RAINFALL INTENSITY-DURATION-FREQUENCY MODELS FOR LOKOJA METROPOLIS, NIGERIA.

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ABSTRACT

Intensity-Duration-Frequency (IDF) relationship is one of the most commonly used tools in water resources engineering. The purpose of this study was to develop rainfall intensity-duration-frequency models/curves for Lokoja Metropolis, Kogi State, Nigeria. Rainfall data was obtained from Nigeria Meteorological Agency (NIMET) and sorted for frequency analysis. Five different frequency analysis techniques namely; Normal, Log-Normal, Gumbel, Pearson Type III and Log-Pearson Type III distributions were used to develop the IDF relationships for Lokoja. Storm durations of 5, 10, 20, 30, 45, 60, 90 and 120 minutes, and return periods of 2, 5, 10, 25, 50 and 100 years were adopted for the derivation of the models. Kolmogorov-Smirnov and Anderson Darling goodness of fit tests were conducted using Easy Fit software to ascertain the best distribution that fits the data. A power-law model was adopted in developing the desired IDF models/ curves for the study area. The results of the goodness of fit showed that all the five distributions were not rejected both at 5% and 1% significance levels except Pearson Type III which recorded Anderson-Darling value of 3.0814 at 30 minutes' duration which is above the critical value of 2.5018 at 5% level of significance. Although, all the distributions gave good results, Log-Pearson Type III distribution was adjudged the best for the study area because of its best ranking. It is recommended that the IDF models/curves derived in this study should be used as tools for prediction of rainfall events for design of hydraulic structures in the study area. Also, more meteorological stations should be created in the country and properly equipped to generate requisite data for planning and design of water resources systems.

KEYWORDS: Rainfall-Intensity-Duration-frequency models, Log-Pearson Type III distribution, Power model, rainfall events, hydraulic structures

INTRODUCTION

Rainfall records are required for planning and development of water resources projects. Rainfall intensity values find useful engineering application as input data in the estimation of design discharge for flood control structures as well as in erosion control studies, where they serve as important parameters in the measurement of erosivity index. Recent devastation caused by flood in different parts of the world in addition to the challenges currently being posed by uncertainties occasioned by climate change phenomenon has made the reliable estimation of rainfall events more imperative. Rainfall-intensity-duration-frequency relationship is one of the most widely used methods in urban drainage design and flood plain management. Earlier works on the establishment of such relationships includes; Meyer (1928), Sherman (1931) and Bernard (1932). Bell (1969) developed IDF relationship using a formula which enabled the computation of depth-duration ratio for certain areas of U.S.S.R. Chen (1983) developed a simple method to derive a generalized rainfall intensity-duration-frequency formula for any location in the United States using three iso-pluvial maps of the U.S Weather Bureau.

Burlando and Rosso (1996) proposed the mathematical framework to model extreme storm probabilities from the scaling properties of observed data of station precipitation, and the simple scaling and the multiple scaling conjectures was thus introduced to describe the temporal structure of extreme storm rainfall. Hadadin (2005) studied Rainfall Intensity–Duration–Frequency Relationship in the Mujib Basin in Jordan. IDF equations were developed for each of the 8 rainfall recording stations in the basin. The 8 IDF equations obtained were compared with the curves obtained by Gumbel method and Water Authority of Jordan (WAJ). The results predicted were close to the measured values.

Bara *et al*. (2009) studied the estimation of IDF curves of extreme rainfall by applying simple scaling theory to the intensity-duration-frequency (IDF) characteristics of short duration rainfall in Slovakia. AlHassoun (2011) developed empirical formulae to estimate rainfall intensity in Riyadh region. He found that there is no much difference in results of rainfall analysis of IDF curves in Riyadh area between Gumbel and LPT III methods. He attributed this to the fact that Riyadh region has semi-arid climate and flat topography where variations of precipitation are not big.

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Elsebaie (2012) conducted a study for the formulation and construction of IDF curves using data from two locations in Saudi Arabia (KSA) by using two distribution methods (Gumbel and Log Pearson type III Distribution). He found that Gumbel method gave some larger rainfall intensities estimates compared to Log Pearson type III Distribution. Also, he derived IDF equations for the two regions (Najran and HafrAlBatin) for durations varying from 10 to 1440 min and return periods from 2 to 100 years. The results obtained using the two approaches were very close at most of the return periods and had the same trend.

Generation of rainfall IDF curves is still in the infant level in Nigeria. Oyebande (1982) derived rainfall intensity-duration-frequency relationship for regions with inadequate data using Gumbel Extreme-Value Type 1 distribution (Gumbel EV-I) and applied to the annual extreme rainfall data sets generated by 11 rainfall zones to estimate the parameters and hence the intensity-duration-frequency (IDF) rainfall. Chi-squared test confirmed the appropriateness of the fitted distribution. Gumbel graphical plots and the computed confidence limits also showed that the Gumbel EV-1 function fits well to the empirical distribution. Nwaogazie and Duru (2002) developed rainfall intensity – duration – frequency models for Port-Harcourt city. Okonkwo and Mbajiorgu (2010) developed rainfall intensity-duration frequency models for South Eastern Nigeria and reported that the IDF curves were in agreement with the

IDF theory for lower return periods of 2 to 10 years, but differ for higher return periods of 50 to 100 years. Rainfall Intensity – Duration – Frequency (IDF) models for Calabar city were developed by Akpan and Okoro (2013). Ologhadien and Nwaogazie (2014) developed IDF models for some selected cities in Southern Nigeria.The Gumbel Extreme Type 1 distribution was applied to estimate 5-, 10-, 15-, 20-, 25-, 35- and 40-year return period maximum values for durations of 0.083 to 24 hrs. The developed models were return period specific. In general, the three-parameter models gave higher R^2 values with a range of 0.915 to1, indicating reliable and useful tools for estimation of storm events in the area.

Akpen *et al*. (2016) developed power and quotient IDF models for Makurdi metropolis based on Gumbel EV-1 distribution. Chi-squared tests performed on the models revealed that the power models fitted the data better than the quotient models.Ologhadien and Nwaogazie (2017) compared IDF equation types for predicting rainfall intensity in Southern Nigeria. They observed significant differences in the rainfall intensities as predicted by the various equations.

IDF models have not been developed for Lokoja town, hence designers of hydraulic structures depends on mere assumption of intensity values for estimation of the design flow. This study was therefore undertaken to bridge the gap of the non-availability of rainfall intensity-duration-frequency (IDF) models for Lokoja town.

THEORETICAL CONCEPT

The magnitude x_T of a hydrologic event may be represented as the mean μ plus the departure Δx_T of the variate from the mean as in Equation (1)(Chow 1951):

$$x_T = \mu + \Delta x_T \tag{1}$$

The departure may be taken as:

$$\Delta x_T = K_T \sigma \tag{2}$$

where, σ is standard deviation and K_T is a frequency factor.

The departure Δx_T and the frequency factor K_T are functions of the return period and the type of probability distribution to be used in the analysis. Equation (1) may therefore be expressed as:

$$x_T = \mu + K_T \sigma \tag{3}$$

which is approximated by:

$$P_T = P_{ave} + K_T \sigma \tag{4}$$

Where P_T is desired rainfall peak value for a specified frequency, P_{ave} is average of maximum rainfall corresponding to a specified duration, K_T is frequency factor and σ is the standard deviation of rainfall data. The rainfall intensity, I(mm/hr) of a specified return period, T is obtained from:

$$I = \frac{P_T}{T_d} \tag{5}$$

where T_d = duration in hours

MATERIALS AND METHODS
STUDY AREA
Lokoja town lies within the middle belt region of Nigeria. The gauging station that provided the rainfall data lies on a latitude $07^0 47'$ N and longitude $06^0 44'E$ at an altitude of 62.4 metres above mean sea level. The annual rainfall in the area is between 1016 mm and 1524 mm with mean annual temperature of 27^oC. The rainy season lasts from April to October while the dry season lasts from November to March. The land rises from about 300 metres along the Niger-Benue confluence, to the heights of between 300 and 600 metres above sea level in the uplands. Lokoja is drained by Niger and Benue rivers and their tributaries. The confluence of the Niger and Benue rivers which could be viewed from the top of Mount Patti is located within the

study area. The main vegetation type in Lokoja is Guinea savanna or parkland savanna with tall grasses and some trees.

RAINFALL DATA COLLECTION AND ANALYSIS
Rainfall data for sixteen years within the period 1976 to 1991 was obtained from the Nigeria Meteorological Agency (NIMET), Oshodi-Lagos. The data was sorted and the annual maximum rainfall amounts at specified durations 5, 10, 20, 30, 45, 60, 90 and 120 minutes were obtained. The observed annual maximum rainfall amounts (see Table 1) were divided by the corresponding durations to obtain the intensities which were ranked in descending order of magnitude as shown in Table 2.

Frequency analysis using different probability distribution functions (PDF) namely; Normal, Log-Normal, Pearson Type III, Log-Pearson Type III and the Gumbel Extreme Value type I distributions was then applied to the data to determine the best fit PDF. The value of required rainfall intensity was obtained by determining the frequency factor K_T according to the procedures outlined below and then using Equations (4) and (5) to evaluate the rainfall intensity.

NORMAL DISTRIBUTION.
Equation (3) was expressed in terms of frequency factor, K_T as:

$$K_T = \frac{x_T - \mu}{\sigma} \tag{6}$$

This is the same as the standard normal variate, z. The value of *z* corresponding to an exceedence probability, p (p = 1/T) can be calculated by finding the value of an intermediate variable w:

$$w = \left[\ln\left(\frac{1}{p^2}\right) \right]^{1/2} \qquad (0 < p \le 0.5) \tag{7}$$

then calculating z using the approximation (Chow *et al.*,1988):

$$z = w - \frac{2.515517 + 0.802853w + 0.010328w^2}{1 + 1.432788w + 0.189269w^2 + 0.001308w^3} \tag{8}$$

LOG-NORMAL DISTRIBUTION.
For the Log-Normal distribution, the same procedure as in the case of Normal distribution was applied except that it was applied to the logarithms of the variables.

EXTREME VALUE TYPE I DISTRIBUTION (GUMBEL)
For the Extreme Value Type I distribution; the frequency factor K_T is given by Equation (9) thus:

$$K_T = -\frac{\sqrt{6}}{\pi}\left\{0.5772 + \ln\left[\ln\left(\frac{T}{T-1}\right)\right]\right\} \tag{9}$$

PEARSON TYPE III DISTRIBUTION
The frequency factor depends on the return period and the coefficient of skewness, C_S. When C_S = 0, the frequency factor is equal to the standard normal variate z. When $C_S \ne 0$, K_T was calculated using Equation (10) (Kite, 1977) as:

$$K_T = z + (z^2 - 1)k + \frac{1}{3}(z^3 - 6z)k^2 - (z^2 - 1)k^3 + zk^4 + \frac{1}{3}k^5 \tag{10}$$

where, k = $C_S/6$
The value of z for a given return period was calculated using Equation (8) and then substituted in Equation (10) to obtain the frequency factor.

LOG-PEARSON TYPE III DISTRIBUTION.
The logarithms of the rainfall data, was taken; the mean, y, standard deviation, σ_y, and coefficient of skewness, C_v were calculated using the logarithms of the data. The procedure for the Pearson III was then repeated to obtain the frequency factor.

GOODNESS OF FIT TESTS
The Anderson-Darling (AD) and Kolmogorov-Smirnov (KS) tests were used for the goodness of fit test in this study. The goodness of fit test was executed in the downloadable software; Easy Fit, available at http://www.mathwave.com/easyfit-distribution-fitting.html. All test values and statistics were produced from this program. The goodness of fit test was used to examine the relationship between observed and expected frequencies, in order to determine the type of probability distribution function (PDF) that best fit the rainfall data of the study area.

DERIVATION OF IDF EQUATION
The power-law model given by Equation (11) (Chow *et al.*, 1988, Koutsoyiannis *et al.*, 1998, and AlHassoun, 2011) was adopted to derive the IDF models.

$$I = \frac{CT_r^m}{T_d^e} \tag{11}$$

where, I is the Intensity in mm/hr, T_r and T_d are Return Period and the rainfall duration respectively. C, m and e are station coefficients.

Taking the logarithm of both sides of Equation (11), we have:
$$\log I = \log K - e\log T_d \tag{12}$$
where,
$$K = CT_r^m \tag{13}$$
and represents the slope of the straight line and log K represent the Y- intercept. The plot of Log I against Log T_d for various recurrence intervals resulted to slope of e and intercept of Log K and the average e value, e_{ave}, was calculated using Equation (14) thus (Elsebaie, 2012):

$$e_{ave} = \frac{\sum e}{n} \tag{14}$$

where n represents recurrence intervals considered (n = 6 in this case).

Equation (13) was linearized by taking natural logarithm on both sides to become:

$$\text{Log } K = \text{Log } C + m \text{ Log } T_r \qquad\qquad (15)$$

A plot of Log K against Log T_r gave a straight line with Y–intercept and slope of log C and m respectively for the various probability distribution functions (PDF) adopted. The values of the station parameters C, e and m derived for the various PDF were inserted into Equations (11) to obtain the required IDF models.

RESULT S AND DISCUSSIONS

Table 1: Ranked Observed Annual Maximum Rainfall Precipitation of Different Durations for Lokoja

Ranking	5 Minutes	10 Minutes	20 Minutes	30 Minutes	45 Minutes	60 Minutes	90 Minutes	120 Minutes
1	198.0	130.0	117.7	114.3	104.6	98.6	66.7	51.3
2	191.0	128.0	109.3	106.1	85.7	80.8	61.0	51.1
3	136.0	124.9	96.5	93.7	70.6	58.0	52.0	41.3
4	134.4	119.5	85.8	81.7	63.9	57.4	43.5	33.5
5	128.7	115.9	83.2	79.3	61.1	55.4	40.2	31.3
6	118.7	113.1	81.0	74.0	60.9	53.4	38.6	30.1
7	118.4	103.0	77.7	71.3	58.9	50.4	37.8	29.5
8	117.0	102.2	76.5	70.6	57.4	47.7	35.4	27.1
9	108.2	101.5	74.8	69.0	49.5	44.9	32.9	26.2
10	106.6	100.0	72.4	66.5	49.4	44.2	32.8	25.5
11	103.0	93.1	61.3	57.3	48.2	43.7	30.2	23.5
12	97.8	91.8	60.2	57.2	46.6	39.8	28.2	22.3
13	96.4	85.7	60.0	55.8	43.9	38.1	28.0	22.1
14	89.0	81.5	59.0	49.2	42.0	36.0	27.9	21.3
15	83.9	79.9	58.6	48.0	39.7	32.2	26.9	20.9
16	67.8	64.6	50.3	47.6	35.5	25.3	22.5	15.7
μ	**118.416**	**102.158**	**76.521**	**71.344**	**57.362**	**50.365**	**37.767**	**29.532**
Σ	**34.922**	**18.927**	**19.023**	**20.001**	**17.918**	**18.149**	**12.548**	**10.345**
Cv	**1.207**	**-0.221**	**0.805**	**0.832**	**1.402**	**1.439**	**1.208**	**1.169**

The frequency analysis results for the five PDFs were as shown in Figs. 1 to 5. The results indicate that rainfall intensity decreased with increase in duration for a given return period and increased with return period for a given duration of rainfall. Generally, only small differences existed between the computed values of rainfall intensity using the 5 PDFs considered. At the return period of 25 years and duration of 45 minutes for instance; computed intensity values were 88.74, 90.68, 92.54, 96.32 and 93.98 mm/hr using Normal, Log-Normal, Pearson Type III (PT III), Log-Pearson Type III (LPT III) and Gumbel distributions respectively. Similar results were obtained for other return periods and durations. Elsebaie (2012) reported a similar agreement between LPT III and Gumbel methods in the prediction of rainfall in two regions in Saudi Arabia.

Table 2: Ranked Observed Annual Maximum Rainfall Intensities (I(mm/hr)) of different Duration for Lokoja.

| Ranking	5 Minutes P (mm)	10 Minutes P (mm)	20 Minutes P (mm)	30 Minutes P (mm)	45 Minutes P (mm)	60 Minutes P (mm)	90 Minutes P (mm)	120 Minutes P (mm)
1	16.5	21.7	39.2	57.2	78.5	98.6	100.0	102.6
2	15.9	21.3	36.4	53.0	64.3	80.8	91.5	102.1
3	11.3	20.8	32.2	46.8	52.9	58.0	77.9	82.5
4	11.2	19.9	28.6	40.9	48.0	57.4	65.2	67.0
5	10.7	19.3	27.7	39.6	45.8	55.4	60.3	62.5
6	9.9	18.8	27.0	37.0	45.7	53.4	57.8	60.2
7	9.9	17.2	25.9	35.7	44.2	50.4	56.7	59.1
8	9.8	17.0	25.5	35.3	43.0	47.7	53.2	54.1
9	9.0	16.9	24.9	34.5	37.1	44.9	49.3	52.3
10	8.9	16.7	24.1	33.3	37.0	44.2	49.2	51.0
11	8.6	15.5	20.4	28.7	36.1	43.7	45.4	47.0
12	8.1	15.3	20.1	28.6	35.0	39.8	42.3	44.6
13	8.0	14.3	20.0	27.9	32.9	38.1	42.0	44.2
14	7.4	13.6	19.7	24.6	31.5	36.0	41.8	42.6
15	7.0	13.3	19.5	24.0	29.8	32.2	40.3	41.8
16	5.6	10.8	16.8	23.8	26.6	25.3	33.7	31.4
μ	9.868	17.026	25.507	35.672	43.021	50.365	56.650	59.065
Σ	2.910	3.154	6.341	10.000	13.438	18.149	18.821	20.690
Cv	1.207	-0.221	0.805	0.832	1.402	1.439	1.208	1.169
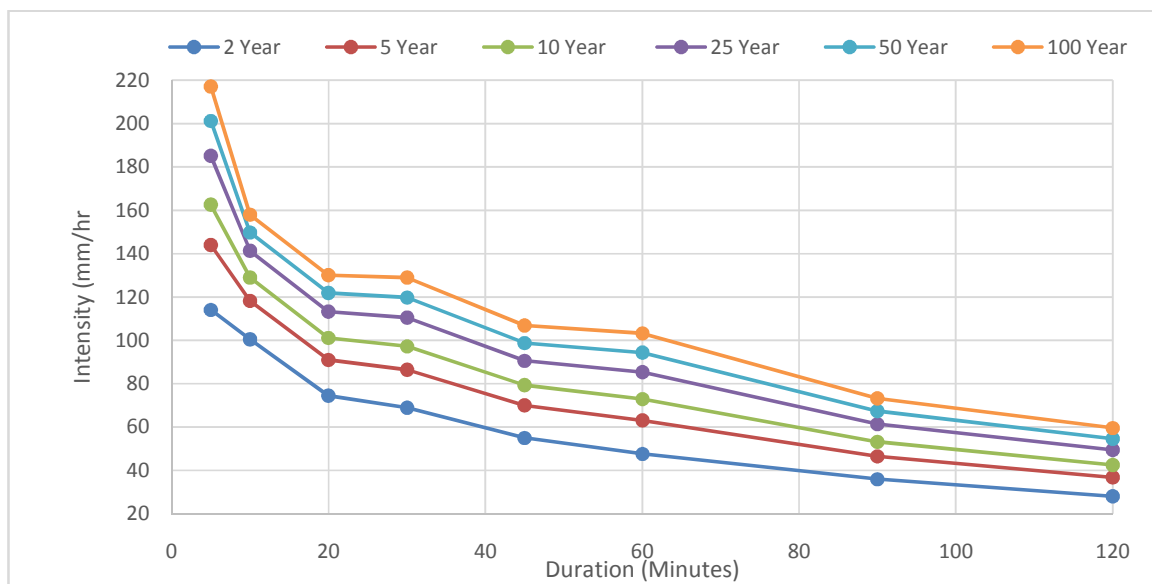
Fig 1. **Computed Rainfall Intensities for different Durations and Return Periods with Normal distribution method**
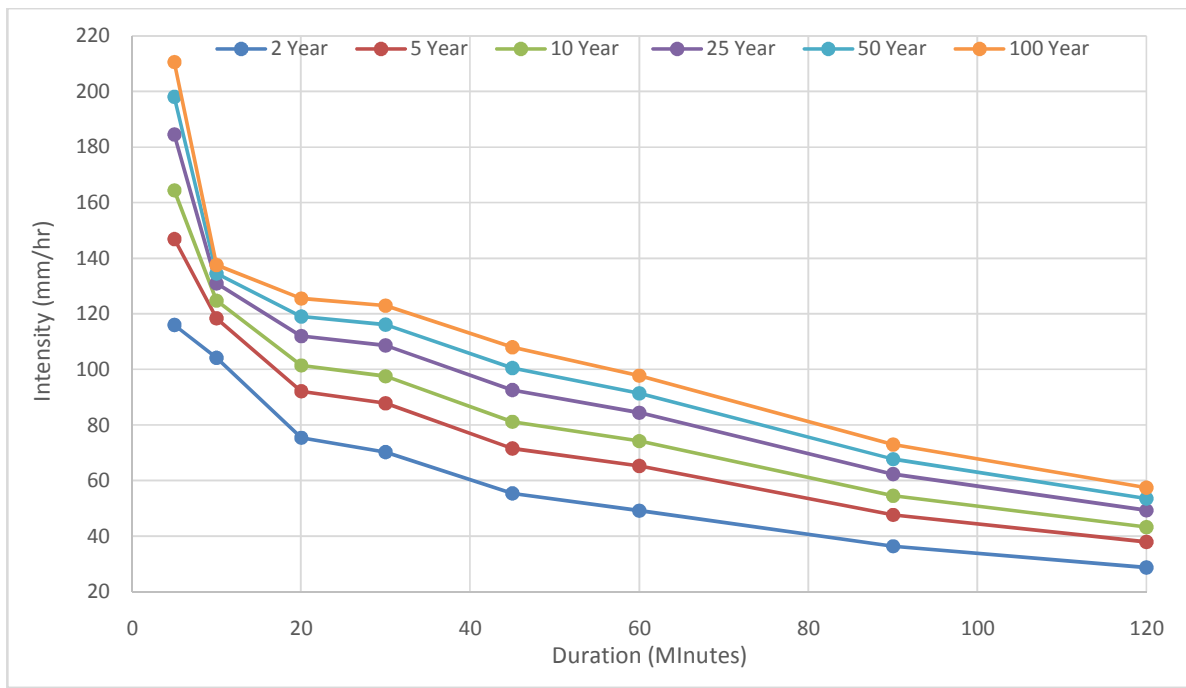
Fig. 2: Computed Rainfall Intensities for different Durations and Return Periods with Log-Normal distribution method
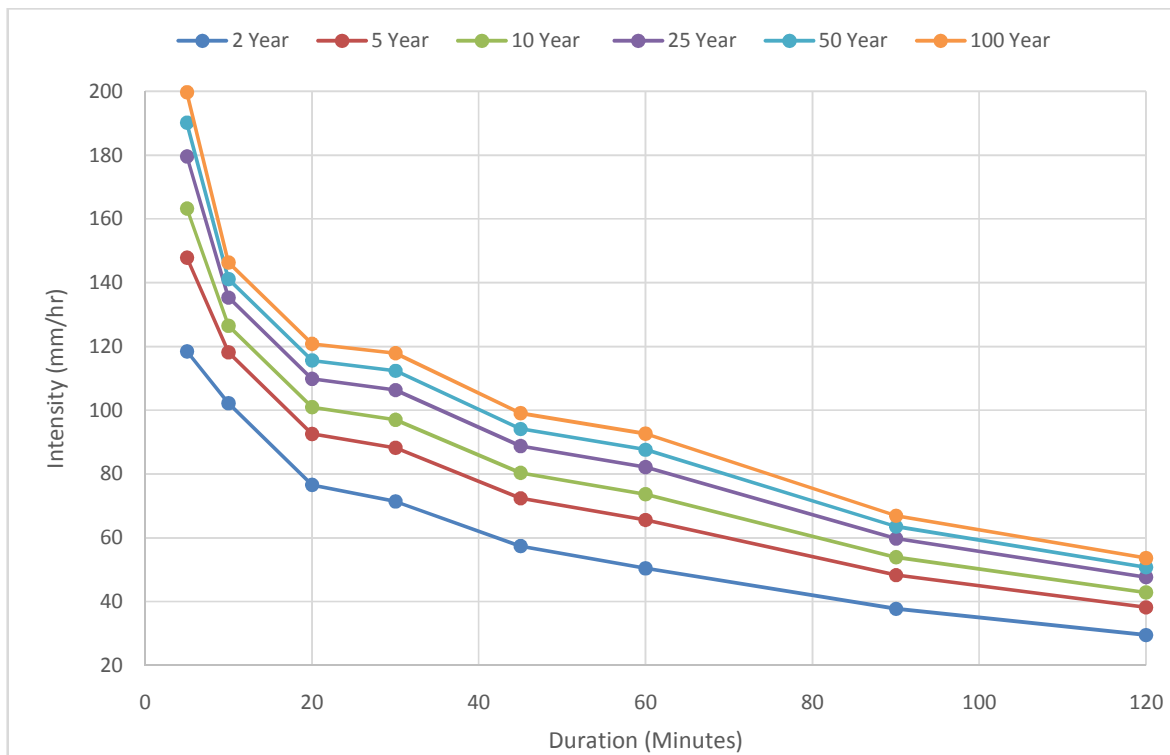

Fig. 3: Computed Rainfall Intensities for different Durations and Return Periods with Pearson III method
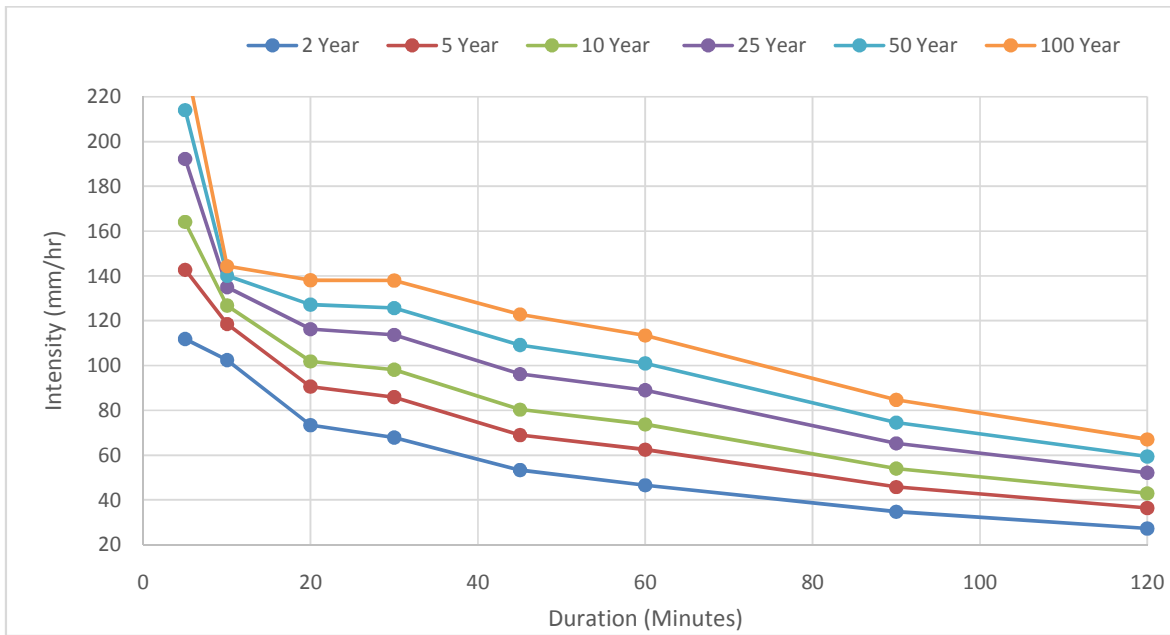
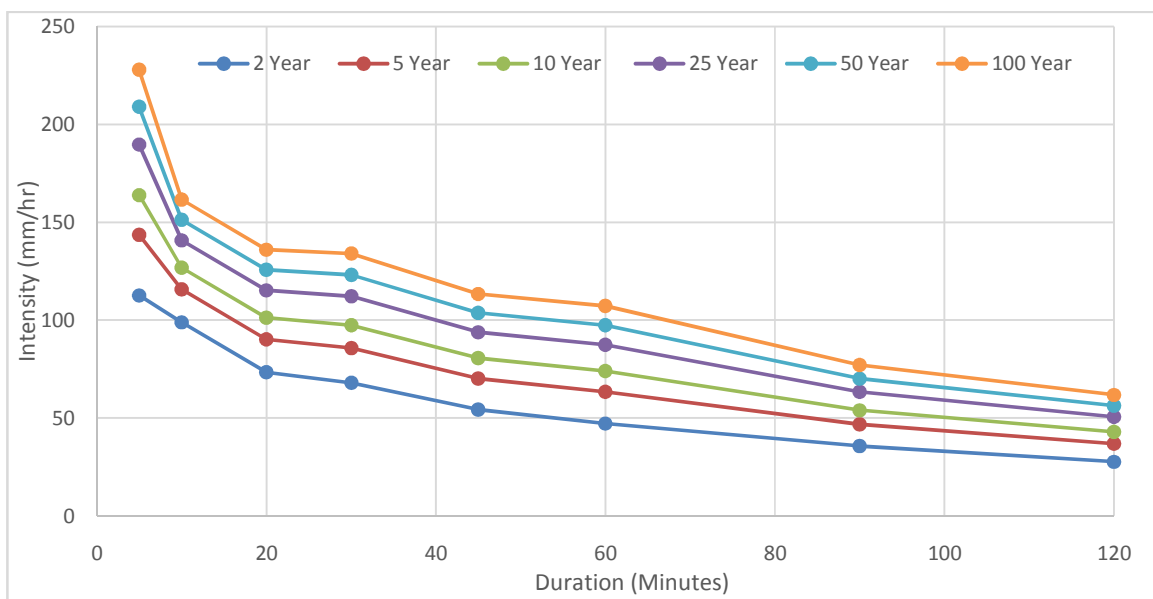**Fig. 4: Computed Rainfall Intensities for different Durations and Return Periods with Log-Pearson III method**

Fig. 5: Computed Rainfall Intensities for different Duration and Return Periods with Gumbel Method.

The goodness of fit test results was as presented in Tables 3 and 4 for Kolmogorov-Smirnov and Anderson-Darling respectively. Almost all of the five distributions were not rejected both at 5% and 1% significance levels, except that; at 30 minutes, PT III recorded Anderson-Darling value of 3.0814 which is above the critical value of 2.5018 at 5% level of significance (see Table 4). Other Anderson-Darling goodness of fit values ranged from 0.1476-0.8259 indicating goodness of fit because the calculated values of the goodness of fit were below the critical values of 2.5018 and 3.9074 at 5% and 1% levels of significance respectively. Kolmogorov-Smirnov values ranged from 0.1043-0.3152 and were all less than the respective critical values of 0.3273 and 0.3920 at 5% and 1% levels of significance.

The ranking (in order of increasing magnitude of the goodness of fit value) of the PDFs was as indicated with superscripted and italicised numerals in Tables 3 and 4. With respect to Anderson-Darling goodness of fit test, the Log Person Type III PDF consistently ranked first for all the durations reported. A critical analysis of Kolmogorov-Smirnov goodness of fit test also revealed that LPT III best described the data having ranked first in 4 and second in 3 durations. The PT III distribution was adjudged the second best PDF based on the same reasoning. The least PDF with respect to goodness of fit of the data was Normal distribution. The developed IDF models based on the power model adopted were as presented in Table 5, while the corresponding curves for the best fit PDF was presented in Fig.6. All the IDF

Table 3: Kolmogorov-Smirnov Goodness of fit test Results.

Distribution	5	10	20	30	45	60	90	120
Normal	0.1844^5	0.1075^2	0.1632^3	0.1348^3	0.1702^5	0.2121^5	0.2121^5	0.1826^5
Log-Normal	0.1309^4	0.1158^4	0.1737^5	0.1354^4	0.1520^4	0.1473^2	0.1473_2	0.1140^1
PT III	0.1306^3	0.1043^1	0.1447^1	0.3152^5	0.1044^1	0.1496^3	0.1496^3	0.1256^3
LPT III	0.1234^1	0.1143^3	0.1612^2	0.1191^1	0.1102^2	0.1392^1	0.1392^1	0.1210^2
Gumbel	0.1301^2	0.1468^5	0.1662^4	0.1241^2	0.1270^3	0.1542^4	0.1542^4	0.1318^4

Critical values: 5% = 0.3273, 1% = 0.3920

Table 4: Anderson-Darling Goodness of fit test Results.

Distribution	5	10	20	30	45	60	90	120
Normal	0.7282^5	0.1968^2	0.4703^5	0.4378^4	0.6482^5	0.7157^5	0.7157^5	0.8259^5
Log-Normal	0.3252^4	0.2643^4	0.3498^4	0.2756^3	0.3729^4	0.2529^2	0.2529^2	0.3766^4
PT III	0.3052^3	0.2194^3	0.3193^2	3.0814^5	0.1841^2	0.2682^3	0.2682^3	0.3122^2
LPT III	0.2568^1	0.1835^1	0.3066^1	0.2318^1	0.1476^1	0.2221^1	0.2221^1	0.2544^1
Gumbel	0.2793^2	0.5474^5	0.3204^3	0.2403^2	0.2314^3	0.2691^4	0.2691^4	0.3394^3

Critical values: 5% = 2.5018, 1% = 3.9074

Table 5: Summary of IDF Models developed for Lokoja

S/N	PDF	IDF Model
1	Normal	$I = \dfrac{260.21 T_r^{0.0842}}{T_d^{0.3905}}$
2	Log-Normal	$I = \dfrac{253.74 T_r^{0.1028}}{T_d^{0.3922}}$
3	Pearson Type III	$I = \dfrac{266.56 T_r^{0.0706}}{T_d^{0.3827}}$
4	Log-Pearson Type III	$I = \dfrac{262.3 T_r^{0.0826}}{T_d^{0.37875}}$
5	Gumbel	$I = \dfrac{244.06 T_r^{0.1178}}{T_d^{0.38845}}$
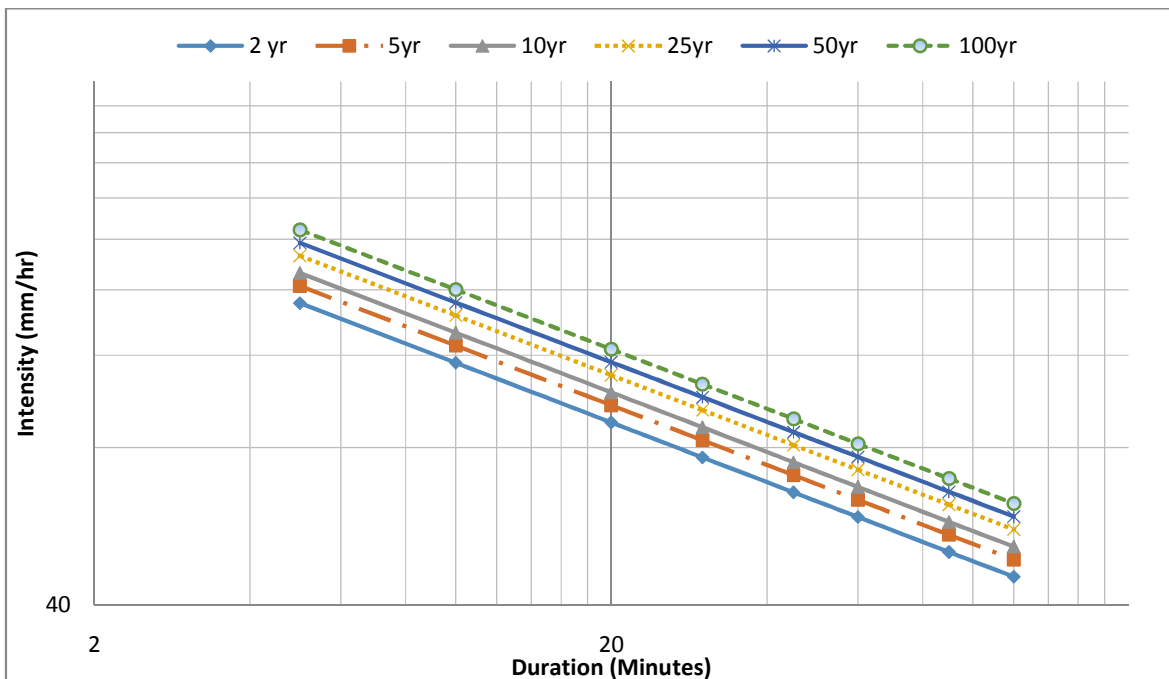
CONCLUSTION

This study has been conducted to derive IDF curves/models for Lokoja metropolis. Rainfall data for the study area was obtained from NIMET and subjected to frequency analysis using five probability distribution methods: Normal, Log-normal, Pearson type III, Log-Pearson type III and the Gumbel distributions. IDF models were derived by calibrating the quotient model proposed by Chow (1988). The results obtained showed a good match between the rainfall intensity computed from the rainfall data using the 5 PDFs and that estimated by the derived models.

All of the five distributions were not rejected at 5% significance level for all duration data tested except Pearson Type III, whose Anderson Darling result indicated a statistical value of 3.0814 while the critical value was 2.5018 at 5% significance level. Although, all the models were good, Log-Perason Type III distribution was adjudged the best distribution for the study area because of its best ranking.

RECOMMENDATIONS

Based on the findings of this study it was recommended that the derived IDF models/curves should be used as tools for prediction of rainfall events for design of hydraulic structures in the study area. Also, more metrological stations should be created within the zone and properly equipped to generate requisite data for planning and design of water resources systems in the region.

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