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BILINEAR MOVING AVERAGE VECTOR MODELS AND ITS APPLICATION TO ESTIMATION OF REVENUE SERIES

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ABSTRACT

This paper intends to establish multivariate time series models for pure moving average vector series which assume linear and nonlinear components. General Bilinear Moving Average Vector (BMAV) models have been established. The three vector series used for the modeling suggested trivariate time series models as a special case of multivariate time series models and estimates obtained from the models are graphically shown in figures 1, 2, and 3.

KEYWORDS: Bilinear Moving Average Vector Models, Vector white noise, Trivariate time series models, Linear and Non linear models

INTRODUCTION

Most time series analysts assume linearity and stationarity, for technical convevience, when analyzing macroeconomic and financial time series data, (Franses, 1998). However, some of the microeconomic and financial data are not linear, due to its dynamic behaviour. Classical linear models are not appropriate for modelling such nonlinear series, (Subba Rao and Gabr, 1984). In most cases, nonlinear forecast is superior to linear forecast. Maravall (1983) used a bilinear model to forecast Spanish monetary data and reported a near 10% improvement in one-step ahead mean square forecast errors over several ARMA alternatives. There is no-gainsaying the fact that most of the economic or financial data assume fluctuations due to certain factors. That is why the use of nonlinear models in forecast gives higher precision than linear models.

Let e_t be a sequence of independently and identically distributed random variables defined on a probability space (Ω,B,P) with $E(e_t) = 0$ and $E(e^2_t) = \sigma^2 < \infty$. The general superdiagonal bilinear model X_t with respect to e_t is

$$X_t = e_t + \sum_{i=1}^{r} a_i X_{t-i} + \sum_{j=1}^{h} b_j e_{t-j} + \sum_{i=1}^{m} \sum_{j=1}^{s} c_{ij} X_{t-i} e_{t-j} \qquad 1.1$$

where a_i, b_j, c_{ij} are fixed time independent parameters, (Akamanam, Bhaskara Rao, and Subramanyam, 1986).

Oyet (1991) defined a process $(X_t)_{t\in Z}$ on a probability space (Ω,ξ,P) as a time varying bilinear process of order (p,q,P,Q) and denoted by BL(p,q,P,Q), if it satisfies the following stochastic difference equation

$$X_t = \sum_{i=1}^{p} a_{i,t}(a) X_{t-i} + \sum_{j=1}^{q} c_{j,t}(c)\varepsilon_{t-j} + \sum_{i=1}^{P} \sum_{j=1}^{Q} b_{ij,t}(b) X_{t-i}\varepsilon_{t-j} + \varepsilon_t$$

where $(a_{i,t}(a))_{1\le i\le p}$, $(c_{j,t}(c))_{1\le j\le q}$, $(b_{ij,t}(b))_{1\le i\le P,\, 1\le j\le Q}$ are time-varying coefficients which depend on finite dimensional unknown parameter vectors a, c and b respectively. The sequence $(\varepsilon_t)_{t\in Z}$ is a heteroscedastic white noise process. That is, $(\varepsilon_t)_{t\in Z}$ is a sequence of independent random variables, not necessarily identically distributed, with mean zero and variance σ_t^2. Moreover ε_t is independent of past X_t. The initial values X_t, $t < 1$, and ε_t, $t < 1$ are assumed to be equal to zero.

Boonchai and Eivind (2005) stated the general form of a multivariate bilinear time series model as

$$X_t = \sum A_i X_{t-i} + \sum M_i e_{t-j} + \sum\sum\sum B_{dij} X_{t-i} e_{dt-j} + e_t.$$

Here the state X_t and noise e_t are n-vectors and the coefficients A_i, M_j, and $B_{dij} = 0$, and we have the class of well-known vector ARMA models. The bilinear models include additional product terms $B_{dij}X_{t-i}e_{dt-j}$, as the name indicates these models are linear in state X_t and in noise e_t separately, but not jointly. From a theoretical point of view, it is therefore natural to consider bilinear models in the process of extending linear theory to non-linear cases. According to Boonchai and Eivind (2005), a particular reason for introducing bilinear time series in population dynamics is that they are suitable for modelling environmental noise. One may start with a deterministic system with (constant) parameters that describe conditions that depend on a fluctuating environment. Boonchai and Eivind (2005) made extension first to univariate and then to multivariate bilinear models. The main results give conditions for stationarity, ergodicity, invertibility, and consistency of least square estimates.

In this paper, we are interested in estimation of Bilinear Moving Average Vector (BMAV) models. We consider three vectors, which consist of a response and two predictor vectors. The data source is a monthly generated revenue (for a period of ten years) of a Local Government Area in Nigeria.

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2. MODELS SPECIFICATION

(a). Linear model:

The general multivariate analogue to the univariate Autoregressive Moving Average (ARMA) model for the vectors is.

$$X_{it} = \sum_{a=1}^{p}\sum_{i=1}^{n}\sum_{f=1}^{k}\gamma_{a\,if}X_{it-a} + C_{jt} + \sum_{b=1}^{q}\sum_{j=1}^{v}\sum_{h=1}^{m}\lambda_{b\,jh}C_{jt-b} \quad \ldots\ldots 2.1$$

Where, $X_{it} = (X_{1t}, X_{2t}, \ldots, X_{nt})$ are vectors, $\gamma_{a\,if}$ are matrices of coefficients of the autoregressive parameters, C_{jt} are the vector white noise, $\lambda_{b\,jh}$ are matrices of coefficients of the moving average parameters, (r = n).

(b). Non-linear model:

The non-linear models for $X_{1t}, X_{2t}, X_{3t}, \ldots, X_{nt}$ is:

$$X_{it} = \sum_{i=1}^{n}\sum_{j=1}^{v}\sum_{a=1}^{p}\sum_{b=0}^{q}\beta_{ab\,ij}\,X_{it-a}\,C_{jt-b} \quad \ldots\ldots 2.2$$

where,

$X_{it} = (X_{1t}, X_{2t}, \ldots, X_{nt})$. $\beta_{ab\,ij}$ are the matrices of coefficients of the respective vector product series, $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, v$, $a = 1, 2, \ldots, p$, $b = 0, 1, \ldots, q$.

(c) Bilinear Moving Average Vector Model:

The general BMAV model may be written in the form:

$$X_{nt} = C_{vt} + \sum_{b=1}^{q}\sum_{j=1}^{v}\sum_{h=1}^{m}\lambda_{b\,jh}C_{jt-b} + \sum_{i=1}^{n}\sum_{j=1}^{v}\sum_{b=1}^{q}\beta_{bij}X_{it}C_{jt-b} \quad \ldots\ldots 2.3$$

where,

Vectors and coefficients are as described above.

3. ESTIMATES for BMAV Model:

The vector moving average bilinear models are models which compose of linear and non-linear parts. The linear part describes a pure moving average vector model, while the non-linear part describes the interactive products of the stationary series and vector white noise such that the subscripts of each coefficient explains the particular pair of interactive vector product at specific lags. The models are generally additive, and each of the response vectors is a function of the distributive lags of vector white noise and the associative products. The ordinary least squares method was adopted for the estimation of the parameters. The large number of predictive vectors in the relationship makes manual calculation of the parameters using inverse of a square matrix difficult. This calls for the use of statistical software (Minitab) for the calculation of the parameters. The regression estimates obtained provides the following models for the three vector series.

$$X_{1t} = C_{1t} - 0.523C_{1t-1} + 0.182C_{2t-1} - 0.334C_{3t-2} + 0.00423X_{1t-0}C_{1t-1} - 0.00038X_{1t-0}C_{2t-1} - 0.00264X_{2t-0}C_{1t-1} + 0.00035X_{2t-0}C_{2t-1} - 0.00236X_{1t-0}C_{3t-2} - 0.00300X_{2t-0}C_{3t-2} \quad \ldots\ldots 3.4$$

From the above model,

$\lambda_{1\,11} = -0.523$, $\lambda_{1\,12} = 0.182$, $\lambda_{2\,13} = -0.334$, $\beta_{01\,11} = 0.00423$, $\beta_{01\,12} = -0.00038$

$\beta_{01\,21} = -0.00264$, $\beta_{01\,22} = 0.00035$, $\beta_{02\,13} = -0.00236$, $\beta_{02\,23} = -0.00300$

$$X_{2t} = C_{2t} - 0.037C_{1t-1} - 0.394C_{2t-1} + 0.097C_{3t-2} + 0.00313X_{1t-0}C_{1t-1} - 0.00121X_{1t-0}C_{2t-1} - 0.00272X_{2t-0}C_{1t-1} + 0.00213X_{2t-0}C_{2t-1} - 0.00210X_{1t-0}C_{3t-2} - 0.00358X_{2t-0}C_{3t-2} \quad \ldots\ldots 3.5$$

From the above model,

$\lambda_{1\,21} = -0.037$, $\lambda_{1\,22} = -0.394$, $\lambda_{2\,23} = -0.097$, $\beta_{01\,11} = 0.00313$, $\beta_{01\,12} = -0.00121$

$\beta_{01\,21} = -0.00272$, $\beta_{01\,22} = 0.00213$, $\beta_{02\,13} = -0.00210$, $\beta_{02\,23} = -0.00358$

$$X_{3t} = C_{3t} - 0.486C_{1t-1} + 0.577C_{2t-1} - 0.432C_{3t-2} + 0.00110X_{1t-0}C_{1t-1} + 0.00083X_{1t-0}C_{2t-1} - 0.00008X_{2t-0}C_{1t-1} + 0.00178X_{2t-0}C_{2t-1} - 0.00025X_{1t-0}C_{3t-2} + 0.00058X_{2t-0}C_{3t-2} \quad \ldots\ldots 3.6$$

From the above model,

$\lambda_{1\,31} = -0.486$, $\lambda_{1\,12} = 0.577$, $\lambda_{2\,13} = -0.432$, $\beta_{01\,11} = 0.00110$, $\beta_{01\,12} = 0.00083$

$\beta_{01\,21} = -0.00008$, $\beta_{01\,22} = -0.00178$, $\beta_{02\,13} = -0.00025$, $\beta_{02\,23} = 0.00058$

The estimated models for the response vectors X_{1t}, X_{2t} and X_{3t} are used to obtained estimates, which are shown in Appendix '2'. The data in Appendix '1' are original differenced data. The actual and estimated data are both plotted in each of figures '1', '2' and '3' for X_{1t}, X_{2t} and X_{3t} response vectors.

Table 1: Stationary Vector Series of Internally Generated Revenue Series

s/n	X_{1t}	X_{2t}	X_{3t}	s/n	X_{1t}	X_{2t}	X_{3t}	s/n	X_{1t}	X_{2t}	X_{3t}
1				41	24.78	0.00	24.78	81	22.59	30.71	-8.12
2	0.39	0.30	0.09	42	-16.93	-1.43	-15.50	82	50.74	-5.69	56.43
3	-1.91	-1.21	-0.70	43	7.02	9.75	-2.73	83	-48.62	11.81	-60.43
4	0.70	2.58	-1.88	44	79.30	90.65	-11.35	84	52.33	8.86	43.47
5	-4.09	-1.22	-2.87	45	3.79	-69.26	73.05	85	-43.30	-31.18	-12.12
6	4.35	3.09	1.26	46	174.75	139.03	35.72	86	75.45	-58.35	133.80
7	1.23	-3.51	4.74	47	-176.52	-101.04	-75.48	87	73.60	136.35	-62.75
8	13.66	6.81	6.85	48	-88.44	-63.53	-24.91	88	-67.25	-14.47	-52.78
9	-4.13	-3.28	-0..85	49	188.36	185.39	2.97	89	-62.83	-29.62	-33.21
10	4.39	4.29	0.10	50	39.11	54.04	-14.93	90	25.49	2.60	22.89
11	3.22	4.79	-1.57	51	-118.25	-230.30	112.05	91	5.25	-58.10	63.35
12	-8.51	-0.98	-7.53	52	-58.26	4.68	-62.94	92	-57.88	13.10	-70.98
13	5.62	1.09	4.53	53	-41.76	-7.71	-34.05	93	30.41	-7.67	38.08
14	-13.03	-6.87	-6.16	54	-14.49	-43.99	29.50	94	58.78	2.85	55.93
15	-1.99	0.36	-2.35	55	27.83	32.84	-5.01	95	-74.07	-4.84	-69.23
16	1.30	-1.28	2.58	56	6.21	-38.24	44.45	96	-76.89	-34.11	-42.78
17	5.76	-2.33	8.09	57	-55.00	-14.08	-40.92	97	101.05	79.85	21.20
18	6.02	2.96	3.06	58	12.36	37.99	-25.63	98	2.19	22.61	-20.42
19	-13.08	-10.00	-3.08	59	42.61	-3.52	46.13	99	100.30	12.58	87.72
20	6.29	1.93	4.36	60	62.42	55.11	7.31	100	84.61	67.20	17.41
21	-5.10	-3.92	-1.18	61	9.91	66.45	-56.54	101	-5.34	-26.02	20.68
22	-2.96	-0.31	-2.65	62	-59.82	-57.16	-2.66	102	-138.61	-34.83	-103.78
23	1.36	4.74	-3.38	63	-22.96	-26.40	3.44	103	78.38	91.41	-13.03
24	6.27	1.86	4.41	64	64.65	88.98	-24.33	104	146.71	-3.82	150.53
25	9.14	13.55	-4.41	65	-66.99	-87.19	20.20	105	-78.43	-44.42	-34.01
26	4.23	1.05	3.18	66	-83.97	-83.22	-0.75	106	64.15	76.37	-12.22
27	22.50	23.70	-1.20	67	43.81	41.31	2.50	107	-130.03	-178.99	48.96
28	4.68	-3.47	8.15	68	38.85	38.03	0.82	108	32.07	125.16	-93.09
29	26.90	15.85	11.05	69	0.00	0.00	0.00	109	-114.67	-61.65	-53.02
30	16.62	13.32	3.30	70	-35.08	-83.39	48.31	110	91.48	22.18	69.30
31	36.64	30.57	6.07	71	4.56	64.99	-60.43	111	37.77	21.07	16.70
32	63.11	53.32	9.79	72	17.49	-22.73	40.22	112	-77.33	-0.78	-76.55
33	-21.69	-32.44	10.75	73	9.10	-19.13	28.23	113	65.86	58.41	7.45
34	-27.73	24.35	-52.08	74	-10.35	38.70	-49.05	114	75.09	-33.06	108.15
35	60.94	49.06	11.88	75	14.03	10.93	3.10	115	-194.68	-118.33	-76.35
36	111.61	115.36	-3.75	76	-59.80	-64.25	4.45	116	169.79	176.04	-6.25
37	-199.85	-188.24	-11.61	77	98.60	112.74	-14.14	117	-95.54	-114.14	18.60
38	-17.57	-41.67	24.10	78	-47.21	-43.20	-4.01	118	26.41	58.55	-32.14
39	-18.23	-16.46	-1.77	79	439.18	436.68	2.50	119	75.56	19.52	56.04
40	54.11	64.66	-10.55	80	-472.61	-473.43	0.82	120	218.32	193.90	24.42

Appendix 2: Regression Estimates Of Stationary Series Obtained From Bilinear Moving Average Vector Models

s/n	X_{1t}	X_{2t}	X_{3t}	s/n	X_{1t}	X_{2t}	X_{3t}	s/n	X_{1t}	X_{2t}	X_{3t}
1				41	30.56	9.00	21.56	81	-71.78	-87.23	15.45
2				42	-32.94	-15.68	-17.26	82	91.52	76.24	15.27
3				43	10.35	9.61	0.75	83	-81.05	-0.64	-80.41
4				44	81.11	96.69	-15.58	84	43.32	0.97	42.36
5				45	25.62	-65.24	90.86	85	-74.77	-47.22	-27.55
6	4.64	2.38	2.26	46	184.92	137.27	47.65	86	60.22	-69.29	129.51
7	2.61	-3.40	6.01	47	-197.27	-78.74	-118.53	87	62.51	127.81	-65.30
8	13.72	6.08	7.65	48	-96.04	-59.50	-36.54	88	-52.46	31.11	-83.57
9	-4.42	-1.99	-2.43	49	182.20	210.96	-28.77	89	-122.34	-65.04	-57.30
10	0.48	3.11	-2.63	50	94.10	90.97	3.14	90	30.14	-1.86	31.99
11	1.03	5.84	-4.81	51	-146.60	-328.93	182.33	91	21.24	-75.50	96.74
12	-9.39	-1.33	-8.06	52	-69.30	-20.13	-49.17	92	-55.94	2.43	-58.37
13	5.95	-0.24	6.19	53	17.72	66.30	-48.58	93	26.78	0.34	26.44
14	-12.30	-7.56	-4.74	54	-50.25	-49.40	-0.85	94	46.76	9.26	37.49
15	0.48	-0.11	0.59	55	31.66	33.79	-2.13	95	-71.93	-1.35	-70.57
16	2.63	-0.53	3.16	56	32.27	-30.84	63.11	96	-60.75	-18.17	-42.57
17	6.92	-2.51	9.43	57	-79.94	-28.97	-50.97	97	83.71	83.16	0.55
18	6.73	3.11	3.62	58	8.48	41.27	-32.79	98	32.84	36.32	-3.48
19	-14.15	-8.83	-5.62	59	42.12	0.78	41.34	99	130.02	1.98	128.05
20	1.59	1.08	0.50	60	72.77	55.84	16.93	100	110.52	86.04	24.49
21	-5.96	-1.52	-4.44	61	12.61	62.81	-50.20	101	3.90	-7.75	11.65
22	-3.36	-0.45	-2.91	62	-70.68	-67.26	-3.43	102	-196.82	-53.14	-143.68
23	1.25	5.41	-4.16	63	-40.91	-48.93	8.02	103	29.97	64.77	-34.80
24	7.01	2.14	4.87	64	97.41	106.07	-8.66	104	204.33	29.03	175.29
25	10.00	12.76	-2.76	65	-57.90	-94.59	36.69	105	-113.38	-93.57	-19.81
26	6.77	1.38	5.39	66	-107.76	-112.56	4.80	106	32.10	36.84	-4.74
27	20.83	21.49	-0.67	67	45.96	46.37	-0.41	107	-158.93	-154.79	-4.14
28	7.86	-2.13	9.99	68	47.93	56.90	-8.97	108	-7.23	79.98	-87.21

29	23.69	11.60	12.10	69	-7.32	-5.08	-2.24	109	-46.00	-2.29	-43.71
30	17.03	15.29	1.74	70	-39.46	-92.23	52.77	110	60.48	-9.90	70.38
31	31.06	27.69	3.37	71	-2.86	56.13	-58.99	111	85.99	43.13	42.86
32	60.79	53.50	7.29	72	33.62	-1.05	34.67	112	-71.03	-11.93	-59.09
33	-24.90	-34.67	9.77	73	-15.10	-34.13	19.03	113	43.19	46.20	-3.01
34	-42.75	10.72	-53.48	74	-2.95	40.12	-43.07	114	89.33	-25.45	114.78
35	59.85	51.44	8.41	75	14.70	15.49	-0.79	115	-239.24	-162.51	-76.73
36	145.22	145.93	-0.71	76	-83.51	-81.23	-2.28	116	61.92	76.37	-14.46
37	-224.51	-232.73	8.22	77	99.34	103.58	-4.23	117	-129.69	-115.40	-14.30
38	-48.90	-81.18	32.28	78	-33.89	-35.95	2.56	118	-0.29	17.02	-17.30
39	5.76	20.64	-14.88	79	433.24	432.08	1.17	119	115.42	45.29	70.13
40	44.84	61.83	-16.99	80	-770.59	-744.06	-26.54	120	262.90	227.69	35.21

CONCLUSION

It is a well known fact that not all series assume linearity in modelling. Some observations are both linear and nonlinear due to certain conditions in which they occur. Bilinear models have two parts, the linear and nonlinear parts. In this special moving average case, the first part is the linear combination of the vectors white noise and its associative parameters, while the second part is sum of products of the vector series and time varying vector white noise. A statistical software (Minitab) is used for the estimation of the parameters. The models established are called Bilinear Moving Average Vector (BMAV) Models. The estimates obtained for X_{1t}, X_{2t}, and X_{3t} shown in appendix '2' are plotted in figures '1', '2' and '3' and these estimates prove reality of the models obtained.

ACTUAL PLOTS WITH BLACK DOTS
ESTIMATES WITH BLACK PLUS

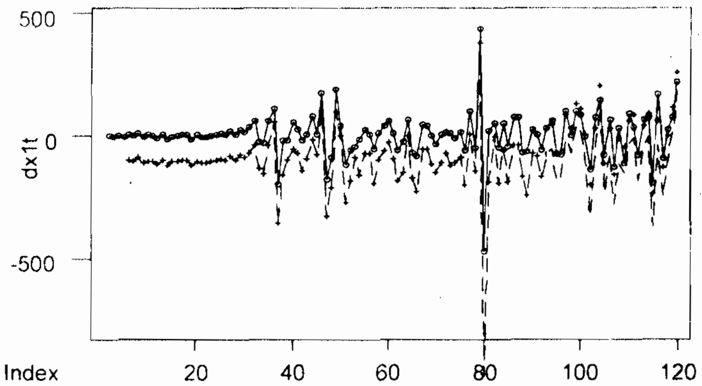

Fig. 1: Plot of Actual and Estimates of X1t

ACTUAL PLOTS WITH BLACK DOTS
ESTIMATES WITH BLACK PLUS

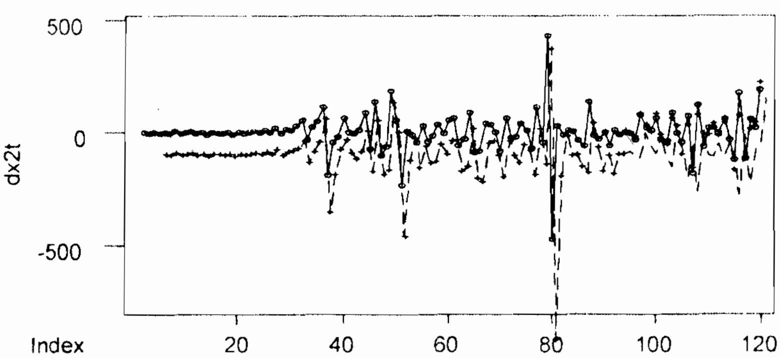

Fig. 2 PLOTS OF ACTUAL AND ESTIMATES OF X2t

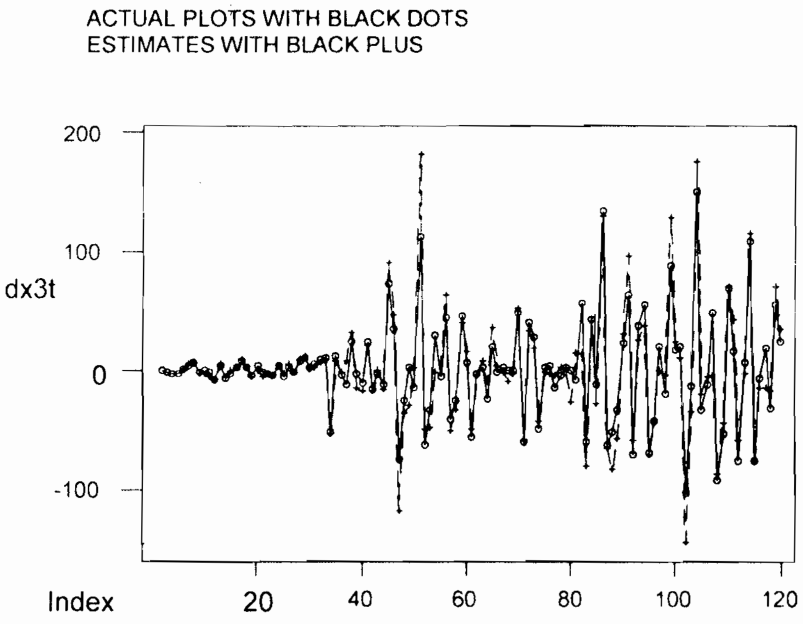

Fig 3: PLOTS OF ACTUAL AND ESTIMATES OF X3t

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