MODEL DEVELOPMENT FOR STREAM QUALITY MANAGEMENT

A. O. IBEJE

ABSTRACT

This study is aimed at finding the level of wastewater treatment at two sites that would achieve the desired concentrations at a minimum total cost for a river that received wastewater effluent from some other two point sources located at two sites. Pollutant concentrations and stream flow in the stream at selected reaches were obtained. Linear programming model was developed incorporating waste transfer coefficients. The model was applied at Nworie River in Imo State, Nigeria. The model solution was obtained using graphical method and the results revealed that 80% treatment efficiencies met the stream standards for the design stream flow and waste load condition at a total minimum cost. The study shows that least-cost waste removal efficiencies could be determined without prior knowledge of the cost functions.

KEYWORDS: Linear programming, wastewater treatment, model, stream

INTRODUCTION

Linear programming deals with the problem of allocating limited resources among competing activities in an optimal manner. Linear programming uses a mathematical model to describe the problem of concern. Linear programming has previously been used to study water quality problems. Lynn et al. (I962) demonstrated the use of linear programming to determine the optimal design for a sewage treatment plant. Loucks and Lynn (I967) have used linear programming models to determine the least-cost plan for waste treatment in a river basin. The decision variables were the degrees of BOD removal to be provided by each discharger for individual waste effluents. The constraints were that each discharger must provide partial or complete secondary treatment and that the dissolved oxygen concentration at any point in the stream must not go below a specified minimum value. A linear programming model has been developed and applied to determine the minimum treatment cost to maintain at least a minimum dissolved oxygen concentration at all points in the Delaware estuary (Thomann, 1963; 1965; Thomann and Sobel, 1964). Hence, this study is focused on finding the level of wastewater treatment that would achieve the desired concentrations at a minimum total cost for a river that received wastewater effluent from two point sources located at two sites.

MATERIALS AND METHOD

STUDY AREA

Nworie River is a first-order stream that runs about 5km course across Owerri metropolis in Imo State, Nigeria before emptying into another river, the Otamiri River (Figure 1). Its watershed is subject to intensive human and industrial activities resulting in the discharge of a wide range of pollutants. The river is used for various domestic applications by inhabitants of Owerri. When the public water supply fails, the river further serves as a source of direct drinking water, especially for the poorer segment of the city.

A. O. Ibeje, Department of Civil Engineering, Imo State University, Owerri, Nigeria.
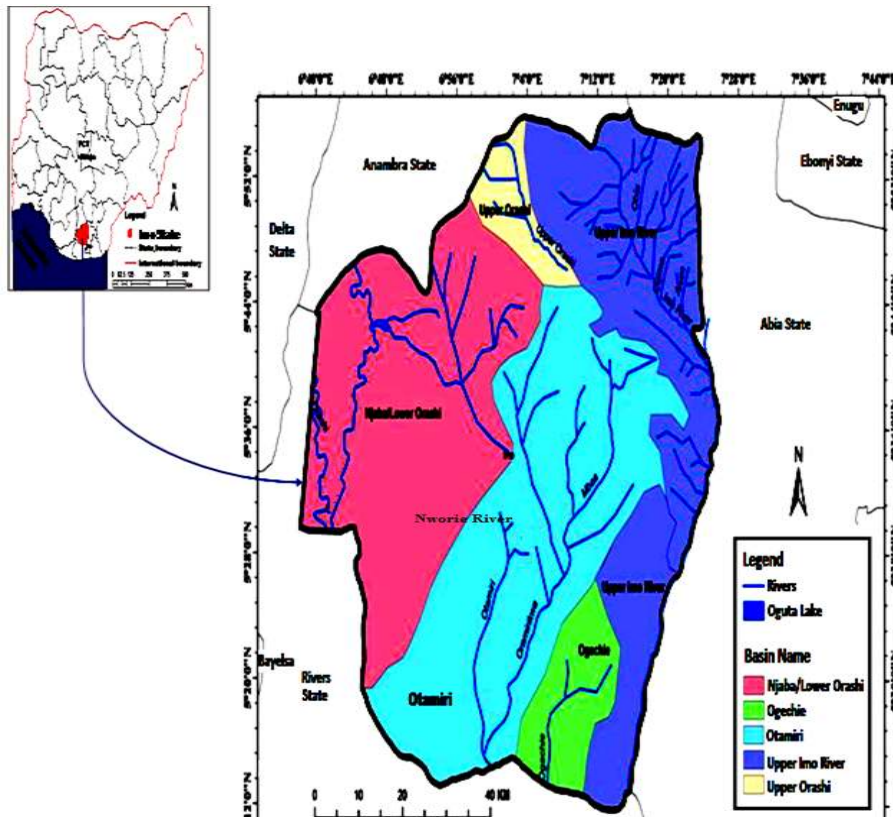
Fig 1: Location of Nworie River in Imo River Basin

Nworie River receives wastewater effluent from two point sources located at Site 1 and 2 (Figure 2). Sites 2 and 3 constitute source of water supply for some community. Thus, without some wastewater treatment at these sites, the concentration of pollutant, at the sites 2 and 3, would continue to exceed the maximum desired concentration specified by the state sanitation authority. The problem is to find the level of wastewater treatment at sites 1 and 2 that would achieve the desired concentrations at sites 2 and 3 at a minimum total cost.

The solution to this problem can be obtained through linear programming.

DATA COLLECTION

At the two sites of point pollution, discharge and velocity measurements were at the river made using current meter. After collecting water samples in plastic bottles per site, the pollutant concentrations at two sites were determined in the laboratory as biochemical oxygen demand (BOD), using standard procedures (APHA, 1998).
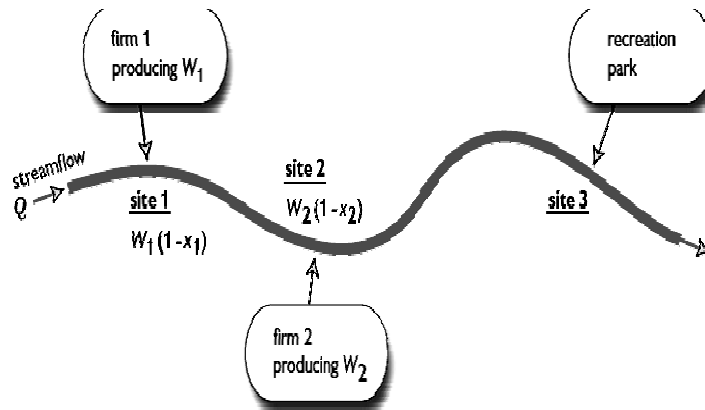

Fig 2: Pollution Sites along Nworie River

ESTIMATION OF TRANSFER COEFFICIENTS

Let P_j (mg/l) be the pollutant concentration in the stream at site j having stream flow Q_j (m³/s). Mass is expressed as kg/day can obtained as.

$$\text{mass at site j} \left(\frac{\text{kg}}{\text{day}}\right) = 86.4 P_j Q_j \tag{1}$$

The fraction a_{12} of the mass at site 1 that reaches site 2 is often assumed to be:

$$a_{12} = e^{-kt_{12}} \tag{2}$$

Where k is a rate constant and t_{12} is the time it takes a particle of pollutant to flow from site 1 to site 2. A similar expression, a_{23}, applies for the fraction of pollutant mass at site 2 that reaches site 3. The fraction of pollutant mass at site 1 that reaches site 3 is obtained as

$$a_{13} = a_{12}a_{23} \tag{3}$$
$$P_2 = P_1 a_{12} \tag{4}$$
$$P_j = P_i a_{ij} \left(\frac{Q_i}{Q_j}\right) \tag{5}$$

The design stream flow condition is the minimum seven-day average flow expected once in ten years and is assumed that the design stream flow just downstream of site1 and just upstream of site 2 are the same and equal to 12m³/s (Ibe et al.,1991). Denote the concentration of each pair of sample measurements s in the first reach (just downstream of site 1 and just upstream of site 2) as SP_{1s} and SP_{2s} and their combined error as E_s, equation (5) becomes

$$SP_{2s} + E_s = SP_{1s}a_{12}\left(\frac{Q_1}{Q_2}\right) \tag{6}$$

In order to obtain the best estimates of the unknown a_{12}, the values of a_{12} and all E_s that minimize the sum of the absolute values of all the error terms E_s, are evaluated. This objective combined could be written as

$$\text{minimze} \sum_s |E_s| \tag{7}$$
$$E_s = PE_s - NE_s \tag{8}$$
$$\text{minimze} \sum_s (PE_s + NE_s) \tag{9}$$

MODEL FORMULATION

To find the factor x_i of waste removal at site i = 1 and 2 that meet the stream quality standards at the downstream sites 2 and 3 at a minimum total cost, thus from equation 2

$$P_2 = \frac{[P_1Q_1 + W_1(1-x_1)]a_{12}}{Q_2} \tag{10}$$
$$P_3 = \frac{[P_2Q_2 + W_2(1-x_2)]a_{23}}{Q_3} \tag{11}$$
$$P_j \le P_j^{max} \quad \text{for j = 2 and 3} \tag{12}$$
$$0 \le x_i \le 1.0 \quad \text{for i = 1 and 2} \tag{13}$$

Hence, the objective is to minimize the total cost of meeting the stream quality standards P_2^{max} and P_3^{max} specified in equation 12. Let $C_i(x_i)$ represent the wastewater treatment cost function for site i, the objective can be written as:

$$\text{minimize } C_1(x_1) + C_2(x_2) \tag{14}$$

There are four unknown decision variables, $x_1, x_2, P_2,$ and P_3. All variables are assumed to be non-negative. Combining Equations 10 and 12,

$$\frac{[P_1Q_1 + W_1(1-x_1)]a_{12}}{Q_2} \le P_2^{max} \tag{15}$$

Combining equations 11 and 12, and using the fraction a_{13} (see equation 2) to predict the contribution of the pollutant concentration at site 1 on the pollutant concentration at site 3:

$$\frac{\{[P_1Q_1 + W_1(1-x_1)]a_{13} + [W_2(1-x_2)]a_{23}\}}{Q_3} \le P_3^{max} \tag{16}$$

Rewriting the water quality management model defined by equations 13 to 17 and substituting the parameter values in place of the parameters, and recalling that kg/day = 86.4 (mg/l)(m³/s):

$$\text{minimize } C_1(x_1) + C_2(x_2) \tag{17}$$

Subject to:
Water quality constraint at site 2:

$$\frac{[P_1Q_1 + W_1(1-x_1)]a_{12}}{Q_2} \le P_2^{max} \tag{18}$$

Water quality constraint at site 3:

$$\frac{\{[P_1Q_1 + W_1(1-x_1)]a_{13} + [W_2(1-x_2)]a_{23}\}}{Q_3} \le P_3^{max} \tag{19}$$

Restrictions on fractions of waste removal:

$$0 \le x_i \le 1.0 \quad for\ sites\ i = 1\ and\ 2 \tag{20}$$

Parameters values selected for the water quality management problem illustrated in Table 1

Table 1: Values of Selected Input Variables for the Model

SOURCE OF PARAMETER	PARAMETER	VALUE	REMARK
FLOW	$Q_1(m^3/s)$	10	Flow just upstream of site 1
	$Q_2(m^3/s)$	12	Flow just upstream of site 2
	$Q_3(m^3/s)$	13	Flow at park
WASTE	$W_1(kg/day)$	250,000	Pollutant mass produced at site 1
	$W_2(kg/day)$	80,000	Pollutant mass produced at site 2
POLLUTANT CONC.	$P_1(mg/l)$	32	Concentration just upstream of site 1
	$P_2(mg/l)$	20	Maximum allowable concentration upstream of 2
	$P_3(mg/l)$	20	Maximum allowable concentration at site 3
DECAY FRACTION	α_{12}	0.25	Fraction of site 1 pollutant mass at site 2
	α_{13}	0.15	Fraction of site 1 pollutant mass at site 3
	α_{14}	0.60	Fraction of site 2 pollutant mass at site 2

RESULTS AND DISCUSSION

Using Lindo software based on the solution for a_{ij} and records of flow measurements given in Tables 1 and 2, $a_{12} = 0.25, a_{23} = 0.60,$ and thus from equation 3, $a_{12}a_{23} = a_{13} = 0.15.$
Also from equation 3, $[(32)(10) + 250000(1 - x_1)/86.4]0.25/12 \leq 20$
That when simplified is $x_1 \geq 0.78$
From equation 3, $\{[(32)(10) + 250000(1 - x_1)/86.4]0.15 + [80000(1 - x_2)/86.4]0.60\}/13 \leq 20$
That when simplified is: $x_1 + 1.28x_2 \geq 1.79$
Rewriting the cost function, equation 17, as a linear function converts the model defined by equations 13 and 14 into a linear programming model.

minimize $c_1x_1 + c_2x_2$ (21)
Subject to:
$\qquad x_1 \geq 0.78$ (22)
$x_1 + 1.28x_2 \geq 1.79$ (23)
$0 \leq x_i \leq 1.0 \qquad$ for $i = 1$ and 2   (24)

MODEL SOLUTION

The feasible combinations of x_1 and x_2 is shown in Figure 3a. This graph is a plot of each constraint, showing the boundaries of the region of combinations of x_1 and x_2 that satisfy all the constraints. The shaded region is called the feasible region. Since the actual cost functions are not known, their general form was assumed, as shown in Figure 3b. Since the wasteloads produced at Site 1 were substantially greater than those produced at Site 2, and given similar site, transport, labour, and material cost conditions, it seems reasonable to assume that the cost of providing a specified level of treatment at Site 1 would exceed the cost of providing the same specified level of treatment at Site 2. It would also seem the marginal cost at Site 1 would be greater than, or at least not less than, the marginal cost at Site 2 for the same amount of treatment. The relative positions of the cost functions shown in Figure 3b are based on these assumptions.

To find the least-cost solution, it was assumed that c equal c_1. Then, let $c_1x_1 + c_2x_2 = c$ and $c/c_1 = 1$. Thus, $x_1 + x_2 = 1.0$. The plot of this line is shown in Figure 3c, as line 'a'. Line 'a' represents equal values for c_1 and c_2, and the total cost, $c_1x_1 + c_2x_2$, equal to 1. Keeping the slope of this line constant and moving it upward, representing increasing total cost, to line 'b', where it covers the nearest point in the feasible region, will identify the least-cost combination of x_1 and x_2, again assuming marginal costs are equal. In this case the solution is approximately 80% treatment at both sites. If the marginal cost of 80% treatment at site 1 is no less than the marginal cost of 80% treatment at site 2, then $c_1 \geq c_2$ and indeed the 80% treatment efficiencies will meet the stream standards for the design streamflow and wasteload condition at a total minimum cost. For any other assumption regarding c_1 and c_2, 80% treatment at both sites will result in a least-cost solution to meeting the water quality standards for those design wasteload and streamflow conditions.
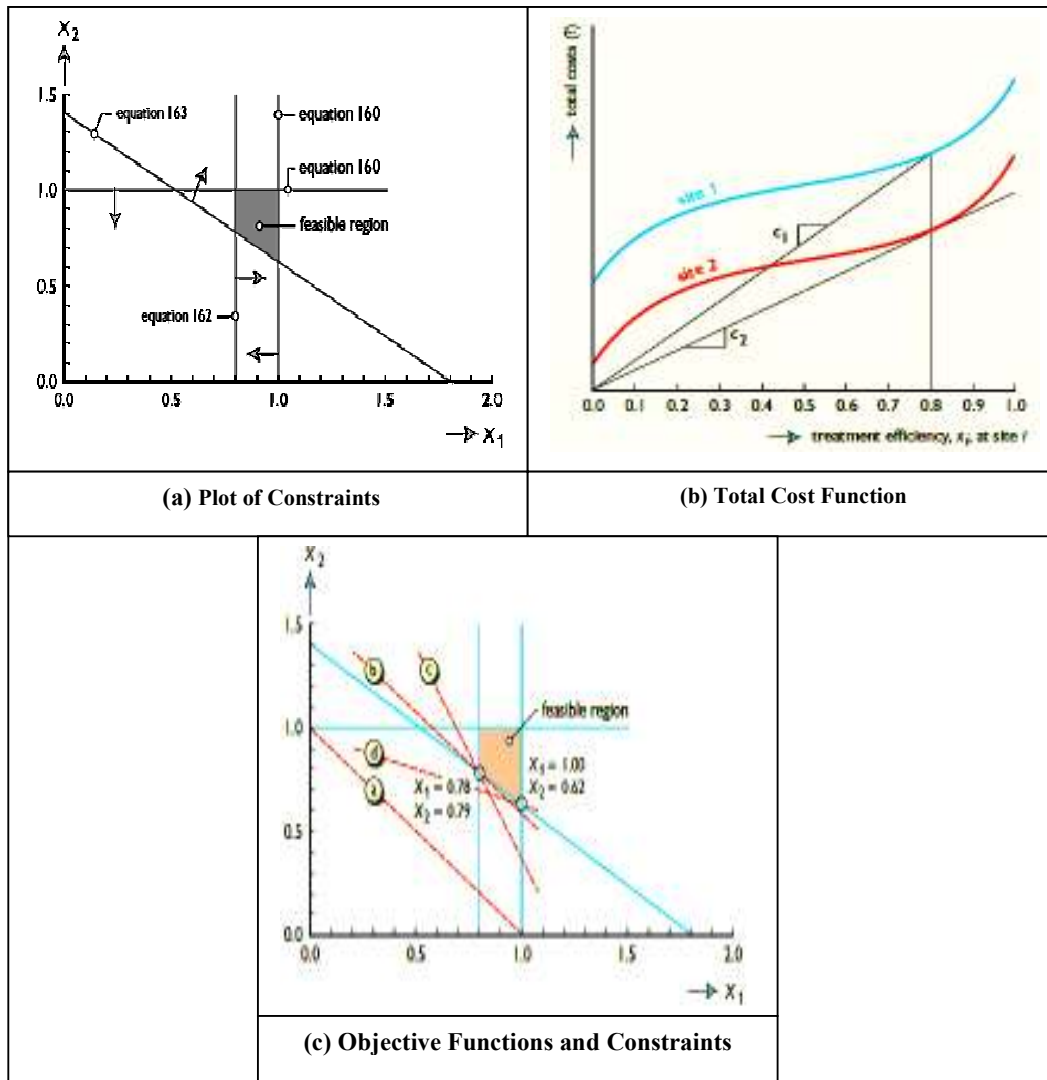
Figure 3: Graphical Solution of the Model

CONCLUSION

The least-cost waste load removal efficiencies have been determined without knowing the cost functions. No doubt the actual cost of installing the least-cost treatment efficiencies of 80% will have to be determined for issuing bonds, or making other arrangements for paying the costs. However, knowing the least-cost removal efficiencies means one does not have to spend money defining the entire cost function $C_i(x_i)$.

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