GLOBAL JOURNAL OF PURE AND APPLIED SCIENCES VOL. 13 NO. 1, JANUARY, 2007 : 125 - 132

MAGNETOHYDRODYNAMIC (MHD) MIXED CONVECTION OF A RADIATING AND VISCOUS DISSIPATING FLUID IN A HEATED VERTICAL CHANNEL

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(Received 27 February, 2006; Revision Accepted 20 July, 2006)

ABSTRACT

The paper investigates the effects of radiation on combined free and forced convection of a fully developed magnetohydrodynamic (MHD) fluid in a heated infinite vertical channel with viscous dissipation, when the temperatures of the channel walls are equal and vary linearly with the heights of the walls. The velocity and temperature fields together with the flow rate and rate of heat transfer are obtained by invoking the optically thin limit for the radiative heat flux in the energy equation and a regular perturbation for small Eckert number, Ec. The effect of radiation and magnetic parameters are presented graphically. Also, the variation of flow rate and the rate of heat transfer are given in tables The results show that increase in magnetic field parameter led to a decrease in the velocity profile (for fixed radiation); while for fixed magnetic field parameter increase in radiation parameter led to increase in the velocity profile. Also, simultaneous increase in radiation and magnetic field parameters led to a progressively flatter velocity. In addition, separate increases in both radiation and magnetic field parameters led to decrease in temperature within the channel with a flatter regime recorded for higher values of radiation and magnetic field parameter. Also, the flow rate increased with increase in radiation parameter, whereas the reverse is the case with increase in the magnetic field parameter Finally, the rate of heat transfer decreased with increase in both radiation and magnetic field parameters

KEYWORDS: *MHD, mixed convection, radiation, viscous dissipation, heated channel*

Nomenclature

(x, y)	dimensional Cartesian coordinates
(u^*, v^*)	dimensional velocity components
T_w	dimensional wall temperature
T	dimensional temperature
T_0	reference temperature
$\mathbf{B}_0$	constant transverse magnetic field
g	acceleration due to gravity
h	half width of the channel
c_p	specific heat capacity
B	Planck's function
$\mathbf{q}_r$	radiative heat flux
M^2	magnetic parameter
N^2	radiation parameter
Ec	Eckert number
Ra	Rayleigh number
Pr	Prandtl number
Greek symbols	
τ	a parameter
β^*	coefficient of volumetric expansion
σ	electrical conductivity
ν	kinematic viscosity
ρ	fluid density
δ_r	radiation absorption coefficient
λ_r	frequency
θ	dimensionless temperature
κ	thermal conductivity
ξ	measure of pressure above its hydrostatic value

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μ viscosity

η dimensionless y coordinate

INTRODUCTION

The study of combined free and forced convection flow in vertical channels has received considerable attention in recent times due to its application in solar collectors, and in the cooling of modern electronic devices. Problems of this nature have been studied by many researchers and excellent reviews on their properties and phenomenon may be found in literature. For example, Ostrach (1954) studied the problem of combined free and forced convection flow between parallel plates with viscous dissipation in which the wall temperature decreased with height; Morton (1960) and Toa (1960) considered the problem of laminar, fully developed flow in vertical pipes and channels with uniform heating of the plates, respectively. Barletta (1988, 1999) investigated the problem of fully developed combined free and forced convection in a vertical channel with viscous dissipation using the perturbation techniques. In another study, Greif et. al. (1971) obtained an exact solution to the problem of fully developed and radiating laminar convection in a heated channel without the effects of viscous dissipation and magnetic field.

The analysis of magnetohydrodynamic channel/duct flows of electrically conducting and viscous dissipating fluid under the action of transversely applied magnetic field is equally important and therefore has received attention in literature. This class of flow has immediate applications in MHD generators, pumps, accelerators, and in flow meters. Channel flows of Newtonian incompressible viscous fluid in the presence of magnetic fields and heat transfer have been studied. For example, Poots (1971) studied the problem of electrically conducting fluid flow between two heated parallel plates taking into consideration the effects of Joule heating and viscous dissipations; while Alagoa et. al. (1999) considered the effects of radiation and free convection on MHD flow through porous medium between two infinite parallel plates with time-dependent suction, but without dissipation. Umavathi and Malashetty (2005) considered the problem of laminar fully developed mixed convection in a vertical channel with symmetric and asymmetric boundary heating in the presence of viscous and Ohmic dissipations, using both numerical and regular perturbation techniques.

The above studies (Poots, 1961; Umavathi and Malashetty, 2005), however neglected the effects of radiation, which is significant in processes involving high temperature phenomena as in space technology. Thus, Gupta and Gupta (1974) studied the effect of radiation on the combined free and forced convection of an electrically conducting fluid inside an open-ended channel in the presence of a uniform transverse magnetic field without the effects of viscous dissipation. The aim of this present paper is to study the effects of radiation and viscous dissipation on laminar fully developed combined free and forced MHD convections of an incompressible viscous fluid inside a heated channel when the temperature of the channel walls increases linearly with height using the optically thin limit of Cogley et. al. (1968) for the radiative heat transfer. This attempt is therefore to complement the earlier works of Greif et. al, (1971) and Gupta and Gupta (1974).

Mathematical Formulation

We consider the two-dimensional steady, laminar, viscous dissipating and radiating magnetohydrodynamic fully developed flow between two heated vertical and non-conducting parallel walls in the Cartesian coordinate (x, y). The parallel walls are located at $y = \pm h$ and extending from $x = 0$ to $x = \infty$. The flow is driven by a constant pressure gradient in the x-direction. The schematic diagram of the physical model is shown in Fig. 1. The Cartesian coordinate system has its origin on the centre line of the channel with the x-axis measured in the vertical upward direction and y-axis normal to it; with fluid velocities u and v in the x and y directions, respectively.

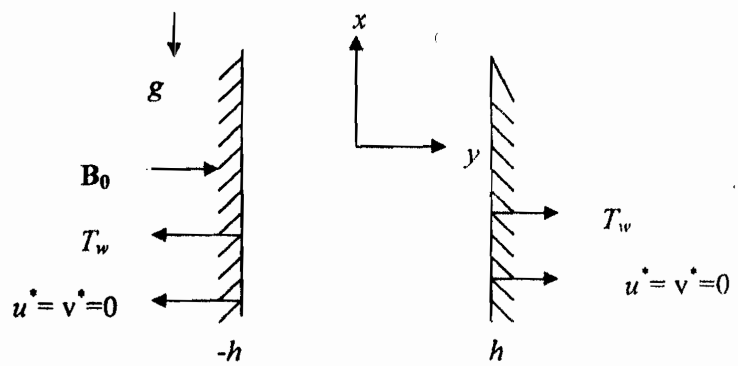

Fig. 1: Schematic diagram of the physical model and coordinate system

A uniform magnetic field with magnetic flux density vector $\mathbf{B}_0$ is applied across the channel in the positive y-direction, while the magnetic Reynolds number is small so that induced magnetic field is neglected. We assume the Boussinesq approximation where the density is constant except where it directly causes a buoyancy force and all other physical properties such as the viscosity, thermal conductivity, specific heat capacity and the coefficient of thermal expansion β^* are all constants and independent of the temperature. The temperatures of the walls are assumed equal and maintained at constant temperature gradient (τ / h) so that the wall temperature is $T_w = T_0 - (\tau / h)x$, where T_0 is the temperature of the wall at the origin and τ a parameter. The flow velocity and temperature profiles are assumed symmetrical about the channel of width $2h$ at $y = 0$. In this study, we note that the conservation of mass together with the no-slip boundary conditions on the walls indicate that there is no horizontal components of the fluid velocity, that is $v^*(x, y) = 0$.

Under these assumptions, the equations which govern the flow of a fully developed combined free and forced convection of an incompressible MHD fluid taking into consideration radiation and viscous dissipation are

$$\frac{\partial u^*}{\partial x} = 0 \tag{1}$$

$$-\left(\frac{1}{\rho}\frac{\partial p}{\partial x} + g\right) + \nu\frac{\partial^2 u^*}{\partial y^2} - g\beta^*(T - T_w) - \frac{\sigma B_0^2}{\rho}u^* = 0 \tag{2}$$

$$\frac{\partial p}{\partial y} = 0 \tag{3}$$

$$\rho c_p u^* \frac{\partial T}{\partial x} = \kappa\frac{\partial^2 T}{\partial y^2} + \mu\left(\frac{\partial u^*}{\partial y}\right)^2 - \frac{\partial q_r}{\partial y} \tag{4}$$

where all the variables and constants are given in the nomenclature.

Now, for an optically thin fluid with relatively low density, that is, a fluid that does not absorb its own emitted radiation but absorbs radiation emitted by the boundary, we take the radiative heat flux in the energy equation in the spirit of Cogley et. al. (1968) as

$$\frac{\partial q_r}{\partial y} = 4\alpha^2(T - T_w), \quad \alpha^2 = \int_0^\infty \delta_r \lambda_r \frac{\partial B}{\partial T} \tag{5}$$

where δ_r is the radiation absorption coefficient, λ_r the frequency and B the Planck's function.

Since we are seeking a solution for a kinematically and thermally developed system where the velocity and temperature fields are independent of x, the temperature inside the fluid can be written as

$$\Theta(y) = T - T_w \tag{6}$$

In view of Eqs. (5) and (6), the governing equations become

$$\nu\frac{\partial^2 u^*}{\partial y^2} - \frac{\sigma B_0^2}{\rho}u^* = \left(\frac{1}{\rho}\frac{\partial p}{\partial x} + g\right) + g\beta^*(T - T_w) \tag{7}$$

$$\kappa\frac{\partial^2 T}{\partial y^2} - 4\alpha^2(T - T_w) = -\mu\left(\frac{\partial u^*}{\partial y}\right)^2 - \rho c_p\left(\frac{\tau}{h}\right)u^* \tag{8}$$

Equations (7) and (8) are to be solved subject to the no-slip boundary conditions

$$u^* = 0, \quad T = T_w \quad \text{at} \quad y = \pm h \tag{9}$$

Introducing the following non-dimensional parameters

$$y = h\eta,\ u^* = \frac{\alpha_m u}{h}, \alpha_m = \frac{\kappa}{\rho c_p}, \theta = \tau\,\Theta,\ \xi = -\frac{h^3}{\alpha_m \nu}\left(\frac{1}{\rho}\frac{\partial p}{\partial x} + g\right),\ Ra = \frac{g\beta^*\tau h^3}{\alpha_m \nu},\ M^2 = \frac{\sigma B_0^2 h^2}{\rho\nu}$$

$$\Pr = \frac{\nu}{\alpha_m},\ N^2 = \frac{4\alpha^2 h^2}{\nu\rho c_p},\ Ec = \frac{\alpha_m^2}{\tau c_p h^2} \tag{10}$$

in Eqs. (7), (8) and the boundary conditions (9) yield the following dimensionless equations

$$\frac{\partial^2 u}{\partial \eta^2} - M^2 u = Ra\theta - \xi \tag{11}$$

$$\frac{\partial^2 \theta}{\partial \eta^2} - F\theta = -u - Ec\Pr\left(\frac{\partial u}{\partial \eta}\right)^2;\ F = N^2\Pr \tag{12}$$

$$u = 0,\ \theta = 0 \quad \text{at} \quad \eta = \pm 1 \tag{13}$$

The mathematical statement of the problem is now complete and depends on the Prandtl number (*Pr*), the magnetic parameter (M^2), the Rayleigh number (*Ra*), the radiation parameter (N^2), the measure of the pressure above its hydrostatic value (ξ) and the Eckert number (*Ec*).

METHOD OF SOLUTION

The problem posed in Eqs. (11) and (12) subject to the no-slip boundary conditions (13) are highly non-linear partial differential equations due to the presence of viscous dissipation in the energy equation. Generally, these coupled equations cannot be solved in a closed form. But for small Eckert number $(Ec \ll 1)$, we can advance by adopting regular perturbation expansion of the form

$$u(\eta) = u_0(\eta) + Ecu_1(\eta) + O(Ec^2); \qquad \theta(\eta) = \theta_0(\eta) + Ec\theta_0(\eta) + O(Ec^2) \tag{14}$$

Substituting Eq.(14) into Eqs. (11)-(12), neglecting the coefficients of $O(Ec^2)$ and simplifying yield the sequence of approximations

$$u_0'' - M^2 u_0 = Ra\theta_0 - \xi \tag{15}$$

$$\theta_0'' - F\theta_0 = -u_0 \tag{16}$$

subject to the conditions

$$u_0 = 0,\ \theta_0 = 0 \ \text{ at } \ \eta = \pm 1 \tag{17}$$

for $O(1)$ equations, and

$$u_1'' - M^2 u_1 = Ra\theta_1 \tag{18}$$

$$\theta_1'' - F\theta_1 = -u_1 - \Pr(u_0')^2 \tag{19}$$

subject to the conditions

$$u_1 = 0,\ \theta_1 = 0 \ \text{ at } \ \eta = \pm 1 \tag{20}$$

for $O(Ec)$ equations. Here, the primes denote differentiation with respect to η

To proceed, we first consider Eqs. (15) and (16) by combining the differential equations for $u_0(\eta)$ and $\theta_0(\eta)$, respectively to form a fourth-order ordinary differential equation given by

$$[D^4 - (F + M^2)D^2 + (FM^2 + Ra)]u_0 = F\xi \tag{21}$$

now subject to the conditions

$$u_0 = 0 = u_0'' \ \text{ at } \ \eta = \pm 1 \tag{22}$$

where $D \equiv \dfrac{d}{d\eta}$. Solving Eq. (21) subject to conditions (22) yield

$$u_0(\eta) = c_0\left(1 + c_1\frac{Cosh[\eta\sqrt{\delta}]}{Cosh[\sqrt{\delta}]} - c_2\frac{Cosh[\eta\sqrt{\lambda}]}{Cosh[\sqrt{\lambda}]}\right) \tag{23}$$

where the constants are given in appendix.

Now, using the expression for $u_0(\eta)$ as given by Eq. (23) in Eq. (16), solving together with boundary condition (17) yield

$$\theta_0(\eta) = d_1\frac{Cosh[\eta\sqrt{F}]}{Cosh[\sqrt{F}]} + c_0\left[\frac{c_1}{F-\delta}\frac{Cosh[\eta\sqrt{\delta}]}{Cosh[\sqrt{\delta}]} - \frac{c_2}{F-\lambda}\frac{Cosh[\eta\sqrt{\lambda}]}{Cosh[\sqrt{\lambda}]} + \frac{1}{F}\right] \tag{24}$$

Following similar procedure used above and using the expressions for $u_0(\eta)$ and $\theta_0(\eta)$, the solutions to Eqs. (18) and (19) subject to conditions (20) are

$$\begin{aligned} u_1(\eta) = {} & \Gamma_1 + \Gamma_3 + c_6\frac{Cosh[\eta\sqrt{\delta}]}{Cosh[\sqrt{\delta}]} + c_7\frac{Cosh[\eta\sqrt{\lambda}]}{Cosh[\sqrt{\lambda}]} + \Gamma_2 Cosh[2\eta\sqrt{\delta}] + \Gamma_4 Cosh[2\eta\sqrt{\lambda}] \\ & + \Gamma_5 Cosh[\eta(\sqrt{\delta} + \sqrt{\lambda})] + \Gamma_6 Cosh[\eta(\sqrt{\delta} - \sqrt{\lambda})] \end{aligned} \tag{25}$$

and

$$\begin{aligned} \theta_1(\eta) = {} & d_9\frac{Cosh[\eta\sqrt{F}]}{Cosh[\sqrt{F}]} - \frac{d_2}{F} + \frac{d_3}{F-\delta}Cosh[\eta\sqrt{\delta}] + \frac{d_4}{F-\lambda}Cosh[\eta\sqrt{\lambda}] + \\ & \frac{d_5}{F-4\delta}Cosh[2\eta\sqrt{\delta}] + \frac{d_6}{F-4\lambda}Cosh[2\eta\sqrt{\lambda}] + \frac{d_7}{(\sqrt{\delta}+\sqrt{\lambda})^2 - F}Cosh[\eta(\sqrt{\delta}+\sqrt{\lambda})] \\ & + \frac{d_8}{(\sqrt{\delta}-\sqrt{\lambda})^2 - F}Cosh[\eta(\sqrt{\delta}-\sqrt{\lambda})] \end{aligned} \tag{26}$$

Hence, from Eq. (14) the velocity and temperature profiles are respectively

$$u(\eta) = c_0\left(1 + c_1\frac{Cosh[\eta\sqrt{\delta}]}{Cosh[\sqrt{\delta}]} - c_2\frac{Cosh[\eta\sqrt{\lambda}]}{Cosh[\sqrt{\lambda}]}\right) +$$
$$+ Ec\left\{\Gamma_1 + \Gamma_3 + c_6\frac{Cosh[\eta\sqrt{\delta}]}{Cosh[\sqrt{\delta}]} + c_7\frac{Cosh[\eta\sqrt{\lambda}]}{Cosh[\sqrt{\lambda}]} + \Gamma_2 Cosh[2\eta\sqrt{\delta}] + \right. \quad (27)$$
$$\left. \Gamma_4 Cosh[2\eta\sqrt{\lambda}] + \Gamma_5 Cosh[\eta(\sqrt{\delta}+\sqrt{\lambda})] + \Gamma_6 Cosh[\eta(\sqrt{\delta}-\sqrt{\lambda})]\right\}$$

and

$$\theta(\eta) = d_1\frac{Cosh[\eta\sqrt{F}]}{Cosh[\sqrt{F}]} + c_0\left[\frac{c_1}{F-\delta}\frac{Cosh[\eta\sqrt{\delta}]}{Cosh[\sqrt{\delta}]} - \frac{c_2}{F-\lambda}\frac{Cosh[\eta\sqrt{\lambda}]}{Cosh[\sqrt{\lambda}]} + \frac{1}{F}\right]$$
$$+ Ec\left\{ d_9\frac{Cosh[\eta\sqrt{F}]}{Cosh[\sqrt{F}]} - \frac{d_2}{F} + \frac{d_3}{F-\delta}Cosh[\eta\sqrt{\delta}] + \frac{d_4}{F-\lambda}Cosh[\eta\sqrt{\lambda}] + \right.$$
$$\frac{d_5}{F-4\delta}Cosh[2\eta\sqrt{\delta}] + \frac{d_6}{F-4\lambda}Cosh[2\eta\sqrt{\lambda}] + \frac{d_7}{(\sqrt{\delta}+\sqrt{\lambda})^2 - F}Cosh[\eta(\sqrt{\delta}+\sqrt{\lambda})] \quad (28)$$
$$\left. + \frac{d_8}{(\sqrt{\delta}-\sqrt{\lambda})^2 - F}Cosh[\eta(\sqrt{\delta}-\sqrt{\lambda})] \right\}$$

The non-dimensional flow rate, w and the heat transfer coefficient, Nu (at $\eta = 1$) are given by

$$w = 2\int_0^1 u(\eta)d\eta$$
$$= 2\left(c_0 + Ec(\Gamma_1 + \Gamma_3) + \frac{c_0c_1 + Ecc_6}{\sqrt{\delta}}\tanh[\sqrt{\delta}] - \frac{(c_0c_2 - Ecc_7)}{\sqrt{\lambda}}\tanh[\sqrt{\lambda}]\right.$$
$$\left. + Ec\left(\frac{\Gamma_2}{2\sqrt{\delta}}\sinh[2\sqrt{\delta}] + \frac{\Gamma_4}{2\sqrt{\lambda}}\sinh[2\sqrt{\lambda}] + \frac{\Gamma_5}{\sqrt{\delta}+\sqrt{\lambda}}\sinh[\sqrt{\delta}+\sqrt{\lambda}] + \frac{\Gamma_6}{\sqrt{\delta}\sqrt{\lambda}}\sinh[\sqrt{\delta}-\sqrt{\lambda}]\right)\right) \quad (29)$$

and

$$Nu(\eta=1) = \left.\frac{d\theta}{d\eta}\right|_{\eta=1} = -(d_1 + Ecd_9)\sqrt{F}\tanh\sqrt{F} - \frac{(c_0c_1 + Ecc_6)}{F-\delta}\sqrt{\delta}\tanh[\sqrt{\delta}]$$
$$+ \frac{(c_0c_2 - Ecc_7)}{F-\lambda}\sqrt{\lambda}\tanh[\sqrt{\lambda}] - \frac{2Ecd_5\sqrt{\delta}}{F-4\delta}\sinh[2\sqrt{\delta}]$$
$$- \frac{2Ecd_6\sqrt{\lambda}}{F-4\lambda}\sinh[2\sqrt{\lambda}] - \frac{Ecd_7(\sqrt{\delta}+\sqrt{\lambda})}{F-(\sqrt{\delta}+\sqrt{\lambda})^2}\sinh[(\sqrt{\delta}+\sqrt{\lambda})] \quad (30)$$
$$- \frac{Ecd_8(\sqrt{\delta}-\sqrt{\lambda})}{F-(\sqrt{\delta}-\sqrt{\lambda})^2}\sinh[(\sqrt{\delta}-\sqrt{\lambda})]$$

The mathematical analysis is now complete.

RESULTS AND DISCUSSION

The problem of magnetohydrodynamic mixed convection in a heated vertical channel with viscous dissipation and radiation of incompressible and optically thin fluid has been solved making fairly realistic assumption. For small Eckert number, Ec the nonlinear problem is solved by regular asymptotic perturbation, giving rise to $O(1)$ solutions on which the $O(Ec)$ are then superimposed. The complete expressions for the velocity $u(\eta)$ and temperature profiles are given by Eqs (27) and (28) respectively. In order to understand the physical situation of the problem, we have computed the numerical values of the velocity, temperature, flow rate and the rate of heat transfer using the software *Mathematica*. The results are shown in Figures 2-5 and tables 1 and 2.

Figures 2 and 3 depict the velocity profiles $u(\eta)$ for various values of the radiation and magnetic parameters. From Fig. 2 we observe that for fixed magnetic parameter ($M = 1$) increase in the radiation parameter, N cause a corresponding increase in the velocity profiles. Also, from Fig. 3 it is observed that for fixed radiation ($N = 5$) increase in magnetic parameter, M is associated with decrease in the velocity profile. Furthermore, simultaneous increases in N and M recorded a progressively flatter velocity. These results are in qualitative agreement with earlier results of Gupta and Gupta (1974).

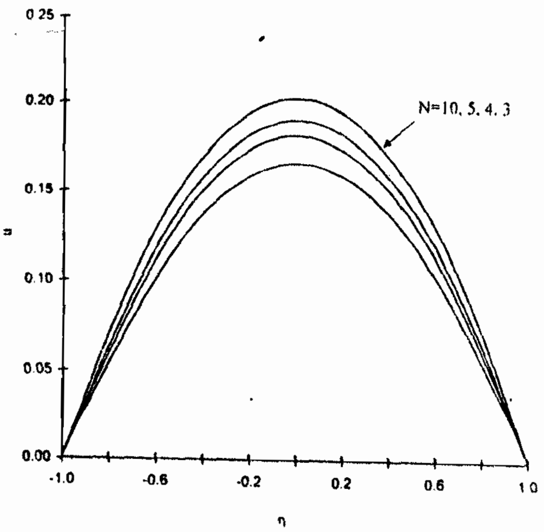

Fig.2 Effect of radiation on the velocity profiles for M=1, ξ=1, Ec=0.01, Ra=1 and Pr=0.71

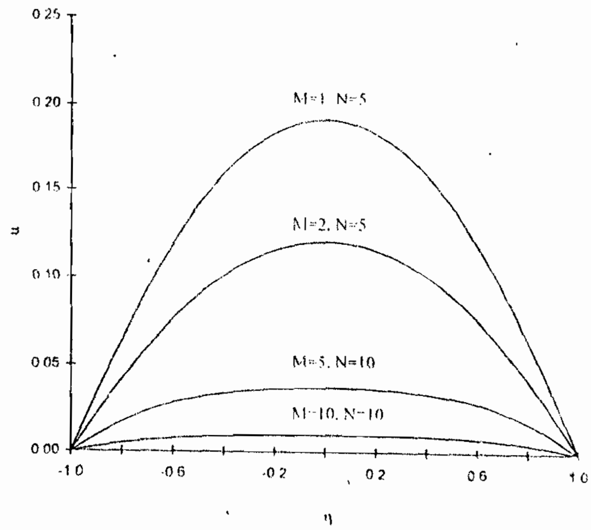

Fig.3 Effects of radiation and magenetic parameters on the velocity profiles for M=1 ξ=1, Ec=0.01, Ra=1 and Pr=0.71

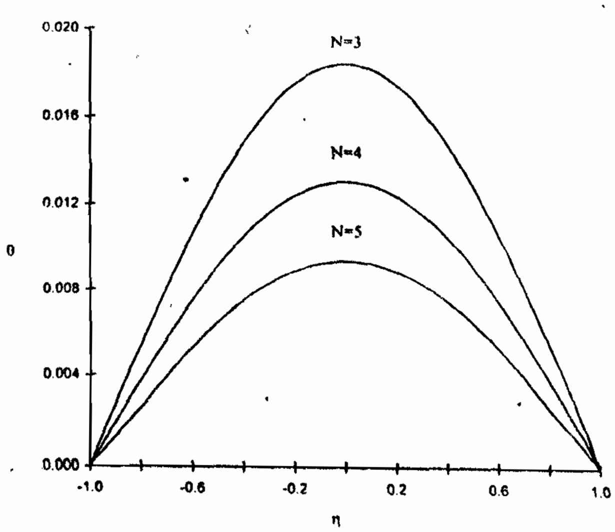

Fig.4 Effect of radiation on the temperature profiles for M=1, ξ=1, Ec=0.01, Ra=1and Pr=0.71

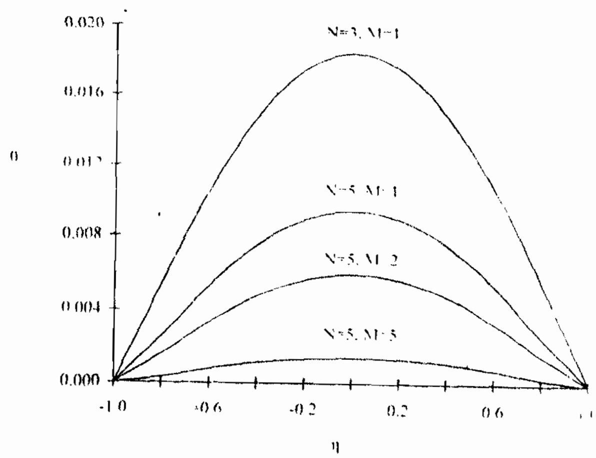

Fig 5 Effects of radiation and magenetic parameters on the velocity profiles for M=1, ξ=1, Ec=0.01, Ra=1 and Pr=0.71

Table1: Variation of flow rate, *w* for $\xi = 1,\ Ec = 0.01,\ Ra = 1,\ \Pr = 0.71$

M	N = 3	N = 4	N = 5
1	.217043	.238196	.250415
2	.120452	.146584	.159377
3	.053855	.086717	.099474

Table 2: Variation of heat transfer Nu at $\eta = 1$ for $\xi = 1,\ Ec = 0.01,\ Ra = 1,\ \Pr = 0.71$

M	N = 3	N = 4	N = 5
1	.0262736	.0172317	.0095682
2	.0057110	.0089420	.0048605
3	.0048737	.0063800	.0028096

In Figs. 4 and 5, we presented the behaviour of the temperature profiles, $\theta(\eta)$ for various values of radiation and magnetic parameters. It is observed that increases in radiation recorded a decrease in the temperature inside the channel. This is as expected since the effect of radiation is to reduce the influence of natural convection by causing a reduction in the temperature difference between the fluid and the channel walls. Equally, increase in magnetic parameter, *M* is associated with a decrease in the temperature inside the channel for fixed radiation; while simultaneous increases in radiation and magnetic parameters cause a progressively flatter fluid temperature. Again, these results are qualitatively consistent with the results of Gupta and Gupta (1974).

Tables 1 and 2 show the flow rate, *w* (given by (29)) and the rate of heat transfer, *Nu* (given by (30), respectively. As expected, the flow increased with increase in radiation for fixed magnetic parameter, while the reverse is the case with increase in magnetic parameter for fixed radiation (see Table 1). Also, the rate of heat transfer at the wall ($\eta = 1$) decreased for separate increases in magnetic as well as for simultaneous increases in these parameters (see Table 2).

CONCLUSIONS

In conclusion therefore, the problem of magnetohydrodynamic mixed convection in a heated vertical channel with viscous dissipation and radiation of incompressible and optically thin fluid is affected as follows

- for fixed radiation, increase in magnetic parameter is associated with decrease in the velocity profile
- for fixed magnetic field, increase in radiation led to an increase in the velocity profile
- simultaneous increases in radiation and magnetic parameters led to a progressively flatter velocity
- increase in both radiation and magnetic parameters led to a decrease in the temperature within the channel with flatter regime being recorded for higher values of the radiation and magnetic parameters
- the flow rate increased with increase in radiation, whereas as the reverse is the case with increase in magnetic parameter
- the rate of heat transfer decreased with increase in both radiation and magnetic parameters.

ACKNOWLEDGEMENTS

One of the authors (C.I) is highly indebted to Mrs. Gladys Israel-Cookey for her patience and excellent typesetting of the manuscript.

Appendix

$$F = N^2 \Pr, \quad c_0 = \frac{F\xi}{FM^2 + Ra},$$

$$c_1 = \frac{\lambda}{\delta - \lambda}, \quad c_2 = \frac{\delta}{\delta - \lambda},$$

$$c_3 = -\frac{\Pr c_0^2 c_1^2 \delta}{2Cosh^2[\sqrt{\delta}]}, \quad c_4 = -\frac{\Pr c_0^2 c_2^2 \lambda}{2Cosh^2[\sqrt{\lambda}]}$$

$$c_5 = \frac{\Pr c_0^2 c_1 c_2}{Cosh[\sqrt{\delta}]Cosh[\sqrt{\lambda}]}$$

$$c_6 = \frac{1}{\delta - \lambda}\Big((\Gamma_1 + \Gamma_3)\lambda + \Gamma_2(\lambda - 4\delta)Cosh[2\sqrt{\delta}] - 3\Gamma_4\lambda Cosh[2\sqrt{\lambda}] - \Gamma_5(\delta + 2\sqrt{\delta\lambda})Cosh[\sqrt{\delta} + \sqrt{\lambda}] - \Gamma_6(\delta - 2\sqrt{\delta\lambda})Cosh[\sqrt{\delta} - \sqrt{\lambda}]\Big)$$

$$c_7 = -\frac{1}{\delta - \lambda}\Big((\Gamma_1 + \Gamma_3)\delta - 3\Gamma_2\delta Cosh[2\sqrt{\delta}] - 4\Gamma_4\lambda Cosh[2\sqrt{\lambda}] - \Gamma_5(\lambda + 2\sqrt{\delta\lambda})Cosh[\sqrt{\delta} + \sqrt{\lambda}] + \Gamma_6(2\sqrt{\delta\lambda} - \lambda)Cosh[\sqrt{\delta} - \sqrt{\lambda}]\Big)$$

$$\delta = \frac{1}{2}(\alpha + \beta), \quad \lambda = \frac{1}{2}(\alpha - \beta),$$

$$\alpha = F + M^2, \quad \beta = \left[(F - M^2)^2 - 4Ra\right]^{1/2} > 0.$$

$$d_1 = c_0\left[\frac{c_2}{F - \lambda} - \frac{c_1}{F - \delta} - \frac{1}{F}\right]$$

$$d_2 = \frac{\Pr c_1^2 c_0^2 \delta}{2Cosh^2[\sqrt{\delta}]} + \frac{\Pr c_0^2 c_2^2}{2Cosh^2[\sqrt{\lambda}]} - \Gamma_1 - \Gamma_3$$

$$d_3 = \frac{c_6}{Cosh[\sqrt{\delta}]},$$

$$d_4 = \frac{c_7}{Cosh[\sqrt{\lambda}]},$$

$$d_5 = \Gamma_2 + \frac{\Pr c_0^2 c_1^2 \delta}{2Cosh^2[\sqrt{\delta}]}$$

$$d_6 = \Gamma_4 + \frac{\Pr c_0^2 c_2^2 \lambda}{2Cosh^2[\sqrt{\lambda}]},$$

$$d_7 = \frac{\Pr c_0^2 c_1 c_2 \sqrt{\lambda\delta}}{Cosh[\sqrt{\lambda}]Cosh[\sqrt{\delta}]} - \Gamma_5,$$

$$d_8 = \frac{\Pr c_0^2 c_1 c_2 \sqrt{\delta\lambda}}{Cosh[\sqrt{\delta}]Cosh[\sqrt{\lambda}]}$$

$$d_9 = \frac{d_2}{F} - \frac{d_3}{F-\delta}Cosh[\sqrt{\delta}] - \frac{d_4}{F-\lambda}Cosh[\sqrt{\lambda}] - \frac{d_5}{F-4\delta}Cosh[2\sqrt{\delta}] - \frac{d_6}{F-4\lambda}Cosh[2\sqrt{\lambda}]$$
$$- \frac{d_7}{(\sqrt{\delta}+\sqrt{\lambda})^2 - F}Cosh[\sqrt{\delta}+\sqrt{\lambda}] - \frac{d_8}{(\sqrt{\delta}-\sqrt{\lambda})^2 - F}Cosh[\sqrt{\delta}-\sqrt{\lambda}]$$

$$\Gamma_1 = \frac{c_3 Ra(4F\delta - Ra - 16\delta^2 + 4M^2\delta - FM^2)}{FM^4(F-4\delta) + Ra(Ra + 4\delta(4\delta - F)) + M^2(2F(Ra + 8\delta^2) - 4F^2\delta - 4Ra\delta)}$$

$$\Gamma_2 = \frac{c_3 Ra(FM^2 + Ra)}{FM^4(F-4\delta) + Ra(Ra + 4\delta(4\delta - F)) + M^2(2F(Ra + 8\delta^2) - 4F^2\delta - 4Ra\delta)}$$

$$\Gamma_3 = \frac{c_4 Ra(4F\lambda - Ra - 16\lambda^2 + 4M^2\lambda - FM^2)}{FM^2(F-4\lambda) + Ra(Ra + 4\lambda(4\lambda - F)) + M^2(2F(Ra + 8\lambda^2) - 4F^2\lambda - 4Ra\lambda)}$$

$$\Gamma_4 = \frac{c_4 Ra(FM^2 + Ra)}{FM^2(F-4\lambda) + Ra(Ra + 4\lambda(4\lambda - F)) + M^2(2F(Ra + 8\lambda^2) - 4F^2\lambda - 4Ra\lambda)}$$

$$\Gamma_5 = \frac{c_5 Ra}{Ra + M^2(F - (\sqrt{\delta}+\sqrt{\lambda})^2) + (\delta + \lambda - F - \sqrt{\lambda\delta})(\sqrt{\delta}-\sqrt{\lambda})^2}$$

$$\Gamma_6 = \frac{c_5 Ra}{Ra + M^2(F - (\sqrt{\delta}-\sqrt{\lambda})^2) + (\delta + \lambda - F - \sqrt{\delta\lambda})(\sqrt{\delta}-\sqrt{\lambda})^2}$$

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