shanmugam, 1979)} \]

Fig. 2. Channel bit error probabilities and word error probabilities for coded and uncoded systems

Shanmugam (1979) showed that:

Channel bit error probability for the coded system \( q_c \) is

\[ q_c = Q \left( \frac{2S_n}{\eta r_b} \right)^{1/2} \]  \hspace{1cm} (2)

Channel bit error probability for the uncoded system \( q_u \) is

\[ q_u = Q \left( \frac{2S_n}{\eta r_b} \right)^{1/2} \]  \hspace{1cm} (3)

Probability of incorrectly decoding a message bit (k-block) in the uncoded system \( P_u \) is

\[ P_u = kq_u \]  \hspace{1cm} (4)

Probability of incorrectly decoding a message bit (k-block) in the coded system \( P_c \) is

\[ P_c = kq_c \]  \hspace{1cm} (5)
THE HAMMING CODE

The Hamming code is a perfect one error detecting/correcting linear block code with a minimum distance $d_{\text{min}} = 3$. In block codes, a block of $k$ information bits is followed by a group of $r$ check bits. Thus, a code $C(N,K)$ is made up of $2^k$ codewords $\{C_i\}$ a set of which forms a group under modulo – 2 addition. A correspondence between a message and a codeword takes the form (Kwaha, 1999).

$$C_i = M_i \cdot G$$

where $i = 1,2,\ldots,2^k$, $m_i = m_{i,0}, \ldots, m_{i,k-1}$ and $C_i = C_{i,0} \ldots 1 \ C_{i,N-1}$

And $G$ is the generator matrix, and $m$ the message bits.

A linear block code can correct up to $(d_{\text{min}} - 1)/2$ and detect up to $d_{\text{min}} - 1$ errors in each codeword (Dupontiel et al, 1983). The Hamming code is well adapted to the American 7-Bit ASCII character used in digital communication systems. The structure for a 7-bit codeword is shown in Table 1.

<table>
<thead>
<tr>
<th>Bit No.</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bit</td>
<td>$P_1$</td>
<td>$P_2$</td>
<td>$X_3$</td>
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$P_1$ is parity bit selected to establish even parity over bits 1, 3, 5 and 7. $P_2$ does the same over bits 2, 3, 6 and 7. While $P_4$ is over bits 4, 5, 6 and 7. $X_3$, $X_5$, $X_6$, and $X_7$ are information bits. The design and implementation of this code was accomplished using three Exclusive OR (XOR) gates each obtained as IC SN 7486 (Fig. 3) and operated on 5V d.c power supply.

![Diagram of Hamming Code](image)

C1, C2 and C3 represent the XOR trees while S1, S2, and S3 represent their respective outputs. The output is read in the sequences S1, S2, S3; the binary correspondence of which identifies the bit in error. Nine Hamming coded messages were received with their corresponding outputs as shown in Table 2.

THE PARITY CHECK CODE

The Parity check code is obtained by including an extra digit with the information bits such that the total number of 1’s in the data is fixed -- even or odd as may be required.

<table>
<thead>
<tr>
<th>0</th>
<th>1</th>
<th>1</th>
<th>1</th>
<th>0</th>
<th>1</th>
<th>1</th>
<th>1</th>
<th>Position of Parity bit</th>
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<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>Even Parity</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
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where $i = 1, 2, \ldots, 2^k$, $m_i = m_{i0}, \ldots, m_{i(k-1)}$ and $C_i = C_{i0}, \ldots, 1 \cdot C_{iN-1}$

And $G$ is the generator matrix, and $M$ the message bits.

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Even Parity    Odd Parity
for both coded and uncoded case and is given as (Shanmugam 1979):

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ARQ scheme for satellite channels.

In the Cyclic and Systematic codes, the generator matrix is used in encoding while the parity check matrix is used in decoding. This implies that the encoder stores the generator matrix and performs the binary arithmetic to produce the check bits. Posa (1981) highlighted that this system adapts well to burst error types making this approach suitable in digital audio and Video tracking.

Error rate in speech transmission by pulse code modulation (PCM), is substantially reduced by using the convolutional codes (Haykin, 1988). This is used in Laser systems using the optical disc and Laser videocassette recorders. This technology has been extended to the compact disc stereo system now. When this system is coupled to a block code, it suits well as in computer back up storage systems.

In the telephone channel link, the overall quality of the link requires an inner and an outer stage coding to protect and control data traffic. Paul and Young (1989) reported that for such systems, the concatenated code of the class of convolutional codes is optimum.

Generally the fundamental principle of error control coding is applicable to all digital systems. Present day technology requires that these coding schemes be inbuilt into various systems compliant to the digital signal processing technique -choosing where necessary coding schemes adaptable to specific systems.

REFERENCES


