ABSTRACT

This paper investigates the combined effects of radiation and Hall current on oscillatory magnetohydrodynamic (MHD) free-convection flow of an electrically conducting, viscous, incompressible rotating Newtonian and optically thin fluid past an infinite heated vertical porous plate with time-dependent suction. The temperature of the porous plate is high enough to initiate radiative heat transfer, while an external magnetic field is applied perpendicular to the plate. By taking the optically thin approximation for the radiative heat flux in differential form and imposing a small sinusoidal time-dependent perturbation, the coupled non-linear partial differential equations governing the flow are solved. It is observed that increase in radiation lead to decrease in temperature in the primary flow, whereas the reverse is the case in the secondary flow. In addition, separate increases in Hall current, magnetic field and radiation parameters led to a decrease in velocity in the primary flow, whereas the reverse occur in the secondary flow. Further, it is seen that increases in Hall current, magnetic field and radiation parameters resulted in increase in the magnitude of the skin friction. Finally, increased radiation resulted in an increase in the rate of heat transfer.

KEYWORDS: Radiation, Hall current, optically thin, rotating fluid, perturbation

INTRODUCTION

The study of an electrically conducting incompressible fluid under the action of a transversely applied magnetic field in a rotating fluid has immediate applications in geophysics and astrophysics. For example, the large scale and moderate motions of the earth's core and atmosphere are greatly affected by the vorticity of the earth's rotation, which in turn is responsible for the main geomagnetic fields. Thus, it can provide some possible explanations for the observed maintenance and secular variations of the actual geomagnetic fields (Hidde and Roberts, 1961). Also, it is important in solar physics associated with the development of Sunspot, the solar cycle and the structure of a rotating star (Dikie, 1967).

In order to provide an insight to the physical understanding of such motions, boundary layer flows with or without the presence of magnetic fields have been studied by many researchers (Gupta, 1972; Pop and Soundegekar, 1975, Debnath, 1975; Smirnov and Shatov, 1982; Page, 1983). However in area of space technology and in processes involving high temperature phenomena (as in ionosphere-plasma region, hypersonic flight, missile reentry, rocket combustion chambers, and power plants for interplanetary flight, gas-cooled nuclear reactors, and interstellar environment) the effects of radiation become significant. Studies involving the interaction of thermal radiation and magnetic fields over vertical plate in rotating fluid have been carried out when the operating environmental temperatures are high for radiative effects to be significant. Thus, Bestman and Adjepong (1988) considered the problem of unsteady hydromagnetic free-convection flow with radiative heat transfer in a rotating fluid, while Israel-Cooke et al. (2002) investigated the same problem with the inclusion of the effects of mass transfer using the method of Laplace transforms. More recently, Israel-Cooke and Alagoga (2003) studied the combined effects of magnetic field, free-convection and radiation on unsteady boundary layer magnetohydrodynamic (MHD) flow past a heated porous plate in a rotating fluid with time-dependent suction and free stream velocity.

However, when the strength of the magnetic field is high and on the assumption that the magnetic Reynolds number is small, Hall currents become significant. In MHD flows, the Hall current effect rotates the current vector away from the direction of the electric field and generally reduces the effect of force that the magnetic field exerts on the flow (Shercliff, 1965, Crummer and Pai, 1973; Sutton and Sherman, 1965). Many works on the effects of Hall current on boundary layer flows over plates in non-rotating fluids have been reported in literature (Singh, 1983, Rao et al., 1983; Hossain, 1986; Hossein and Rashid, 1985; Hossain and Mohammad, 1988). For example, Singh (1983) studied the free convection effects on the oscillatory flow of an incompressible, viscous and electrically conducting fluid past an infinite vertical porous plate in the presence of a strong transverse magnetic field with constant suction velocity when the plate is moving impulsively in its own plane. More recently, Takhar et al. (2002) considered the effects of Hall currents on a steady non-similar boundary layer flow over a moving surface in a rotating fluid taking into consideration the Coriolis force in the presence of a magnetic field, and free stream velocity.

The aim of this present study is to investigate the combined effects of radiation and Hall currents on oscillatory MHD free-convection flow past an infinite perfectly conducting heated porous plate in a rotating fluid with time-dependent suction, when the free stream velocity oscillates periodically in time about a constant mean value. This attempt is to complement the earlier works of Israel-Cooke and Alagoga (2003), and in turn widen the applicability of problems of this nature.

MATHEMATICAL FORMULATION

We consider the oscillatory MHD flow of an incompressible, viscous and electrically- conducting and rotating Newtonian fluid past an infinite porous heated vertical plate in the presence of a strong magnetic field of strength, $\Phi_0$ with simultaneous
effects of Hall current and radiation. Suppose that both the fluid and the plate are in a state of rigid rotation with constant angular velocity, $\Omega$ about the $\hat{z}$ axis, taken positive in the vertically upward direction. The infinite porous is assumed to coincide with the plane $x^* = 0$; while the imposed magnetic field is maintained in the $y^*$ direction. At time $t^* > 0$ the plate moves impulsively in its own plane with velocity $U_0$ and its temperature is instantly raised from the fluid temperature $T_*$ to $T_w$ (wall temperature) and thereafter maintained constant. Further, we assume that the velocity encountered in the free stream is small, hence the Joule and viscous dissipations are neglected.

Under these assumptions the governing equations including Maxwell’s equations are (Bestman and Adjepong, 1988; Takhar et al., 2002; Israel-Cookey and Alagoda, 2003):

Continuity equation:
\[ \nabla \cdot \mathbf{V}^* = 0 \]  \hspace{1cm} (1)

Momentum equation:
\[ \frac{D \mathbf{V}^*}{D t} + \frac{2}{\rho} \Omega \times \mathbf{V}^* = \nu \nabla^2 \mathbf{V}^* + g \beta (T^* - T_w) + \frac{1}{\rho} \mathbf{J} \times \mathbf{B} \]  \hspace{1cm} (2)

Generalized Ohm’s law:
\[ \mathbf{J} = \sigma (\mathbf{E} + \mathbf{V} \times \mathbf{B} - \beta H \mathbf{J} \times \mathbf{B}) \]  \hspace{1cm} (3)

Maxwell equation:
\[ \nabla \cdot \mathbf{E} = 0, \ \nabla \times \mathbf{E} = 0, \ \nabla \cdot \mathbf{B} = 0 \]  \hspace{1cm} (4)

Energy equation:
\[ \frac{DT^*}{Dt} = \frac{\kappa}{\rho c_p} \left( \nu \nabla^2 T^* - \frac{1}{\kappa} \nabla \cdot \mathbf{q}_v \right) \]  \hspace{1cm} (5)

where $\mathbf{V}^* = (u^*, v^*, w^*)$ is the velocity vector, $\mathbf{E}$ is the electric field intensity, $\mathbf{B} = (0, 0, B_0)$ is the magnetic field induction vector, $\mu_\nu$ is the magnetic permeability, $\mathbf{J}$ is the electric current density vector, $\beta_H$ is the hall factor. Also, $g$ is the gravitational acceleration, $\beta$ the coefficient of volumetric thermal expansion, $\Omega = (0, 0, \Omega)$ is the uniform angular velocity of the fluid and plate, $\nu$ is the kinematic viscosity, $\rho$ the fluid density, $\kappa$ the thermal conductivity, $\mathbf{q}_v$ the radiative heat transfer flux, $\sigma$ the electrical conductivity and $c_p$ the specific heat capacity.

From the relation $\nabla \cdot \mathbf{E} = 0$, which indicates the absence of any excess charge and the fact that the magnetic Reynolds number is very small together with $\nabla \times \mathbf{E} = 0$, the induced magnetic field can be ignored (Shercliff, 1965; Sutton and Sherman, 1965). Now for a non-conducting plate the relation $\nabla \times \mathbf{J} = 0$ implies that $\mathbf{J}_F = 0$ at the plate and everywhere in the fluid. Under these assumptions, the generalized Ohm’s law (Eq. 3) reduces to
\[ \mathbf{J} = \frac{\sigma B_0}{1 + m^2} \left( m u^* - w^*, 0, u^* + mw^* \right) \]  \hspace{1cm} (6)

and
\[ \mathbf{J} \times \mathbf{B} = \frac{\sigma B_0}{1 + m^2} \left( u^* + mw^*, 0, w^* - mu^* \right) \]  \hspace{1cm} (7)

where $m(= \sigma \beta H B_0)$ is the Hall parameter. It is to be mentioned that for weakly ionized plasma, the value of $m$ is less than unity [13].

Further, on assumption that the medium is optically thin with relatively low density, the radiative heat flux, $\nabla \cdot \mathbf{q}_v$ in the energy equation can be written in differential form as (Cogley et al. 1968)
\[ \frac{\partial q_v}{\partial y^*} = 4 \alpha^2 (T^* - T_w) \]  \hspace{1cm} (8)

where $\alpha^2 = \int \delta \lambda \frac{\partial B_\nu}{\partial T^*}$ and $\delta \lambda, B_\nu$ denote the radiation coefficient, frequency of radiation and Plank’s constant, respectively.

Under the above assumptions and the usual Boussinesq approximation the boundary layer problem represented by Eqs. (1-5) become
\[ \frac{\partial u^*}{\partial t^*} + v^* \frac{\partial u^*}{\partial y^*} - 2 \Omega_0 w^* = \nu \frac{\partial^2 u^*}{\partial y^* 2} + g \beta (T^* - T_w) - \frac{\sigma B_0^2}{\rho (1 + m^2)} (u^* + mw^*) \]  \hspace{1cm} (9)

Further, on assumption that the medium is optically thin with relatively low density, the radiative heat flux, $\nabla \cdot \mathbf{q}_v$ in the energy equation can be written in differential form as (Cogley et al. 1968)
\[ \frac{\partial q_v}{\partial y^*} = 4 \alpha^2 (T^* - T_w) \]  \hspace{1cm} (8)

where $\alpha^2 = \int \delta \lambda \frac{\partial B_\nu}{\partial T^*}$ and $\delta \lambda, B_\nu$ denote the radiation coefficient, frequency of radiation and Plank’s constant, respectively.
\[
\frac{\partial \nu^*}{\partial t^*} + v \frac{\partial \nu^*}{\partial y^*} + 2 \Omega_0 u^* = \frac{\partial^2 \nu^*}{\partial y^*^2} - \frac{\sigma B_r^2}{\rho(1 + m')} (u^* - w^*)
\]  
(11)

\[
\frac{\partial T^*}{\partial t^*} + v \frac{\partial T^*}{\partial y^*} = \frac{\kappa}{\rho C_p} \left( \frac{\partial^2 T^*}{\partial y^*^2} - \frac{4 \alpha^2}{\kappa} \left(T^* - T_w\right) \right)
\]  
(12)

Now, since the plate is assumed to oscillate when in state of motion, the appropriate boundary conditions are

\[
\begin{cases}
    u^* + iw^* = U_0(1 + \epsilon \omega \sin \omega t^*), & T^* = T_w, \quad \text{on } y^* = 0 \\
    u^* \rightarrow 0, w^* \rightarrow 0, & T^* \rightarrow T_w, \quad y^* \rightarrow \infty
\end{cases}
\]  
(13)

where \( \omega^* \) is the frequency of oscillation, \( \epsilon U_0 \) the amplitude of oscillation, \( i = \sqrt{-1} \) and \( \epsilon \) is a small positive parameter.

From the continuity equation (i.e. Eq. (9)) it is clear that the suction velocity, \( v^* \) is a function of time, \( t^* \) only and so we represent it by

\[
v^* = -v_0 \left(1 + \epsilon A \exp[i \omega t^*]\right)
\]  
(14)

where the minus sign indicates that the suction is towards the plate, \( v_0 \) is a constant suction at the plate and \( \epsilon A \) is a positive parameter such that \( \epsilon A < 1 \). On introducing \( p^* = u^* + iw^* \), and using Eq.(14) together with the following non-dimensional variables

\[
I = \frac{v_0^2}{4v}, \quad y = \frac{y^*}{\nu}, \quad u = \frac{u^*}{U_0}, \quad w = \frac{w^*}{U_0}, \quad p = \frac{p^*}{\rho U_0^2}, \quad E = \frac{\rho v_0^3}{\rho U_0^3}, \quad \omega = \frac{4v \omega^*}{\nu v_0}, \quad M^2 = \frac{\sigma B_r^2}{\rho v_0^2},
\]

\[
Gr = \frac{\nu g \beta(T_w - T_o)}{U_0 v_0^2}, \quad \theta = \frac{T^* - T_o}{T_w - T_o}, \quad Pr = \frac{\mu c_p}{\nu}, \quad \mu = \frac{\mu}{\rho}, \quad N = \frac{4 \alpha^2 \nu^2(1 + m')}{v_0^2}
\]  
(15)

the governing equations represented by Eqs. (10)-(12) and the corresponding boundary conditions become

\[
\frac{1}{4} \frac{\partial \rho}{\partial t} = \frac{\partial^2 \rho}{\partial y^*^2} + (1 + \epsilon \alpha \text{sin} \omega t^*) \frac{\partial \rho}{\partial y^*} - \left(\frac{M}{1 + m^2} + i(2E - \frac{m M^2}{1 + m^2})\right) \rho + Gr \rho \theta \theta
\]  
(16)

\[
Pr \frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^*^2} + Pr(1 + \epsilon \alpha \text{sin} \omega t^*) \frac{\partial \theta}{\partial y^*} - N \theta
\]  
(17)

\[
p = 1 + \epsilon \alpha \text{sin} \omega t^*, \quad \theta = 1 \quad \text{on } y^* = 0
\]  
(18a)

\[
p \rightarrow 0, \quad \theta \rightarrow 0 \quad \text{as } y^* \rightarrow \infty
\]  
(18b)

Here, \( Pr \) is the Prandtl number, \( M^2 \) the magnetic parameter, \( E \) the rotation parameter (Ekman number), \( Gr \) is the Grashof number, \( \theta \) is the non-dimensional temperature field and \( N \) is the radiation parameter.

**METHOD OF SOLUTION**

The problem posed in Eqs. (16)-(17) subject to boundary conditions (18) are highly non-linear partial differential equations and generally will involve a numerical solution. However, since the unsteadiness is characterized by a sinusoidal perturbation in the flow of the order \( \epsilon \alpha \text{sin} \omega t^* \), approximate solution is possible by asymptotic expansion.

\[
p(y, t) = p_0(y) + \epsilon \alpha \text{sin} \omega t \ p_1(y), \quad \theta(y, t) = \theta_0(y) + \epsilon \alpha \text{sin} \omega t \ \theta_1(y)
\]  
(19)

Substituting Eqs. (19) into Eqs. (16)-(17) and the boundary conditions (18), while neglecting the coefficients of \( O(\epsilon^2) \) yield the sequence of approximations

\[
\frac{d^2 p_0}{dy^2} + \frac{dp_0}{dy} - L_1 p_0 = -Gr \theta_0
\]  
(20)

\[
\frac{d^2 \theta_0}{dy^2} + Pr \frac{d \theta_0}{dy} - N \theta_0 = 0
\]  
(21)

subject to the boundary conditions

\[
p_0 = 0, \ \theta_0 = 1 \quad \text{on } y = 0
\]  
(22a)

\[
p_0 \rightarrow 0, \ \theta_0 \rightarrow 0 \quad \text{as } y \rightarrow \infty
\]  
(22b)

for \( O(1) \) equations, and

\[
\frac{d^2 p_1}{dy^2} + \frac{dp_1}{dy} - L_1 p_1 = -A \frac{dp_0}{dy} - Gr \theta_1
\]  
(23)
\[
\frac{d^2\theta}{dy^2} + \Pr \frac{d\theta}{dy} - \frac{L_2}{\theta} = -A \frac{d\beta}{dy}
\]
subject to the boundary conditions:

\[
p_1 = 0, \quad \theta_1 = 0 \quad \text{on} \quad y = 0
\]

\[
p_1 \to 0, \quad \theta_1 \to 0 \quad \text{as} \quad y \to \infty
\]

for \( O(\varepsilon) \) equations. Here,

\[
L_1 = \frac{M^2}{1 + m^2} - \frac{1}{i + m^2}, \quad L_2 = N + \frac{i \omega \Pr}{4}, \quad L_3 = L_1 + \frac{i \omega}{4}
\]

Solving the \( O(1) \) equations and substituting into \( O(\varepsilon) \) equations give the expressions for the velocity and temperature profile, respectively as

\[
p(y, t) = (1 - A_1)e^{-\alpha_1 y} + A_1 e^{-\alpha_2 y} + \varepsilon e^{i \omega t} \left( A_6 e^{-\alpha_3 y} + A_7 e^{-\alpha_4 y} + A_8 e^{-\alpha_5 y} \right)
\]

\[
\theta(y, t) = e^{-\alpha_1 y} + \varepsilon e^{i \omega t} A_2 (e^{-\alpha_3 y} - e^{-\alpha_4 y})
\]

where

\[
\alpha_1 = \frac{1}{2} (\Pr + \sqrt{\Pr^2 + 4N})
\]

\[
\alpha_2 = \frac{1}{2} (1 + \sqrt{1 + 4L_1})
\]

\[
\alpha_3 = \frac{1}{2} (\Pr + \sqrt{\Pr^2 + 4L_3})
\]

\[
\alpha_4 = \frac{1}{2} (1 + \sqrt{1 + 4L_3})
\]

\[
A_1 = \frac{\alpha_1}{\alpha_1^2 - \alpha_1 - L_3}
\]

\[
A_2 = \frac{\alpha_2}{\alpha_2^2 - \alpha_2 - L_2}
\]

\[
A_3 = \frac{A_1 \alpha_1 - \text{Gr} A_2}{\alpha_1^2 - \alpha_1 - L_3}
\]

\[
A_4 = \frac{A_2 (1 - A_1)}{\alpha_2^2 - \alpha_2 - L_1}
\]

\[
A_5 = \frac{\text{Gr} A_2}{\alpha_2^2 - \alpha_2 - L_1}
\]

\[
A = 1 - A_3 - A_4 - A_5
\]

Now that we have the expressions for the velocity and temperature profiles of the flow problem, we can compute the skin-friction and heat transfer parameters of the flow. The local skin-friction coefficient is given by

\[
\tau = \frac{\partial p}{\partial y} \bigg|_{y=0} = (A_1 - 1)\alpha_2 - \varepsilon e^{i \omega t} (\alpha_4 A_6 + \alpha_1 A_1 + \alpha_2 A_4 + \alpha_3 A_3)
\]

while the local heat transfer coefficient, \( Nu \) is

\[
Nu = \frac{\partial \theta}{\partial y} \bigg|_{y=0} = -\alpha_1 + \varepsilon e^{i \omega t} (\alpha_3 - \alpha_1) A_2
\]

RESULTS AND DISCUSSION

In the preceding sections, we have formulated and solved approximately the problem of oscillatory MHD free convection flow past a heated vertical porous plate in rotating fluid under the combined effects of radiation and Hall currents when the free stream velocity and the plate temperature oscillates periodically in time about a constant mean. In order to understand the physical situation of the problem, we have computed the numerical values of the velocity, temperature, local skin-friction and local heat transfer using the software Mathematica. In addition, the Prandtl number, \( \Pr = 0.71 \) which corresponds to air and various values of the material parameters are used. Our results are shown graphically in Figs. 1-12.
Fig. 1: Effect of radiation on the temperature profiles in the primary flow.

Fig. 2: Effect of radiation on the temperature profiles in the secondary flow.
Fig. 3: Effect of Hall parameter, \( m \) on the velocity profiles in the primary flow.

Fig. 4: Effect of Hall parameter, \( m \) on the velocity profiles in the secondary flow.
Fig. 5: Effect of magnetic field parameter, \( M \) on the velocity in the primary flow

Fig. 6: Effect of magnetic field parameter, \( M \) on the velocity in the secondary flow
Fig. 7: Effect of radiation parameter, $N$ on the velocity in the primary flow.

Fig. 8: Effect of radiation parameter, $N$ on the velocity in the secondary flow.
Fig. 9: Surface skin friction against the suction parameter, $\Lambda$ for different values of Hall parameter.

Fig. 10: Surface skin friction against the suction parameter, $\Lambda$ for different values of magnetic parameter.
Fig. 11: Surface skin friction against the suction parameter, \( A \) for different values of radiation parameter.

\[ A=0.2, \quad E=1, \quad Pr=71, \quad M=2, \quad Gr=2.1 \times 3143, \quad \omega=5, \quad c=0.01, \quad m=0.5 \]

Fig. 12: Surface heat transfer against the suction parameter, \( A \) for different values of radiation parameter.
The effect of radiation on the temperature profiles within the fluid in the primary and secondary flows are shown in Figs. 1 and 2, respectively. From Fig. 1 it is observed that the temperature within the plasma in the primary flow when the plate is cooling through the convection currents ($\gamma > 0$) decreases rapidly with distance away from the plate and converge to a steady value near $y = 1$. Also, increase in radiation parameter, $N$ led to a decrease in temperature. From Fig. 2 we observe that the temperature profiles in the secondary flow decreased rapidly to some minimum value near $y = 1$, rose and then converged close to $y = 4$. In this case increase in radiation is associated with increase in temperature. These results are in good qualitative agreement with earlier results of Israel-Cooksey and Alagoa (2003).

In Figs. 3-8 the behaviors of the velocity profiles in the primary and secondary flows are shown for various material parameters. In the following discussions attention is restricted to values of Hall parameter less than unity (Sutton and Sherman, 1965). It is observed that in the primary flow (see Figs. 3, 5, 7), the velocity profiles decreased rapidly with distance away from the plate and then converged to steady value near the free stream value, whereas, in the secondary flow (see Figs. 4, 6, 8) the velocity profiles decreased initially to some minimum value, rose steadily and then converged close to the free stream value in the absence of Hall parameter, in the results are in good qualitative agreement with earlier results of Israel-Cooksey and Alagoa (2003). Also, from Figs. 2-3 it is seen that increase in Hall parameter resulted in increase in velocity profiles in the primary and secondary flows. Further, it is observed that increases in the material parameters $M$ and $N$ lead to decreases in the velocity profiles in the primary flow (see Figs. 5 and 7), whereas the reverse is the case in the secondary flow (see Figs. 4 and 6). Also, the velocity profiles in the primary flow show reversal pattern, which in turn may be valid for re-entry problems.

Figure 12 illustrates the variation of surface heat transfer coefficient (Nusselt number) with varying values of suction parameter, $A$ and various values of radiation parameter, $N$. It is observed that for given material parameters, the surface heat transfer from the porous plate increased with increase in radiation.

In Figs. 9-11, the behavior of the surface skin friction, $[\tau_1]$ for different values of suction parameter, $A$, and given material parameters are shown. It is observed that the surface skin friction decreased slightly with increase in the magnitude of suction velocity of the porous plate. Further, separate increases in Hall, magnetic field and radiation parameters are associated with increase in the surface skin friction.

CONCLUSIONS

We have examined the problem of combined effects of radiation and Halls on oscillatory MHD free convection flow past an infinite vertical porous plate in a rotating fluid with time dependent suction when the free stream velocity and plate temperature oscillates periodically in time about a constant mean values. The solutions for the flow variable are obtained by imposing a sinusoidal time dependent perturbation. It is observed that increase in radiations lead to decrease in temperature in the primary flow, whereas the reverse is the case in the secondary flow. In addition, separate increases in Hall current, magnetic field and radiation parameters led to a decrease in velocity in the primary flow, whereas the reverse occur in the secondary flow. Further, it is seen that increases in Hall current, magnetic field and radiation parameters resulted in increase in the magnitude of the skin friction. Finally, increased radiation resulted in an increase in the rate of heat transfer.

ACKNOWLEDGEMENTS

The authors are highly indebted to Professor Akane Oguni for useful suggestions and to Mrs. Giadyis Israel-Cooksey for excellent typesetting of the manuscript.

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