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MODIFIED PRODUCT ESTIMATOR UNDER TWO–PHASE SAMPLING

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ABSTRACT

In this paper, modification of product estimator under two-phase sampling was suggested. The modified product estimator was obtained through transformation in two cases using sample mean of auxiliary variables. Case one was when the second sample was drawn from the first sample while case two was when the second sample was drawn from the population. The bias and mean square error (MSE) of the modified product estimator was obtained. The theoretical and numerical validity of the modified product estimator under the two cases were determined to show it superiority to some considered existing product estimators. Numerical results shows that the modified product estimator under the two cases were more efficient than the considered existing estimators.

KEYWORDS: Product estimator, Two-Phase Sampling, Bias, Mean Square Error

INTRODUCTION

In sample surveys, auxiliary information is used at both selections as well as estimation stages to improve the efficiency of the estimators. The use of auxiliary information has become indispensable for improving the precision of the estimators of population parameters such as the mean and variance of a variable under study Cochran (1940) and Okafor (2002). A great variety of techniques such as the ratio, product and regression methods of estimation are commonly known in this regard. Auxiliary information can be used either at the design stage or at the estimation stage or at both the stages. Use of auxiliary information has been in practice to increase the efficiency of the estimators. When the population means of an auxiliary variate is known, so many estimators for population parameter(s) of study variate have been discussed in the literature. When correlation between study variate and auxiliary variate is positive (high) ratio method of estimation is used Cochran (1940). On the other hand, if the correlation is negative, product method of estimation is preferred (Robson (1957) and Murthy(1967). In practice, information on coefficient of variation (CV) of an auxiliary variate is seldom known. Sisodia and Dwivedi (1981) suggested a modified ratio estimator for population mean of the study variate. Later on Upadhyaya and Singh (1999) derived another ratio and product type estimators using coefficient of variation and coefficient of kurtosis of the auxiliary variate. Singh (1967) utilized information on two auxiliary variate x_1 and x_2 and suggested a ratio-cum-product estimator for population mean. Singh and Tailor (2005) utilized known correlation coefficient

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between auxiliary variates $(p_{x_1x_2})$ x_1 and x_2.

Singh and Tailor (2005) motivates authors to suggest ratio-cum-product estimators of population mean utilizing the information on co-efficient of variation of auxiliary variates i.e. C_{x_1} and C_{x_2} and co-efficient of kurtosis of auxiliary variates $\beta_2(x_1)$ and $\beta_2(x_2)$ besides the population means ($\bar{X}_1$ and $\bar{X}_2$) of auxiliary variates x_1 and x_2. Murthy (1964) suggested the use of ratio estimator $\bar{y}_p$ when $\frac{pc_y}{c_x} > \frac{1}{2}$ and unbiased estimator $\bar{y}$ when $-\frac{1}{2} \le p\frac{c_y}{c_x} \le \frac{1}{2}$, where $c_{y,}$ c_x and p are coefficients of variation of y, x and correlation between y and x respectively.

Suppose that simple random sample without replacement SRSWOR of n units is drawn from a population of N units to estimate the population mean $\bar{Y} = \frac{1}{N}\sum_{i=1}^{N} y_i$ of the study variable Y. All the sample units are observed for the variables Y and X. Let y_i and x_i denote the set of the observation for the study variable Y and X. Let the sample means ($\bar{x}, \bar{y}$) be unbiased of the population means of the auxiliary variable $\bar{X}$ and study variable $\bar{Y}$ based on the n observations. The use of auxiliary information in sample surveys was extensively discussed in well-known classical text books such as Cochran (1977), Sukhatme and Sukhatme (1970), Sukhatme et al (1984), Murthy (1967) and Yates (1960) among others. Cochran (1940) was the first to show the contribution of known auxiliary information in improving the efficiency of the estimator of population mean $\bar{Y}$ in survey sampling. Assuming the population mean $\bar{X}$of the auxiliary variable is known, he introduced a ratio estimator of population mean $\bar{Y}$.

If an auxiliary variate X has a negative correlation with study variate Y in SRSWOR scheme and $\bar{X}$ is known, then the product estimator is:

$$\bar{y}_p = \frac{\bar{y}}{\bar{X}}\bar{x}, \tag{1}$$

Murthy (1964) has derived expression for the bias and mean squared error of the product estimator as:

$$B(\bar{y}_p) = \bar{Y}(\frac{1}{n} - \frac{1}{N})\rho_{yx}S_yS_x, \tag{2}$$

and

$$M(\bar{y}_p) = \bar{Y}^2(\frac{1}{n} - \frac{1}{N})(S_y^2 + S_x^2 + 2\rho_{yx}S_yS_x). \tag{3}$$

Singh and Choudhury (2012,suggested estimators of population under double sampling scheme. The suggested estimators, biases and mean squared errors are given as:

$$\bar{y}_{pdR}^{(d)} = \bar{y}[\alpha\frac{\bar{x}}{\bar{x}_1} + (1-\alpha)\frac{\bar{x}^*}{\bar{x}_1}] \tag{4}$$

$$B(\bar{y}_{pdR}^{(do)}) = \frac{1-f^*}{n}\bar{y}C^2C_x^2 \tag{5}$$

$$M(\bar{y}_{pdR}^{(do)} = \bar{y}^2C_y^2\{\frac{1-f}{n} - \frac{1-f^*}{n}\rho^2\} \tag{6}$$

Theoretical and empirical studies were carried out and the conditions for the efficiency of their estimator over some existing estimators were established and the numerical results revealed that their estimators performed better than the related existing estimators tested.

Ray and Sahai (1980) developed the family of ratio-type and product-type estimators for estimating population mean based on simple random sampling and using one concomitant (auxiliary) variable. Using some auxiliary information, it is shown that the family contain estimator which has mean squared error less than the usual ratio, product and mean per unit estimators. Ray and Sahai (1980) proposed a family of ratio-type and product-type estimators as:

$$t_{RK\theta} = \bar{y}[\frac{K\bar{X} + \theta\bar{x}}{\bar{x} + (K + \theta - 1)\bar{X}}], \quad (7)$$

and

$$t_{PK\theta} = \bar{y}[\frac{K\bar{x} + \theta\bar{X}}{\bar{x} + (K + \theta - 1)\bar{X}}] \quad (8)$$

where K is a non-negative constant and $0 \le \theta < 1.$
The biases are given as:

$$B(t_{RK\theta}) = \bar{Y}(\frac{1}{n} - \frac{1}{N})[Q(S_x^2(K+\theta)^{-1} - \rho_{yx}S_yS_x)] \quad (9)$$

and

$$B(t_{PK\theta}) = \bar{Y}(\frac{1}{n} - \frac{1}{N})[Q\rho_{yx}S_yS_x(S_x^2(K+\theta)^{-1}(K+\theta-1))] \quad (10)$$

The mean squared errors are respectively given as:

$$M(t_{RK\theta}) = \bar{Y}^2(\frac{1}{n} - \frac{1}{N})[(S_x^2 + Q(Q - 2\beta)S_x^2)] \quad (11)$$

and

$$M(t_{PK\theta}) = \bar{Y}^2(\frac{1}{n} - \frac{1}{N})[(S_x^2 + Q(Q + 2\beta)S_x^2)] \quad (12)$$

where $Q = [\frac{1-\theta}{K+\theta}],$ and $\beta = \rho_{yx}\frac{S_y}{S_x}$

THE PROPOSED ESTIMATOR

Having studied the estimator suggested by Adebola and Adegoke (2015) defined as;

$$\bar{y}_{pd}^* = \bar{y}\frac{\bar{X}}{\bar{x}^*} + \alpha(\bar{X} - \bar{x}^*) \quad (13)$$

Thus, the modified general product estimator under two-phase sampling is given as:

$$\bar{y}_{pdIA2}^* = \bar{y}_2\frac{\bar{x}_2^*}{\bar{x}_1} + \alpha_2(x_2^* - \bar{x}_1) \quad (14)$$

where

$$\bar{x}_2^* = \frac{n_1\bar{x}_1 - n_2\bar{x}_2}{n_1 - n_2} \quad (15)$$

$\bar{y} =$ Sample mean of the study variable

$\bar{x}_1 =$ Sample mean of the auxiliary variable

$\bar{x}_2^* =$ Sample mean based on sample yet to drawn

$\alpha_1 =$ Unknown weight (0< α_2 <1)

The above modified genera product estimators under the two cases are based on the following assumptions:

(i) $\bar{x}_1 \neq 0$

(ii) $\rho_{xy} < 0$

(iii) $0 < \alpha_2 < 1$

(iv) $n_2 < \frac{1}{2} n_1$

BIAS, MSE AND OPTIMUM OF $\bar{y}^*_{pdIA2}$

$$\bar{Y}^*_{pdIA2} = \bar{Y}_2 \frac{\bar{x}^*_2}{\bar{x}_1} + \alpha_2(\bar{x}^*_2 - \bar{x}_1)$$

We write $e_0 = \frac{\bar{y}_2 - \bar{Y}}{\bar{Y}}$ $e_1 = \frac{\bar{x}_1 - \bar{X}}{\bar{X}}$ $e_2 = \frac{\bar{x}_2 - \bar{X}}{\bar{X}}$

$\bar{y}_2 = (1+e_0)\bar{Y}$ $\bar{x}_1 = (1+e_1)\bar{X}$ $\bar{x}_2 = (1+e_2)\bar{X}$

By substituting $\bar{x}_1 = (1+e_1)\bar{X}$ and $\bar{x}_2 = (1+e_2)\bar{X}$ in to equation (15), we have

$$\bar{x}^*_2 = \frac{n_1(1+e_1)\bar{X} - n_2(1+e_2)\bar{X}}{n_1 - n_2}$$

$$= \frac{\bar{X}(n_1 + n_1 e_1 - n_2 - n_2 e_2)}{n_1 - n_2}$$

$$= \bar{X}[\frac{(n_1 - n_2) + n_1 e_1 - n_2 e_2)}{n_1 - n_2}]$$

$$= \bar{X}[1 + \frac{n_1}{n_1 - n_2} e_1 - \frac{n_2}{n_1 - n_2} e_2]$$

$$= \bar{X}[1 + h_1 e_1 - h_2 e_2]$$

where

$$h_1 = \frac{n_1}{n_1 - n_2} \quad h_2 = \frac{n_2}{n_1 - n_2}$$

Case 1: Where $S_2 \subset S_1$ (Sample 2 drawn from Sample 1)

$$E(e_0) = E(e_1) = E(e_2) = 0$$

$$E(e_0^2) = (\tfrac{1}{n_2} - \tfrac{1}{n_1})\tfrac{S_y^2}{\bar{Y}^2} = f_0 \tfrac{S_y^2}{\bar{Y}^2} = f_0 C_y^2$$

$$E(e_1^2) = (\frac{1}{n} - \frac{1}{N})\frac{S_x^2}{\bar{X}^2} = f_1 \frac{S_x^2}{\bar{X}^2} = f_1 C_x^2$$

$$E(e_2^2) = (\tfrac{1}{n_2} - \tfrac{1}{n_1})\tfrac{S_x^2}{\bar{X}^2} = f_0 \tfrac{S_x^2}{\bar{X}^2} = f_0 C_x^2$$

$$E(e_0 e_1) = f_1 \ell_{xy} \tfrac{S_x S_y}{\bar{X}\bar{Y}} = f_1 \ell_{xy} C_y C_x$$

$$E(e_0 e_2) = f_0 \ell xy \tfrac{S_x S_y}{\bar{X}\bar{Y}} = f_0 \ell_{xy} C_y C_x$$

$$E(e_1 e_2) = f_1 \tfrac{S_x^2}{\bar{X}^2} = f_1 C_x^2$$

Case 2: $S_2 \subset \Omega$ **(Sample 2 drawn from Population)**

$$E(e_0)=E(e_1)=E(e_2)=0$$
$$E(e_0^2)=f_0\tfrac{S_y^2}{\bar{Y}^2}=f_0C_y^2$$
$$E(e_1^2)=f_1\tfrac{S_x^2}{\bar{X}^2}=f_1C_x^2$$
$$E(e_2^2)=f_0\tfrac{S_x^2}{\bar{X}^2}=f_0C_x^2$$
$$E(e_0e_1)=0$$
$$E(e_0e_2)=f_1\ell_{xy}\tfrac{S_xS_y}{\bar{X}\bar{Y}}=f_1\rho_{xy}C_xC_y$$
$$E(e_1e_2)=0$$

$$\bar{Y}^*_{pdIA2}=\bar{Y}_2\frac{\bar{x}_2^*}{\bar{x}_1}+\alpha_2(\bar{x}_2^*-\bar{x}_1)$$
$$=(1+e_0)\bar{Y}\frac{\bar{X}(1+h_1e_1-h_2e_2)}{(1+e_1)\bar{X}}+\alpha_2[\bar{X}(1+h_1e_1-h_2e_2)-(1+e_1)\bar{X}]$$
$$=\bar{Y}(1+e_0)(1+h_1e_1-h_2e_2)(1+e_1)^{-1}+\alpha_2\bar{X}[h_1e_1-h_2e_2-e_1]$$
$$=\bar{Y}(1+e_0)(1+h_1e_1-h_2e_2)(1-e_1+e_1^2)+\alpha_2\bar{X}[h_1e_1-h_2e_2-e_1]$$
$$=\bar{Y}(1+e_0)[1-e_1+e_1^2+h_1e_1-h_1e_1^2-h_2e_2+h_2e_1e_2]+\alpha_2\bar{X}[(h_1-1)e_1-h_2e_2]$$
$$=\bar{Y}[1+(h_1-1)e_1+(1-h_1)e_1^2-h_2e_2+h_2e_1e_2+e_0+(h_1-1)e_0e_1-h_2e_0e_2]+\alpha_2\bar{X}[(h_1-1)e_1-h_2e_2]$$
$$\bar{Y}^*_{pdIA2}-\bar{Y}=\bar{Y}[e_0+(h_1-1)e_1-h_2e_2+(1-h_1)e_1^2+h_2e_1e_2+(h_1-1)e_0e_1-h_2e_0e_1]+\alpha_2\bar{X}[(1-h_1)e_1-h_2e_2]$$

The bias $\bar{Y}^*_{pdIA2}$ is given as

$$Bias(\bar{Y}^*_{pdIA2})=\bar{Y}[(1-h_1)f_1C_x^2+h_2f_1C_x^2+(h_1-1)f_1\ell_{xy}C_xC_y-h_2f_0\rho_{xy}C_xC_y] \quad (16)$$

The MSE of $\bar{Y}^*_{pdIA2}$ is given as

$$MSE(\bar{Y}^*_{pdIA2})=\{\bar{Y}^2[f_0C_y^2+(h_1-1)^2f_1C_x^2-h_2^2f_0C_x^2]+\alpha_2^2\bar{X}^2[(h_1-1)^2f_1C_x^2+h_2^2f_0C_X^2]+$$
$$2\bar{Y}^2[(h_{1-}1)f_1\ell_{xy}C_xC_y-h_2f_0\rho_{xy}C_xC_y-(h_1-1)h_2f_1C_x^2]$$
$$-2\alpha_2^2\bar{X}^2[(h_1-1)h_2f_1C_x^2]+2\bar{Y}\bar{X}\alpha_2[(h_1-1)f_1\rho_{xy}C_xC_y-h_2f_0C_x^2$$
$$+(h_1-1)^2f_1C_x^2-(h_1-1)h_2f_1C_x^2-h_2(h_1-1)f_1C_x^2+h_2^2f_0C_x^2\}$$
$$=\bar{Y}^2[f_0C_y^2+((h_1-1)^2f_1-h_2^2f_0-2(h_1-1)h_2f_1)C_x^2+2((h_1-1)f_1-h_2f_0)\rho_{xy}C_xC_y]+$$
$$\alpha_2^2\bar{X}^2[(h_1-1)^2f_1+h_2^2f_0-2(h_1-1)h_2f_1]C_x^2+2\bar{Y}\bar{X}\alpha_2[(-h_2f_0+(h_1-1)^2f_1-(h_1-1)h_2f_1-h_2(h_1-1)f_1+h_2^2f_0)C_x^2+$$
$$(h_1-1)f_1\rho_{xy}C_xC_y]$$

The optimum value of the MSE ($\bar{Y}^*_{pdIA2}$) is obtained as follows,

$$\frac{\partial MSE(\bar{Y}^*_{pdIA2})}{\partial \alpha_2} = 2\alpha_2 \bar{X}^2[(h_1-1)f_1 + h_2^2 f_0 - 2(h_1-1)h_2 f_1]C_x^2 + 2\bar{X}\bar{Y}[-h_2 f_0 + (h_1-1)^2 f_1 - (h_1-1)h_2 f_1 - h_2(h_1-1)f_1 + h_2^2 f_0)C_x^2 + (h_1-1)f_1\rho_{xy}C_xC_y] = 0$$

$$\alpha_2 = \frac{\bar{Y}[(-h_2 f_0 + (h_1-1)^2 f_1 - (h_1-1)h_2 f_1 - h_2(h_1-1)f_1 + h_2^2 f_0)C_x + (h_1-1)f_1\rho_{xy}C_y]}{\bar{X}[(h_1-1)^2 f_1 + h_2^2 f_0 - 2(h_1-1)h_2 f_1]C_x} \quad (17)$$

$$MSE(\bar{Y}^*_{pdIA2})^{opt} = \frac{\bar{Y}^2\{[f_0C_y^2 + ((h_1-1)^2 - h_2^2 f_0 - 2(h_1-1)h_2 f_1)C_x^2 + 2((h_1-1)f_1 - h_2 f_0)\rho_{xy}C_xC_Y]\} - [(-h_2 f_0 + (h_1-1)^2 f_1 - (h_1-1)h_2 f_1 - h_2(h_1-1)f_1 + h_2^2 f_0)C_x + (h_1-1)f_1\rho_{xy}C_y]^2}{(h_1-1)^2 f_1 + h_2 f_0 - 2(h_1-1)h_2 f_1} \quad (18)$$

Under Case 2

$$(\bar{Y}^*_{pdIA2})_{II} = \bar{Y}[e_0 + (h_1-1)e_1 - h_2e_2 + (1-h_1)e_1^2 + h_2e_1e_2 + (h_1-1)e_0e_1 - h_2e_0e_1] + \alpha_2\bar{X}[(1-h_1)e_1 - h_2e_2]$$

Take expectation to obtain bias.

$$Bias(\bar{Y}^*_{pdIA2})_{II} = \bar{Y}[(1-h_1)f_1C_x^2] \quad (19)$$

Square $(\bar{Y}^*_{pdIA2})_{II}$ and take expectation, the MSE of $(\bar{Y}^*_{pdIA2})_{II}$ under case 2 is obtained as:

$$MSE(\bar{Y}^*_{pdIA2})_{II} = \bar{Y}^2[f_0C_y^2 + (h_1-1)^2 f_1C_x^2 + h_2^2 f_0C_x^2 - 2h_2 f_1\ell_{xy}C_yC_x] + \alpha_2^2\bar{X}^2[(1-h_1)^2 f_1C_x^2 + h_2^2 f_0C_x^2]$$
$$+2\overline{YX}\alpha_2[f_1\ell_{xy}C_yC_x - (1-h_1)^2 f_1C_x^2 + h_2^2 f_0C_x^2]$$

$$\frac{\partial MSE(\bar{Y}^*_{pdIA2})_{II}}{\partial \alpha_2} = 2\alpha_2\bar{X}_2[(1-h_1)^2 f_1C_x^2 + h_2^2 f_0C_x^2] + 2\overline{YX}[f_1\ell_{xy}C_yC_x - (1-h_1)^2 f_1C_x^2 + h_2^2 f_0C_x^2] = 0$$

$$\alpha_2 = \frac{\bar{Y}[f_1\ell_{xy}C_yC_x - (1-h_1)^2 f_1C_x^2 + h_2^2 f_0C_x^2]}{\bar{X}[(1-h_1)^2 f_1C_x^2 + h_2^2 f_0C_x^2]} \quad (20)$$

$$MSE(\bar{Y}^*_{pdIA2})_{II}^{Opt} = \bar{Y}[\frac{[f_0C_y^2 + (1-h_1)^2 f_1C_x^2 + h_2^2 f_0C_x^2 - 2h_2 f_1\ell_{xy}C_yC_x] - [f_1\ell_{xy}C_yC_x - (1-h_1)^2 f_1C_x^2 + h_2^2 f_0C_2^2]^2}{(1-h_1)f_1C_x^2 + h_2^2 f_0C_x^2}] \quad (21)$$

EMPIRICAL STUDY

To analyze the performance of the modified estimator under the two cases in comparison to some other related estimators, three natural data sets are being considered. The sources of the data, the nature of the variates y and x and the values of the various parameters are given as follows

Data 1: Yadav, Gupta, Mishra and Shukla (2016)

$\bar{Y} = 42, C_Y = 0.1303, C_X = 0.0458, \rho = -0.73, N = 10, n_2 = 2, n_1 = 4$

Data 2: Yadav, Gupta, Mishra and Shukla (2016)

$\bar{Y} = 52, C_Y = 0.1562, C_X = 0.0458, \rho = -0.94, N = 10, n_2 = 2, n_1 = 4$

Data 3: Singh, Sharma and Tarray (2015)

$\bar{Y} = 4.066, C_Y = 0.3126, C_X = 0.2313, \rho = -0.718, N = 12, n_2 = 3, n_1 = 8$

EFFICIENCY COMPARISONS

Table 1: Bias, MSE and PRE of $\overline{y}^*_{pdIA2}$ and some Related Product Estimators using data 1:

ESTIMATORS	BIAS	MSE	PRE
Sample mean	0	11.97974	100
Murthy (1964)	-0.03794801	7.312013	163.8364
Singh and Choudhury (2012)	0.09500005	7.989738	149.9391
Modified $\overline{y}^*_{pdIA2}$ (case 1)	-0.02744564	5.02502	238.4018
Modified $\overline{y}^*_{pdIA2}$ (case 2)	-0.03524035	39.76718	30.12469

Table 1 shows the biases, mean square errors and percentage relative efficiency of the modified and some related product estimators using data 1.The results also revealed that the modified estimator has minimum MSE and highest PRE among the considered estimators in case 1, with exception of case 2. This implies that the modified estimator is more efficient under case 1.

Table 2: Bias, MSE and PRE of $\overline{y}^*_{pdIA2}$ and some Related Product Estimators using Data 2:

ESTIMATORS	BIAS	MSE	PRE
Sample mean	0	26.38935	100
Murthy (1964)	-0.09624331	14.11124	187.0094
Singh and Choudhury (2012)	0.28026	11.81583	223.3389
Modified $\overline{y}^*_{pdIA2}$ (case 1)	-0.05245283	10.80354	244.2657
Modified $\overline{y}^*_{pdIA2}$ (case 2)	-0.04363091	112.3052	23.4979

Table 2 shows the biases, mean square errors and percentage relative efficiency of the modified and some related product estimators using data 2. The results also revealed that the modified estimator has minimum MSE and highest PRE among the considered estimators in case 1, with exception of case 2. This implies that the modified estimator is more efficient under case 1.

Table 3: Bias, MSE and PRE of $\overline{y}^*_{pdIA2}$ and some Related Product Estimators using Data 3:

ESTIMATORS	BIAS	MSE	PRE
Sample mean	0	0.4038803	100
Murthy (1964)	0.0016113	0.1958644	206.204
Singh and Choudhury (2012)	0.04267298	0.230372	175.3166
Modified $\overline{y}^*_{pdIA2}$ (case 1)	-0.005277113	0.1358868	297.2182
Modified $\overline{y}^*_{pdIA2}$ (case 2)	-0.03262946	1.5225	26.52744

Table 3 shows the biases, mean square errors and percentage relative efficiency of the modified and some related product estimators using data 3. The results also revealed that the modified estimator has minimum MSE and highest PRE among the considered estimators in case 1, with exception of case 2. This implies that the modified estimator is more efficient under case 1.

DISCUSSION OF RESULTS

Table 1-3 shows the biases, mean square errors and percentage relative efficiency of the modified and some related product estimators using data 1- 3. The results also revealed that the modified estimator has minimum MSE and highest PRE among the considered estimators in case 1, with exception of case 2. This implies that the modified estimator is more efficient under case 1.

CONCLUSION

In this paper, we proposed a product estimator for the estimation of the sample mean using two phase sampling scheme under two cases. The numerical comparison of the estimators under the two cases with some other related existing estimators are shown in Tables 1 to 3where, we observed that the performance of the modified product estimator under case 1 was better than the existing considered related estimator and even better than the estimator under case 2. Hence, recommended for usage in sample survey.

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