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SOME FIXED POINT THEOREMS FOR CONTRACTIVE CONDITIONS IN A G-METRIC SPACE

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ABSTRACT

In this paper, some fixed point theorems were proved, to show the existence and uniqueness of a fixed point under some weaker contractive conditions in a complete G-metric space settings. Moreover, we obtain the G-Cauchy sequence for the unique fixed point. Our results extend and refine some recent results in the literature.

KEYWORDS AND PHRASES: G-metric, G- Cauchy, G- limit, G-continuous and weak contractive conditions.

INTRODUCTION

Most of the problems that occur in life are nonlinear in nature but fixed point theory depends on the linear structure of normed linear spaces or Banach spaces setting. However, a nonlinear framework for fixed point theory is a metric space embedded with a structure.

In 2017, Rauf et.al. [10] introduced some new implicit Kirk-type iterative schemes in generalized convex metric spaces in order to approximate fixed points for general class of quasicontractive type operators. The strong convergence, T-stability, equivalency, data dependence and convergence rate of these results were explored. Their iterative schemes are faster and better, in term of speed of convergence, than their corresponding results in the literature. The results also improved and generalized several existing iterative schemes in the literature and they provided analogues of the corresponding results of other spaces, namely: normed spaces, CAT(0) spaces and so on.

Mustafa and Sim in 2006, [8] Introduced a new notion of generalized metric space called G-metric space, after proving that most of the result concerning the topological properties of D-metric space were incorrect. To repair this setback, they gave a more appropriate notion of a generalized metrics, called G-metric space. For more details on G-metric space, see [6 & 9] and the reference therein.

2. PRELIMINARIES AND DEFINITIONS

In this section, we recollect some basic definitions and overview of the fundamental results.

Definition 1 (Mustafa and Sim [8]): A G-metric space is a pair (X, G) where X is a nonempty set and $G : X \times X \times X \to [0, \infty)$ is a function such that, for all $x, y, z, a \in X$, the following conditions are fulfilled:

(G1) $G(x, y, z) = 0 \;\; if\; x = y = z$ -- **(2.1)**
(G2) $G(x, x, y) > 0 \; for\; all\; x, y \in X\; with\; x \neq y;$ --- **(2.2)**
(G3) $G(x, x, y) \leq G(x, y, z)\; for\; all\; x, y, z \in X\; with\; z \neq y$ --- **(2.3)**
(G4) $G(x, y, z) = G(x, z, y) = G(y, z, x)$ -- **(2.4)**
(G5) $G(x, y, z) \leq G(x, a, a) + G(a, y, z)$ -- **(2.5)**

In such a case, the function G is called a G-metric on X.

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Lemma 2.1 (Mustafa and Sim [8]): Let (X,G) be a G-metric space. Then, for any $x,y,z,a \in X$, the following properties hold:

1. $G(x,y,z) \le G(x,x,y) + G(x,x,z)$;
2. $G(x,y,z) \le G(x,a,a) + G(y,a,a) + G(z,a,a)$;
3. $|G(x,y,z) - G(x,y,a)| \le \max\{G(a,z,z), G(z,a,a)\}$;
4. If $n \ge 2$ and $x_1, x_2 \dots x_n \in X$, then

$$G(x_1,x_n,x_n) \le \sum_{i=1}^{n-1} G(x_i,x_{i+1},x_{i+1})$$

and

$$G(x_1,x_1,x_n) \le \sum_{i=1}^{n-1} G(x_i,x_i,x_{i+1}).$$

5. If $G(x,y,z) = 0$, then $x = y = z$;
6. $G(x,y,z) \le G(x,a,z) + G(a,y,z)$;
7. $G(x,y,z) \le \frac{2}{3}[G(x,y,a) + G(x,a,z) + G(a,y,z)]$;
8. If $x \in X\{z,a\}$, then $|G(x,y,z) - G(x,y,a)| \le G(a,x,z)$; and
9. $G(x,y,y) \le 2G(x,y,z)$.

Definition 2.2: (Agarwal et.al. [1]): Let (X,G) be a G-metric space, let $x \in X$ be a point and let $\{x_n\} \subseteq X$ be a sequence. We say that:

1. $\{x_n\}$ G-converges to x, and we write $\{x_n\} \to x$, if $\lim_{n,m\to\infty} G(x_n,x_m,x) = 0$, that is, for all $\varepsilon > 0$ there exists $n_0 \in \mathbb{N}$ satisfying $G(x_n,x_m,x) \le \varepsilon$ for all $n,m \in \mathbb{N}$ such that $n,m \ge n_0$ (in such case, x is the G-limit of $\{x_n\}$;

2. $\{x_n\}$ is G-Cauchy if $\lim_{n,m\to\infty} G(x_n,x_m,x_k) = 0$, that is, for all $\varepsilon > 0$ there exists $n_0 \in \mathbb{N}$ satisfying $G(x_n,x_m,x_k) \le \varepsilon$ for all $n,m,k \in \mathbb{N}$ such that $n,m,k \ge n_0$;

3. (X,G) is complete if every G-Cauchy sequence in X is G-convergent in X.

Definition 2.3: (Agarwal et.al. [1]): Let (X,G) be a G-metric space, let $\{x_n\} \subseteq X$ be a sequence and let $x \in X$. Then the following conditions are equivalent.

1. $\{x_n\}$ G-convergents to x;
2. $\lim_{n,m\to\infty} G(x_n,x_m,x_m) = 0$;
3. $\lim_{n,m\to\infty, m\ge n} G(x_n,x_m,x_n) = 0$;
4. $\lim_{n,m\to\infty, m>n} G(x_n,x_m,x_m) = 0$;
5. $\lim_{n,m\to\infty} G(x_n,x_n,x_m) = 0$;
6. $\lim_{n,m\to\infty, m\ge n} G(x_n,x_n,x_m) = 0$;
7. $\lim_{n,m\to\infty, m>n} G(x_n,x_n,x_m) = 0$;
8. $\lim_{n\to\infty} G(x_n,x_{n+1},x_{n+1}) = 0$ and $\lim_{n,m\to\infty, m>n} G(x_n,x_{n+1},x_m) = 0$.

Definition 2.3: (Agarwal et.al. [1]): Let (X,G) be a G-metric space. We say that:

a. A mapping $T : X \to X$ is G-continuous at $x \in X$ if $\{Tx_m\} \to Tx$ for all sequence $\{x_n\} \subseteq X$ such that $\{x_m\} \to x$;
b. A mapping $F : X^n \to X$ is G-continuous at $(x_1, x_2, \dots, x_n) \in X^n$ if

$$\{F(x_1^m, x_2^m, \dots, x_n^m) \to F(x_1, x_2, \dots, x_n)$$

For all sequence $\{(x_1^m, x_2^m, \dots, x_n^m)\} \subseteq X^n$ such that $\{x_i^m\} \to x_i$ for all $i \in \{1,2,\dots,n\}$;
c. a mapping $H : X^n \to X^m$ is G-continuous at $(x_1, x_2, \dots, x_n) \in X^n$ if $\pi_i^m \circ H : X^n \to X$ is G-continuous at $(x_1, x_2, \dots, x_n)$ for all $i \in \{1,2,\dots,m\}$, where $\pi_i^m : X^m \to X$ is the ith-projection of X^m onto X (that is, $\pi_i^m(a_1, a_2, \dots, a_m) = a_i$ for all $(a_1, a_2, \dots, a_m) \in X^m$).

Theorem 2.1: (Agarwal et.al. [1]): If (X,G) be a G metric space. Then the function $G(x,y,z)$ is jointly continuous in all three of its variables, that is, if $x,y,z \in X$ and $\{x_n\}, \{y_n\}, \{z_n\} \subseteq X$ are sequences in X such that $\{x_n\} \xrightarrow{G} x, \{y_n\} \xrightarrow{G} y$ and $\{z_n\} \xrightarrow{G} z$, then $\{G(x_m, y_m, z_m)\} \to G(x,y,z)$.

3. MAIN RESULT

Here, we study some results related to unique fixed point p, and show that mapping T is G- continuous at p. We shall establish the results in sequel as follows.

Theorem 3.1: Let (X,G) be a complete G-metric space and let $T:X\to X$ be a mapping satisfying the following conditions:

$$G(Tx,Ty,Tz)\le \alpha G(x,Tx,Tx)+\varphi G(x,y,z) \quad \text{------} \quad 3.1$$
$$G(Tx,Ty,Tz)\le \alpha G(x,x,Tx)+\varphi G(x,y,z) \quad \text{------} \quad 3.2$$

For all $x,y,z\in X$ where $0\le\alpha+\varphi<1$, then T has a unique fixed point $p\ in\ T$; $Tp=p$.

Proof: Let $\alpha\ and\ \varphi$ be a contraction constant of the mapping T; let x_0 be an arbitrary but fixed element in X. Define the sequence of iterate $\{x_n\}$ in X as

$$x_n=T^n x_0 \qquad \text{For all } n\ge 1 \quad \text{------} \quad 3.3$$

If $x_{n+1}=y_n=z_n$ and since T is a contraction satisfying (3.1) or (3.2) we have

$$G(x_n,x_{n+1},x_{n+1})=G(Tx_{n-1},Tx_n,Tx_n) \quad \text{------} \quad 3.4$$
$$\le \alpha G(x_{n-1},x_n,x_n)+\varphi G(x_n,x_{n+1},x_{n+1})$$
$$=\alpha G(Tx_{n-2},Tx_{n-1},Tx_{n-1})+\varphi G(Tx_{n-1},Tx_n,Tx_n)$$
$$\le \alpha^2 G(x_{n-2},x_{n-1},x_{n-1})+\varphi^2 G(x_{n-1},x_n,x_n) \quad \text{------} \quad 3.5$$
$$=\alpha^2 G(Tx_{n-3},Tx_{n-2},Tx_{n-2})+\varphi^2 G(Tx_{n-2},Tx_{n-1},Tx_{n-1})$$
$$\le \alpha^3 G(x_{n-3},x_{n-2},x_{n-2})+\varphi^3 G(x_{n-2},x_{n-1},x_{n-1})$$

Continue iteratively leads to

$$\le \alpha^n G(x_0,x_1,x_1)+\varphi^n G(x_1,x_2,x_2) \quad \text{------} \quad 3.6$$

Hence

$$G(x_n,x_{n+1},x_{n+1})\le \alpha^n G(x_{n-1},x_n,x_n)+\varphi^n G(x_n,x_{n+1},x_{n+1})$$

and

$$G(x_n,x_{n+1},x_{n+1})\le \frac{\alpha^n}{1-\varphi^n}\, G(x_{n-1},x_n,x_n) \quad \text{------} \quad 3.7$$

If we let $k^n=\frac{\alpha^n}{1-\varphi^n}$ For all $n\in\mathbb{N}$ where $0<k^n<1$ we have

$$G(x_n,x_{n+1},x_{n+1})\le k^n G(x_{n-1},x_n,x_n)$$
$$\le k^n G(x_0,x_1,x_1) \quad \text{------} \quad 3.8$$

For all $n,m\in\mathbb{N}, m>n$, we have

$$G(x_n,x_m,x_m)\le G(x_n,x_{n+1},x_{n+1})+G(x_{n+1},x_{n+2},x_{n+2})+\cdots+G(x_{m-1},x_m,x_m)$$
$$\le k^n G(x_0,x_1,x_1)+k^{n+1}G(x_0,x_1,x_1)+\cdots+k^{m-1}G(x_0,x_1,x_1)$$
$$\le (k^n+k^{n+1}+\cdots+k^{m-1})G(x_0,x_1,x_1)$$

Applying sum of geometric progression we have

$$G(x_n,x_{n+1},x_{n+1})\le \frac{k^n}{1-k}G(x_0,x_1,x_1) \quad \text{------} \quad 3.9$$

$\lim G(x_n,x_m,x_m)=0\ \ as\ n,m\to\infty$. Therefore the sequence $\{x_n)$ is G-Cauchy sequence.

To show the uniqueness, suppose $p\neq q$ such that $Tq=q$. Then

$$G(p,q,q)\le \alpha G(p,Tp,Tp)+\varphi G(p,q,q) \quad \text{------} \quad 3.10$$

This implies $p=q$. ■

Theorem 3.2: Let (X,G) be a complete G-metric space, and let $T:X\to X$ be a mapping satisfying (3.11) for which there exist a monotonically decreasing function a,b,c from $(0,\infty)$ into $[0,1]$ satisfying $a(t)+b(t)+c(t)<1$ such that, for each $x,y\in X;\ \ x\neq y$.

$$G(Tx,Ty,Ty)\le aG(x,y,y)G(x,Tx,Tx)+bG(x,y,y)G(y,Ty,Ty)+cG(x,y,y)G(x,y,y) \quad \text{------} \quad 3.11$$

Equivalently,

$$G(Tx,Ty,Ty)\le aG(x,x,y)G(x,x,Tx)+bG(x,x,y)G(y,y,Ty)+cG(x,x,y)G(x,x,y) \quad \text{------} \quad 3.12$$

Then, there exists a unique fixed point p in T: $Tp=p$.

Proof: For all $x_0 \in X$ and Picard iteration $x_n = T^n x_0 \quad n \geq 1$, let $x_{n+1} = y_n$ and since T is a contraction satisfying (3.11) or (3.12) we have

$$\begin{aligned} G(x_n, x_{n+1}, x_{n+1}) &= G(Tx_{n-1}, Tx_n, Tx_n) \\ &\leq aG(x_{n-1}, x_n, x_n)G(x_{n-1}, x_n, x_n) + bG(x_{n-1}, x_n, x_n)G(x_n, x_{n+1}, x_{n+1}) + cG(x_{n-1}, x_n, x_n)G(x_{n-1}, x_n, x_n) \\ &= aG(Tx_{n-2}, Tx_{n-1}, Tx_{n-1})G(Tx_{n-2}, Tx_{n-1}, Tx_{n-1}) + bG(Tx_{n-2}, Tx_{n-1}, Tx_{n-1})G(Tx_{n-1}, Tx_n, Tx_n) \\ &\quad + cG(Tx_{n-2}, Tx_{n-1}, Tx_{n-1})G(Tx_{n-2}, Tx_{n-1}, Tx_{n-1}) \\ &\leq a^2G(x_{n-2}, x_{n-1}, x_{n-1})G(x_{n-2}, x_{n-1}, x_{n-1}) + b^2G(x_{n-2}, x_{n-1}, x_{n-1})G(x_{n-1}, x_n, x_n) \\ &\quad + c^2G(x_{n-2}, x_{n-1}, x_{n-1})G(x_{n-2}, x_{n-1}, x_{n-1}) \\ &= a^2G(Tx_{n-3}, Tx_{n-2}, Tx_{n-2})G(Tx_{n-3}, Tx_{n-2}, Tx_{n-2}) + b^2G(Tx_{n-3}, Tx_{n-2}, Tx_{n-2})G(Tx_{n-2}, Tx_{n-1}, Tx_{n-1}) \\ &\quad + c^2G(Tx_{n-3}, Tx_{n-2}, Tx_{n-2})G(Tx_{n-3}, Tx_{n-2}, Tx_{n-2}) \\ &\leq a^3G(x_{n-3}, x_{n-2}, x_{n-2})G(x_{n-3}, x_{n-2}, x_{n-2}) + b^2G(x_{n-3}, x_{n-2}, x_{n-2})G(x_{n-2}, x_{n-1}, x_{n-1}) \\ &\quad + c^2G(x_{n-3}, x_{n-2}, x_{n-2})G(x_{n-3}, x_{n-2}, x_{n-2}) \end{aligned}$$

Continue iteratively in this manner, then

$$\begin{aligned} &G(x_n, x_{n+1}, x_{n+1}) \\ &\leq a^nG(x_0, x_1, x_1)G(x_0, x_1, x_1) + b^nG(x_0, x_1, x_1)G(x_1, x_2, x_2) + c^nG(x_0, x_1, x_1)G(x_0, x_1, x_1) \quad \text{------} \quad 3.13 \\ &\leq \frac{(a^n + c^n)G(x_0, x_1, x_1)G(x_0, x_1, x_1)}{1 - b^nG(x_1, x_2, x_2)} \end{aligned}$$

By equating $j^n = a^n + c^n$, $g^n = 1 - b^n$ and $\alpha^n = \frac{j^n}{g^n}$. *where* $0 \leq \alpha^n < 1$ We have

$$G(x_n, x_{n+1}, x_{n+1}) \leq \alpha^n G(x_0, x_1, x_1) \quad \text{------} \quad 3.14$$

For all $n, m \in \mathbb{N}$, $m > n$, we have

$$\begin{aligned} G(x_n, x_m, x_m) &\leq G(x_n, x_{n+1}, x_{n+1}) + G(x_{n+1}, x_{n+2}, x_{n+2}) + \cdots + G(x_{m-1}, x_m, x_m) \\ &\leq \alpha^n G(x_0, x_1, x_1) + \alpha^{n+1}G(x_0, x_1, x_1) + \cdots + \alpha^{m-1}G(x_0, x_1, x_1) \\ &\leq (\alpha^n + \alpha^{n+1} + \cdots + \alpha^{m-1})G(x_0, x_1, x_1) \\ G(x_n, x_{n+1}, x_{n+1}) &\leq \frac{\alpha^n}{1-\alpha}G(x_0, x_1, x_1) \quad \text{------} \quad 3.15 \end{aligned}$$

$\lim G(x_n, x_m, x_m) = 0 \quad as\ n, m \to \infty$. Therefore, the sequence $\{x_n\}$ is G-Cauchy sequence.

To show the uniqueness, suppose $p \neq q$ such that $Tq = q$. Then

$$G(p, q, q) \leq aG(p, q, q)G(p, Tp, Tp) + bG(p, q, q)G(q, Tq, Tq) + cG(p, q, q)G(p, q, q) \quad \text{------} \quad 3.16$$

This implies $p = q$. ■

Theorem 3.4. Let (X, G) be a complete G-metric space, and let $T : X \to X$ be a mapping satisfying (3.17) for which there exist nonnegative number a, b, c satisfying $a + b + c < 1$ such that, for each $x, y \in X$,

$$G(Tx, Ty, Tz) \leq aG(x, Tx, Tx) + bG(y, Ty, Ty) + cG(x, y, z) \quad \text{------} \quad 3.17$$

Equivalently

$$G(Tx, Ty, Tz) \leq aG(x, x, Tx) + bG(y, y, Ty) + cG(x, y, z) \quad \text{------} \quad 3.18$$

For all $x, y, z \in X$ where $0 \leq a + b + c < 1$, then T has a unique fixed point (say p, that is $Tp = p$) and T is G-continuous at p.

Proof: Suppose that Tsatisfies condition (3.17), then for all $x, y \in X, we\ have$

$$\begin{aligned} G(Tx, Ty, Ty) &\leq aG(x, Tx, Tx) + bG(y, Ty, Ty) + cG(x, y, y) \\ & \quad \text{------} \quad 3.19 \\ G(Tx, Tx, Ty) &\leq aG(x, x, Tx) + bG(y, y, Ty) + cG(x, x, y) \end{aligned}$$

Let $x_{n+1} = y_n$ and since T is a contraction satisfying (3.11) or (3.12) we have

$$\begin{aligned} G(x_n, x_{n+1}, x_{n+1}) &= G(Tx_{n-1}, Tx_n, Tx_n) \\ &\leq aG(x_{n-1}, x_n, x_n) + bG(x_n, x_{n+1}, x_{n+1}) + cG(x_n, x_{n+1}, x_{n+1}) \\ &= aG(Tx_{n-2}, Tx_{n-1}, Tx_{n-1}) + bG(Tx_{n-1}, Tx_n, Tx_n) + cG(Tx_{n-1}, Tx_n, Tx_n) \\ &\leq a^2G(x_{n-2}, x_{n-1}, x_{n-1}) + b^2G(x_{n-1}, x_n, x_n) + c^2G(x_{n-1}, x_n, x_n) \\ &= a^2G(Tx_{n-3}, Tx_{n-2}, Tx_{n-2}) + b^2G(Tx_{n-2}, Tx_{n-1}, Tx_{n-1}) + c^2G(Tx_{n-2}, Tx_{n-1}, Tx_{n-1}) \\ &\leq a^3G(x_{n-3}, x_{n-2}, x_{n-2}) + b^3G(x_{n-2}, x_{n-1}, x_{n-1}) + c^3G(x_{n-2}, x_{n-1}, x_{n-1}) \quad \text{------} \quad 3.20 \end{aligned}$$

Continue iteratively implies

$$G(x_n, x_{n+1}, x_{n+1}) \leq a^n G(x_0, x_1, x_1) + b^n G(x_1, x_2, x_2) + c^n G(x_1, x_2, x_2)$$
$$\leq \frac{a^n}{1-b^n-c^n} G(x_0, x_1, x_1) \quad \text{-----} \quad 3.21$$

$$let\ \alpha^n = \frac{a^n}{1-b^n-c^n} \quad where\ 0 \leq \alpha^n < 1$$

Then

$$G(x_n, x_{n+1}, x_{n+1}) \leq \alpha^n G(x_0, x_1, x_1) \quad \text{-----} \quad 3.22$$

For all $n, m \in \mathbb{N},\ m > n$

But

$$G(x_n, x_m, x_m) \leq G(x_n, x_{n+1}, x_{n+1}) + G(x_{n+1}, x_{n+2}, x_{n+2}) + \cdots + G(x_{m-1}, x_m, x_m)$$
$$\leq \alpha^n G(x_0, x_1, x_1) + \alpha^{n+1} G(x_0, x_1, x_1) + \cdots + \alpha^{m-1} G(x_0, x_1, x_1)$$
$$\leq (\alpha^n + \alpha^{n+1} + \cdots + \alpha^{m-1}) G(x_0, x_1, x_1)$$
$$G(x_n, x_{n+1}, x_{n+1}) \leq \frac{\alpha^n}{1-\alpha} G(x_0, x_1, x_1) \quad \text{-----} \quad 3.23$$

$\lim G(x_n, x_m, x_m) = 0 \quad as\ n, m \to \infty$. Therefore, the sequence $\{x_n\}$ is G-Cauchy sequence.

To show the uniqueness, suppose $p \neq q$ such that $Tq = q$. Then

$$G(p, q, q) \leq aG(p, Tp, Tp) + bG(q, Tq, Tq) + cG(p, q, q) \quad \text{-----} \quad 3.24$$

This implies $p = q$. ■

CONCLUSION

This work is an extension of Banach fixed point theorem to G-metric space. The existence and uniqueness of some fixed points under some weaker contractive conditions in complete G-metric space settings were obtained. The results are therefore refinements and generalizations of some recent results in the literature.

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