

# MULTIPLE ATTENUATION USING EIGENVALUE DECOMPOSITION

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## ABSTRACT

Multiple reflections constitute one of the most troublesome forms of coherent noise in seismic exploration, especially in marine surveys. There are many approaches to attenuating or suppressing multiples, but none can remove all multiple reflections under all conditions. We have developed two new methods to attenuate multiple reflections using the Karhunen-Loeve transform which extract coherent information from multichannel input data in a least-squares sense. They supplement conventional methods.

The first is the 'localized multiple attenuation' method. Its aim is to attenuate a single (typically water bottom) multiple reflection in deep water or multisource marine acquisition. It is similar to velocity filter methods but is not as prone to spatial aliasing and edge effects. The criterion of local coherency gives the method the flexibility to discriminate between primary and multiple reflections.

The second is the 'targeted multiple attenuation' method. It uses the similarity between a primary reflection and its multiple in both the spatial and time directions. Its aim is to remove a particular multiple reflection associated with a specified primary rather than all the multiple energy for an entire trace. The method can be applied pre- or post-stack, and to 2D or 3D data.

## INTRODUCTION

The Karhunen-Loeve (K-L) transform is well known in image processing (Oppenheim 1978). It decomposes an image into principal components that are ordered on the basis of spatial correlation. The K-L transform has many applications in seismic data processing (e.g., Hemon and Mace, 1978; Jones and Levy, 1987; Zhang et al., 1998). These typically reconstruct the data from a subset of the principal components. Spatially uncorrelated components are ignored, leaving a clear and coherent image. For the purpose of multiple suppression, we utilize the energy packing property of the K-L transform. The idea is to segregate the energy associated with the multiples onto a few eigenvectors. A data subtracted with the reconstructed multiple from these eigenvectors where uncoherent energy is muted should be essentially multiple-free.

In this paper we discuss two different approaches for attenuating multiple reflections using the K-L transform. The first, which we call localized multiple attenuation, aims to attenuate a single, typically water bottom, multiple reflection in deep water from pre-stack data. The localization comes from that the method operates on a localization comes from that the method operates on a local window and uses local energy on eigenvectors. The second, which we call targeted multiple attenuation, aims to remove pegleg multiple reflections from pre- or post-stack data. It is an interactive target-oriented process and has no restriction to flat water bottom (Kneib and Bardan, 1994; Manin and Spitz, 1995).

## METHOD

$$\phi_j(t) = \sum_{i=1}^n a_{ij} x_i(t), j = 1 \dots n, \quad (1)$$

Assume that we have a set of real signal  $x_i(t)$ ,  $i = 1 \dots n$ . Define a transformed set  $\phi_j(t)$  and a transformation (rotation) matrix  $A$  such that

Where  $a_{ij}$  are the elements of  $A$ . The signals  $\phi_j(t)$  are chosen such that they form an orthogonal basis, so that each signal  $x_i(t)$  can be expressed exactly as

$$x_i(t) = \sum_{j=1}^n b_{ij} \phi_j(t), i = 1 \dots n, \quad (2)$$

Where  $b_{ij}$  are the elements of  $B$ . Equations (1) and (2) are the forward and inverse transform relations of the Karhunen-Loeve transformation. The transformation matrices  $A$  and  $B$  are data dependent. Their rows consist of the normalized eigenvectors of the data covariance matrix  $T$ , with elements

$$\gamma_{ij} = \int_0^T x_i(t) x_j(t) dt. \quad (3)$$

The size of an eigenvalue is a measure of the amount of coherent present in its associated eigenvector. 'Coherent' here means events which are horizontally similar in a trace-to-trace sense.

Like many multiple-suppression processes, the forward K-L transform projects the data into a model parameter space where crossing primary and multiple events are better separated. Multiple events are windowed in the model parameter space and reconstructed in the data space using an inverse K-L transform. Now the modeled multiples are subtracted from the original data to obtain a 'primary only' gather.

### Localized multiple attenuation

Velocity filtering is the most common method for suppression deep water bottom multiple reflections.

After the forward transform the flattened multiples are concentrated on the first eigenvector, while unflattened primaries span many eigenvectors.

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Different normal moveout, but can also distort the primaries, particularly where the normal moveout differences between primary and multiple are small (i.e., the near offset data). Velocity filtering can also perform poorly when multiple reflections are spatially aliased even after NMO correction. This problem often occurs in multisource marine acquisition. Localized multiple attenuation addresses the problem by using a K-L transform operating on a small window. The method therefore, is not as subject to spatial aliasing as velocity filtering and is able to discriminate between primaries and multiple reflections in regions of the data gather where velocity filtering may fail. The procedure is:

(1) From a velocity analysis, determine the stacking velocity of the multiple and correct for it using constant velocity NMO. The multiple arrivals are now more or less flat, whereas the primary events are under- or overcorrected and have increased curvature.

(2) Choose the target region for multiple suppression. Typically this is the entire seismic record below the water bottom reflection. Divide the target region into application windows and perform a forward K-L transform on each. Coherent multiple energy now appears predominantly on a single or a few eigenvectors.

(3) Window the coherent multiple energy on those dominant eigenvectors and reconstruct them using an inverse K-L transform. This reconstructed multiple is then averaged over application windows in both the trace and time directions to smooth out boundaries between operation windows.

(4) Subtract the reconstructed multiple from the input data and remove the NMO correction from step 1.

After the localized multiple attenuation any visible water bottom multiples have been removed and the amplitudes of the primaries (particular the middle event) have been restored.

**K-L multiple attenuation**

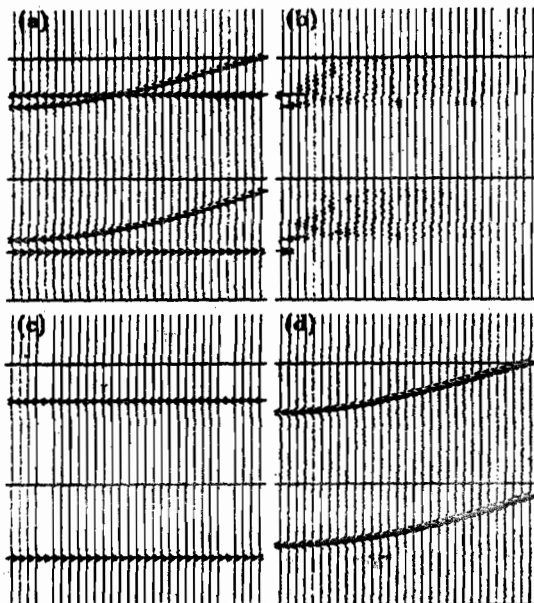


Fig. 1: Multiple-removal methodology using the K-L transform. (a) NMO-corrected input data using multiple velocity. (b) After the forward transform the flattened multiples are concentrated on the first eigenvector, while unflattened primaries span many eigenvectors. (c) After muting of the primaries, an inverse K-L produces an estimate of the multiples. (d) Estimated primaries after subtracting (c) from (a).

**Targeted multiple attenuation**

Targeted multiple attenuation uses the similarity of primary and multiple in both the spatial and time directions. The aim is to remove a particular multiple, rather than all the multiple energy for an entire trace. The dominant eigenvectors determined from the covariance matrix of the targeted multiple and its primary are used to reconstruct the multiple and primary in the region of the interest. To protect primary events, the packed multiple energy on those dominant eigenvectors must be windowed before reconstruction. To further protection primary events, a matching filter is designed which allows only those reconstructed multiples which adaptively match their primaries to be subtracted from the original data.

Targeted multiple attenuation is an interactive target oriented 'prediction and subtract' method whose objective is to remove pegleg multiples, although it could in principal remove multiples of any type, such as free surface, pegleg, or interbed. If we assume that arrive times follow the hyperbolic relationship, multiple and primary velocities are related by the Dix equation

$$(T_0 + nT_w)V_{MULT}^2 = T_{OV}^2 V_{NMO}^2 + nT_w V_w^2 \quad (4)$$

where  $n$  and  $V_{MULT}$  are the order and NMO velocity of the targeted pegleg multiple;  $t_0$  and  $V_{NMO}$  are the two-way vertical travel-time and NMO velocity of the associated primary reflecting; and  $T_0$  and  $V_w$  are the two-way vertical travel-time and velocity of water layer.

The method is as follows;

- 1) Identify the targeted multiple reflection and its associated primary in order to define corresponding temporal and spatial windows and

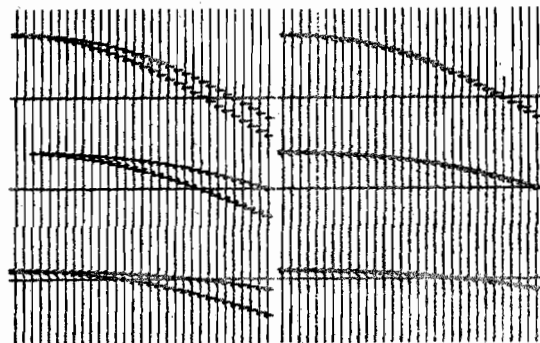


Fig. 2: Model data where the primary and water bottom multiple events interfere with each other. Before (left) and after (right) localized multiple attenuation.

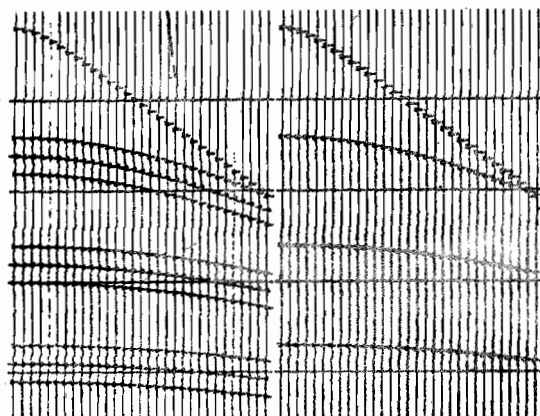


Fig. 3: Model data containing only primaries and their short-period pegleg multiples. Before (left) and after (right) targeted multiple attenuation.

- to derive the arrival times for both the multiple and primary.
- 2) Flatten both the multiple and primary reflections using the estimate arrive time, and perform an eigenvalue decomposition of the data covariance matrix computed at each window.
  - 3) Select a subset of eigenvectors corresponding to the most dominant eigenvalues, and window the packed primary and multiple energy on those eigenvectors.
  - 4) Reconstruct both the primary and multiple reflections using the windowed eigenvectors and build an adaptive filter between the reconstructed multiple and primary reflections in a piecewise manner. To compensate for inexact travel times, perform statics corrections to align multiple and primary.
  - 5) Obtain the residual signal by subtracting the adaptively filtered multiple reflection from the original data in the window. The output data is now averaged with overlapping adjacent data windows.

By using the multiple arrive time calculated with Equation (4) the targeted multiple attenuation method has removed all the pegleg multiples.

#### CONCLUSIONS

We have described the application of the K-L transform to suppress multiple reflections in seismic data. To protect the amplitudes of primary reflections, we window the packed multiple energy on the dominant eigenvectors before using them to reconstruct the multiple reflections.

By operating on a small window, the localized multiple attenuation method can discriminate between primary and multiple reflections at different time and offsets while not being subject to spatial aliasing and edge effects. Although horizon picking is involved, the targeted multiple attenuation method is self-adaptive in the sense that it accounts for lateral amplitude variations. Furthermore, the adaptive filter prevents distortions to primaries having lateral patterns similar to the multiples.

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