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ERROR CONTROL CODING SCHEMES IN DIGITAL SYSTEMS

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ABSTRACT

Error control coding is a scheme adopted in detection, correction and prevention of errors due to distortions and perturbations accruing from modern defects, regeneration defects, interferences, linear and non linear distortions encountered in digital communication networks. Five major classes of codes namely Block codes, Systematic codes. Cyclic codes, Convolutional codes and Independent error control codes were reviewed in this study. The Hamming code of the class of Block codes was designed and implemented using Exclusive OR (XOR) gates obtained as IC SN7486. Also, the Parity check code of the independent error control class was designed and implemented using the AND-OR-INVERT (AOI) gates obtained as IC SN 74LS280.

Results indicate that coded transmission yields a lower word error probability than uncoded transmission; the effect of which is remarkable in excess of average signal power S_{av} per noise spectral density ηr_b ($S_{av}/\eta r_b$) =8.

Key words: Errors, Coding Schemes, Digital Systems

INTRODUCTION

The digital communication system is a network of wires and mechanical parts whose voltage and current positions convey coded information (Ehiorobo, 1997). A codeword can only take a finite set of discrete levels in accordance with the binary code 0 and 1.

The digital communication system is versatile due to the impact of the digital computer flexibility and compatibility of wide band channels provided by geostationary satellites and the ever increasing integrated circuit (IC) technology all in a cost effective manner (Haykin, 1988).

There are various potential sources of distortions and perturbations encountered in digital communications. These accrue from:

i. the propagation within the physical medium (atmosphere, cables, space etc),.
ii. the electromagnetic environment created by signal bearing lines,
iii. the inverse operation that takes place within the receiver, and
iv. the Additive White Gaussion Noise (AWGN).

All these, among many others, contribute in various ways to make it almost impossible to produce an exact replica of a digitized analogue signal from its digital representation. Errors in digital systems are linked to impairments in modem equipment, noise, imperfect filtering, selective and deep fading, attenuation due to regeneration defects, non-linearities within amplifiers operated near saturation points (hard limiters) and Interferences – viz – Inter symbol Interference (ISI), co-channel Interference (CCI), Adjacent channel Interference (ACI), and Inter Channel Interference (ICI) (Haykin, 1988; Akande and Chijioke, 1997).

An early approach to error control analyses assesses performances in bit error rate (BER) using parameters such as the Q-function, the error probability function and the signal to noise ratio (S/N) (Gray, 1979). Error control coding involves the calculated use of redundancies using functional blocks such as the channel encoder and the channel decoder.

However, the redundancies, when introduced, reduce the effective data rate through the channel. In practice, the channel encoder systematically adds digits to the transmitted message bits while the channel decoder uses the added redundancies to detect and correct any errors in the information bearing digits (Posa, 1981).

Errors in digital systems can be classified into two-Random errors and Burst errors. Random errors are due to Guassian noise sourced from thermal, shot and flicker types. Burst errors are due to impulse noise, which are characterized by long quiet intervals followed by high amplitude burst. Such errors are

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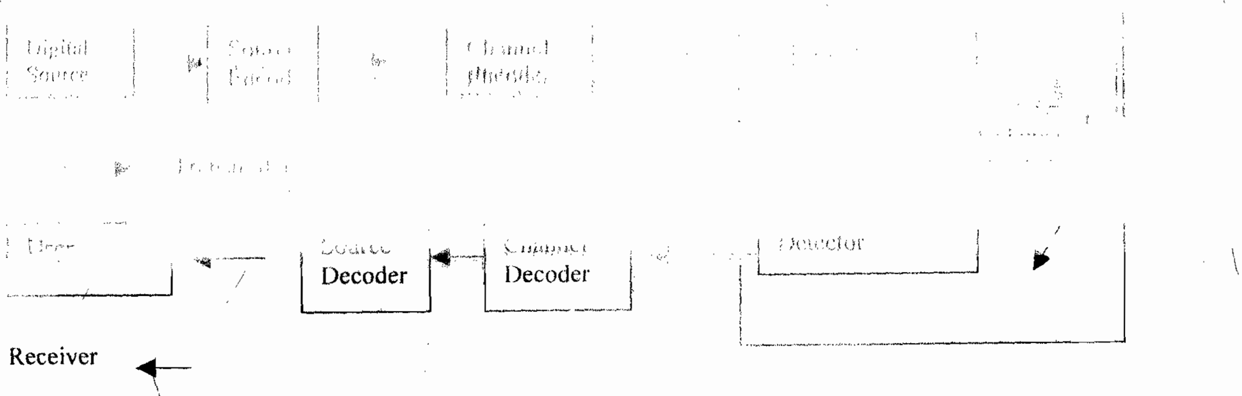

Fig. 1 Block diagram of a digital communication system emp[illegible] the coding scheme.

reduced using two prominent methods – the Forward-acting error correction (FEC) method and the Automatic retransmission request (ARQ) method.

With FEC, errors are corrected at the receiver end while the ARQ accepts or discards a received message sequence depending on its authenticity and reply the transmitter through a feedback channel. These two are used separately but sometimes cascaded when necessary as in digital audio tracking system (Posa 1981).

The general features of any error control code include the code length-which specifies the length of a codeword, coderate-which states the rate at which codewords are transmitted, and the correction properties associated with the distance.

The thrust of this paper is to study error control coding schemes including the design and implementing techniques as applicable to a wide range of message formats in digital communication. The designed error control codes – Hamming code and Parity check code, were selected because of their flexibility and versatility in application with most digital systems

ERROR CONTROL CODING SCHEMES

Error control codes are classified into five – Block codes, systematic codes, Cyclic codes, Convolutional codes and the Independent error control codes (Dupontiel et. al. 1988; Haykin, 1988).

In Block codes, each k-block of message bits is encoded into a block of n > k bits by adding redundancies called check bits. Error control codes in this class include the Repetition code, Maximum length coded and the Hamming code.

In Systematic codes, the stream, of transmitted data, which comprises the k information bits and the n-k checkbits, are grouped into blocks of 23. Thus, a Systematic code operates on multiple bits. One of such is the Reed-Solomon (R-S) code.

A Cyclic code is formed when a circular permutation of S-codewords is carried out to yield another codeword. Error control codes in this class include the Cyclic redundancy code, Bose-Chandhuri-Hocquenghen (BCH) code and the Expurgated code.

In convolutional codes, check bits are continuously interleaved with information bits. This implies that code digits generated by an encoder at a time depend not only on the k-message digits at that time but also on the preceding operation. Codes in this class include the Concatenated code, and the Trellis code.

The independent error control codes have their properties sparsely distributed in the four aforementioned classes. They have no restriction with respect to codelength and coderate. Codes in this class include the Constant ratio code, Parity check code and the Lattice code.

Shanmugam (1979) reported that Shannon theory states that for coded or uncoded information, a block of k message digits is transmitted in a duration time Tw given as $Tw = k/r_b$ where r_b is the message bit rate. The bit rate into the channel is given as $r_c = r_b(n/k)$. This implies that the transmitted bit duration in the coded case must be less than the bit duration in the uncoded case. If the transmitter power S_{av} (also called the signal strength) is held constant, then the energy per bit is decreased by the use of coding and the probability of decoding a transmitted but increases.

It is expedient to determine whether coding results in a significant reduction in the probability of incorrectly decoding a message bit. This is expressed as a probability of error P_e of a binary systematical with a fairly constant average signal power, S_{av}, over a defined noise spectral density η for both coded and uncoded case and is given as (Shanmugam 1979):

$$P_e = Q\left[\frac{2Sav}{\eta r_b}\right]^{1/2} \quad \text{..} \quad (1)$$

where r_b is the message bit rate and Q is a constant (≈ 0.0023) (Shanmugam, 1979).

Where error control is employed, the probability of error is less than one. For arbitrary values of 10^{-1} to 10^{-4} and a constant ratio of signal power to noise power or range 2 to 10dB, plots of q_u, q_c, P_u and P_c are shown in Fig. 5.

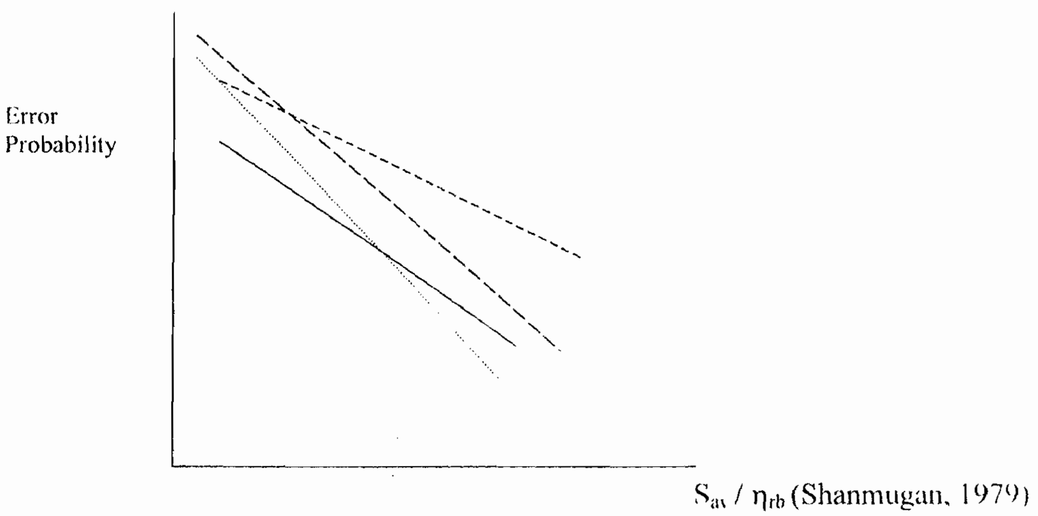

Fig. 2. Channel bit error probabilities and word error probabilities for coded and uncoded systems

Shanmugam (1979) showed that:
Channel bit error probability for the coded system q_c is

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Channel bit error probability for the uncoded system q_u is

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Probability of incorrectly decoding a message bit (k-block) in the uncoded system P_u is.

$$P_u = kq_u \quad \text{..} (4)$$

Probability of incorrectly decoding a message bit (k-block) in the coded system P_c is

$$P_c = kq_c \quad \text{..} (5)$$

DESIGN AND IMPLEMENTATION

THE HAMMING CODE

The Hamming code is a perfect one error detecting/correcting linear block code with a minimum distance d_{min} = 3. In Block codes, a block of k information bits is followed by a group of r check bits. Thus, a code ε(N,K) is made up of 2^k codewords {C_i} a set of which forms a group under modulo – 2 addition. A correspondence between a message and a codeword takes the form (Kwaha, 1999).

$$C_i = M_i G \qquad (6)$$

where $i = 1,2,\ldots\ldots 2^k$, $m_i, = m_{i,0}, \ldots . m_{i,k-1}$ and $C_i = C_{i,0}, \ldots 1\ C_{i,N-1}$

And G is the generator matrix, and m the message bits.

A linear block code can correct up to (d_{min} - 1)/2 and detect up to d_{min} –1 errors in each codeword (Dupontiel et al, 1988). The Hamming code is well adapted to the American 7-Bit ASCII character used in digital communication systems. The structure for a 7-bit codeword is shown in table 1.

Table 1. The Hamming codeword structure for a 7-bit ASCII character.

Bit No.	1	2	3	4	5	6	7
Bit	P_1	P_2	X_3	P_4	X_5	X_6	X_7

P_1 is parity bit selected to establish even parity over bits1,3,5 and 7. P_2 does the same over bits 2,3,6and 7. While P_4 is over bits 4,5,6 and 7. X_3,X_5,X_6 and X_7 are information bits. The design and implementation of this code was accomplished using three Exclusive OR (XOR) gates each obtained as IC SN 7486 (Fig. 3) and operated on 5v d.c power supply.

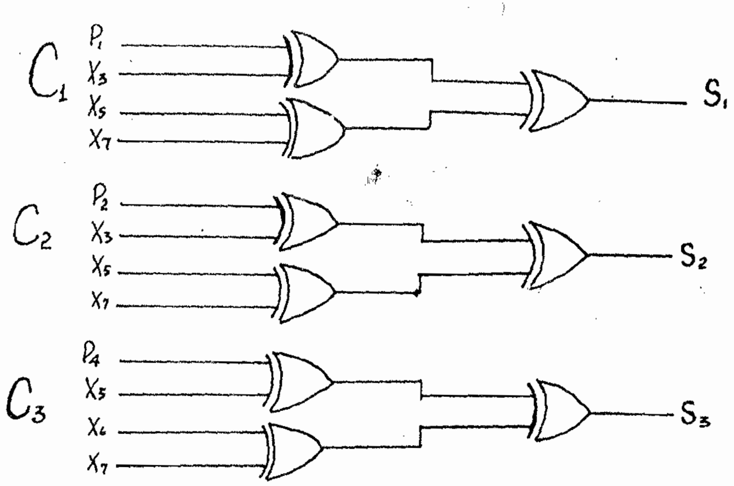
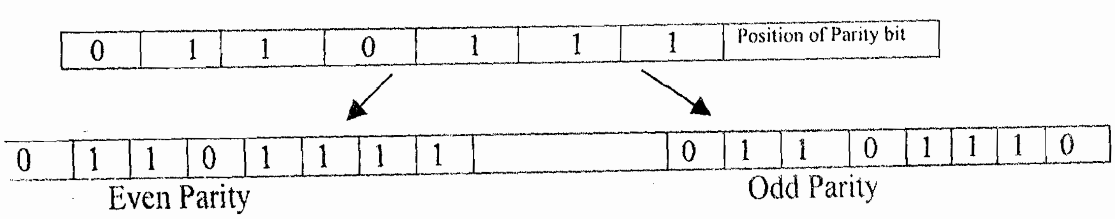


Fig. 3. Design and Implementation of the Hamming Code.

C_1, C_2 and C_3 represent the XOR trees while S_1, S_2, and S_3 represent their respective outputs. The output is read in the sequences S_3, S_2, S_1; the binary correspondence of which identifies the bit in error. Nine Hamming coded messages were received with their corresponding outputs as shown in Table 2.

THE PARITY CHECK CODE

The Parity check code is obtained by including an extra digit with the information bits such that the total number of 1's in the data is fixed – **even** or **odd** as may be required.



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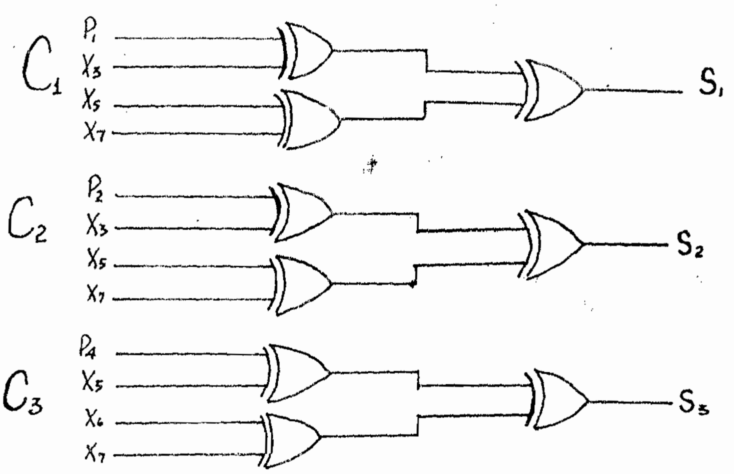

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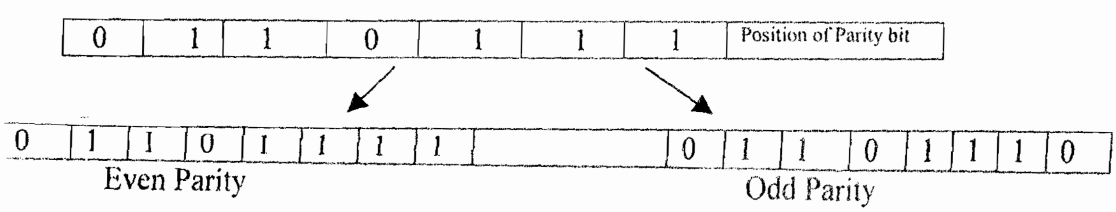

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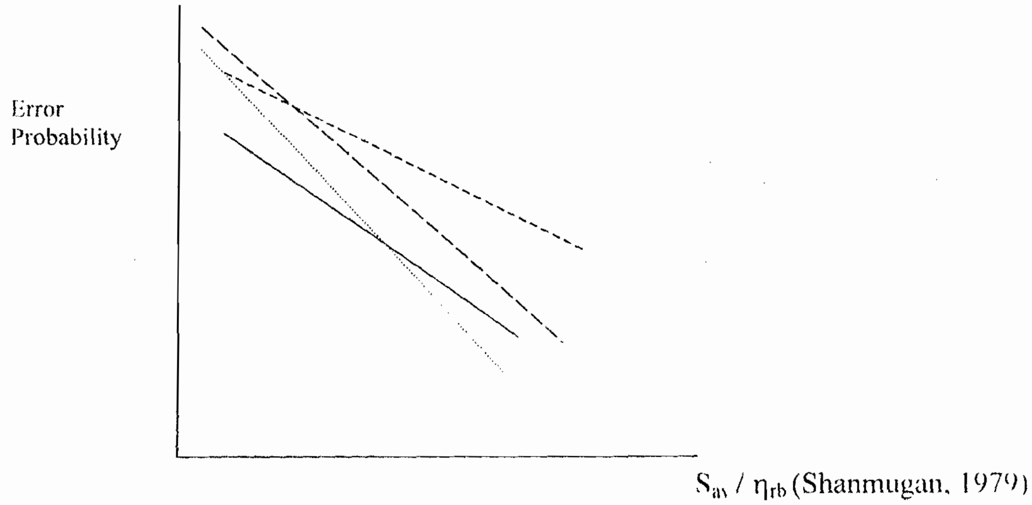

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ARQ scheme for satellite channels.

In the Cyclic and Systematic codes, the generator matrix is used in encoding while the parity check matrix is used in decoding. This implies that the encoder stores the generator matrix and performs the binary arithmetic to produce the check bits. Posa (1981) highlighted that this system adapts well to burst error types making this approach suitable in digital audio and Video tracking.

Error rate in speech transmission by pulse code modulation (PCM), is substantially reduced by using the convolutional codes (Haykin, 1988). This is used in Laser systems using the optical disc and Laser videocassette recorders. This technology has been extended to the compact disc stereo system now. When this system is coupled to a block code, it suits well as in computer back up storage systems.

In the telephone channel link, the overall quality of the link requires an inner and an outer stage coding to protect and control data traffic. Paul and Young (1989) reported that for such systems, the concatenated code of the class of convolutional codes is optimum.

Generally the fundamental principle of error control coding is applicable to all digital systems. Present day technology requires that these coding schemes be inbuilt into various systems compliant to the digital signal processing technique -choosing where necessary coding schemes adaptable to specific systems.

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