A TWO-PRONGED TRUNCATED DEFERRED SAMPLING PLAN (TTDSP) FOR LOG-LOGISTIC DISTRIBUTION

ELAKHE, S. O., BRAIMAH J. O., OGBEIDE, E. M. AND IKPOTOKIN, O

(Received 16 November 2020; Revision Accepted 30 November 2021)

ABSTRACT

This paper is aimed at developing a new truncated sampling plan that uses information from precedent and successive lots for lot disposition with a pretention that the life-time of a particular product assumes a Log-logistic distribution. A new Two-pronged Truncated Deferred Sampling Plan (TTDSP) for Log-logistic distribution is proposed when the testing is truncated at a precise time. The best possible sample sizes are obtained under a given Maximum Allowable Percent Defective (MAPD), Test Suspension Ratios (TSR) and acceptance numbers (c). A formula for calculating the operating characteristics of the proposed plan is also developed. The operating characteristics and mean-ratio values were used to measure the performance of the plan. The findings of the study show that: Log-logistic distribution has a decreasing failure rate; furthermore, as mean-life ratio increase, the failure rate reduces; the sample size increase as the acceptance number, test suspension ratios and maximum allowable percent defective increases. The study concludes that the new minimum sample sizes were smaller which makes the plan a more economical plan to adopt when cost and time of production is costly and the experiment being destructive.

KEYWORDS: Consumers Risk, Mean life, Minimum Sample size, Operating Characteristics, Producers Risk.

1.0 INTRODUCTION

Acceptance sampling inspection is a very important field in quality assurance. It is used to reject or accept products presented for inspection. This field was made popular by Dodge and Roming [1]. Acceptance sampling inspection is the process of investigating samples of lot to confirm whether it meets certain minimum quality requirement so that the lot can be accepted or rejected if otherwise. Dodge and Roming summarize the process of inspecting sampling plan as follows: a sample is selected at random from the lot and the chances of the products being accepted or rejected depends on the information obtained from this inspected sample [2]. Therefore, acceptance sampling inspection is all about evaluation and decision making as regards products in quality assurance. Acceptance inspection sampling is one of the key mechanism in the field of quality control, and is mainly used for incoming products’ inspection. In inspection sampling plans, such as those developed by [3 and 4], a lot under inspection is said to be accepted if the number of failures in the inspected samples is less or equal to the acceptance number.

There are different distributions that model these life products. In order to minimize the producer and consumer's risks, the choice of sample size (n) and other parameters (c,t, $\hat{t}$, $\mu$, $\hat{\mu}$, $P^*$) is done in a logical way, through ‘trial and error’ method adopted by different authors like [1, 5, 6 and 7]. These authors did not put into consideration the two types of risks in their developed plans but considered either the producer’s or consumer’s risk. Furthermore, the failure rate of these life distributions was not put into consideration. A prior knowledge of failure rate/pattern of a product will help the producer to further reduce the failure rate his products. These are the drive behind this study.

2.0 METHODOLOGY

2.1 The Log-logistic Distribution

[8] introduced a new parameter to an existing distribution and called it log-logistic distribution. This distribution has also played an important role in reliability testing of Electronic component, Computer components and in area of Survival analysis.

Elakhe, S. O., Department of Mathematics and Statistics, P.M.B 14, Ambrose Alli University, Ekpoma, Edo State, Nigeria

Braimah J. O., Department of Mathematics and Statistics, P.M.B 14, Ambrose Alli University, Ekpoma, Edo State, Nigeria

Ogbeide, E. M., Department of Mathematics and Statistics, P.M.B 14, Ambrose Alli University, Ekpoma, Edo State, Nigeria

Ikpotokin, O., Department of Mathematics and Statistics, P.M.B 14, Ambrose Alli University, Ekpoma, Edo State, Nigeria

© 2021 Bachudo Science Co. Ltd. This work is Licensed under Creative Commons Attribution 4.0 International License.
The probability density function (pdf) and cumulative distribution function of a Log-Logistic distribution is shown below:

\[ f(t; \alpha, \mu_0) = \frac{(\frac{t}{\mu_0})^{\alpha-1} e^{-\frac{t}{\mu_0}}}{1 + e^{-\frac{t}{\mu_0}}} \]  

(1)

\[ F(t; \alpha, \mu_0) = \frac{1 - e^{-\frac{t}{\mu_0}}}{1 + e^{-\frac{t}{\mu_0}}} \]  

(2)

where \( \alpha > 0 \) and \( \mu > 0 \) are the shape and scale parameters respectively.

### 2.2 Failure Rate Function of Lomax Products’ Distribution

Even if the probability density function (pdf) describes the time until an item will fail totally, it does not give the detailed probability of how the item will continue to function for a given period of time or how the probability of failure depends on the quality of raw materials. The failure function is therefore defined mathematically as:

\[ R(t) = Pr(T > t) = \int_t^{\infty} f(x)dx \]  

(3)

= 1 - F(t) = probability of an item meeting requirement for at least till age (time t), where F(t) is the cumulative distribution function (cdf). Thus, a useful function used in life time analysis is the failure rate. It is defined as:

\[ h(t) = \frac{f(t)}{1-F(t)} \]  

(4)

Equation (4) is the rate of failure for a specified testing till age t, where f(t) is the pdf and F(t) is the cdf respectively. Therefore, the behaviour of failure rate h(t) can be used to describe product performance over time.

### 2.3 Minimum Sample Size (n)

Assuming the probability of accepting a lot is fixed and the lot size \( N \) is large enough, the binomial distribution is used [9 and 10]. Thus, the acceptance and rejection criteria for such lot are equivalent to the decisions of either accepting or rejecting the hypothesis \( H_0: \mu \geq \mu_0 \). Suppose we want to obtain the sample size (n) such that:

\[ \sum_{i=0}^{c} \binom{N}{i} p^i (1-p)^{N-i} \leq 1 - \beta \]  

(5)

If \( p = F(t, \mu) \) which is an increasing function of \( \frac{t}{\mu_0} \), it is therefore sufficient to specify this ratio. [1] said that if the lot is very large and \( p \) is not too small, therefore, equation (5) can be rewritten as:

\[ \sum_{i=0}^{c} e^{-\mu p i / \mu} \leq 1 - \beta \]  

(6)

where \( \mu = np = nF(t; \mu) \).

We therefore have

\[ \sum_{i=0}^{c} e^{-\mu p i / \mu} = 1 - G_k(\mu, \mu) \]  

(7)

where \( k = c + 1 \) and \( (\alpha, \mu) \) denotes the cumulative distribution function of a gamma distribution with the scale and shape parameters \( \alpha \) and \( \mu \) respectively. [11] gave the sample size formula as:

\[ n = \left[ \frac{\gamma_{c+1, p^*}}{p} \right] + 1 \]  

(8)

where \( p^* = \frac{q}{1-q} \) and \( q \) is the specified probability of failure, \( y_{c+1, p^*} \) is the \( P^* \) percentage point at a standardized gamma variable with shape parameter. This approximate was later discussed in [12]. Using the association between Gamma and Chi-square random variable, equation (8) becomes:

\[ n = \left[ \frac{\chi^2_{c+2, p^*}}{2P} \right] + 1 \]  

(9)

where \( P = \frac{\beta}{\alpha} \) and \( \beta \) is the failure probability and can also be taken as the producer’s risk, that is, the cumulative life distribution function. Assuming the Chi-square \( (\chi^2) \) random variables, equation (9) is modified as:

\[ n = \left[ \frac{\chi^2_{c+2, p^*}}{2F(t, \mu)} \right] + 1 \]  

(10)

where \( \beta \) represents the consumer’s risk, \( F(t; \mu) \) is the failure probability and can also be taken as the consumer’s risk. \( \chi^2 \) denotes the consumer’s risk of a \( \chi^2 \) variable with \( v = 2(c + 1) \) degrees of freedom. Since one of the objectives of this study is to arrive at a values of \( n \) that will result to a plan with reduced sample size needed to be selected from the lot for inspection and result to reduced inspection cost and time, the approximate value of \( n \) can then be reduce by introducing parameter \( p \) (shape parameter of the failure rate \( F(t; \mu) \)). When \( \rho < 1.5 \), the sample size values become very large and when \( p > 2.5 \), the sample size become approximately one irrespective of the combination of the parameters.

On replacing \( F(t, \mu) \) in (10) with \( \rho F(t, \mu) \), the resulting equation becomes

\[ n = \left[ \frac{\chi^2_{c+2, p \rho}}{2F(t, \mu)} \right] + 1 \]  

(11)

Therefore, equation (11) is the approximate of the improved sample size \( n \).

### 2.4 Proposition of Two-pronged Truncated Deferred Sampling Plan (TTDSP)

The Operating Characteristics (OC) function for this Sampling Plan (TTDSP) is derived. The algorithm is also given and the Sampling Plan will be valid when the lot quality is not constant. A Computer program (R) is also written to compute the table values which will help quality engineers or testers in applying the proposed Sampling Plan in industries, so as to control the products’ quality. The advantage of this Sampling Plan over ordinary Truncated Chain and the deferred Chain sampling plans is that the surrounding lot are zero defectives.
2.4.1 Condition for Using Bilateral Isolated Truncated Chain Deferred Sampling Plan (TTDSP)

The conditions for the adoption of the proposed sampling scheme are:
i. Testing is destructive and costly such that a relatively small sample size is needed.
ii. The quality of products to be inspected is not constant.

2.4.2 Operating Procedure of Two-pronged Truncated Deferred Sampling Plan (TTDSP)

The algorithm for sentencing a lot or batch is as follows:
i. Select a sample of size \( n \) units from each lot and test each unit for conformance to the specified attribute requirements. Count the number of defectives \( d \).
ii. Accept the current lot if the observed number of defectives \( d \) is zero in the selected sample of \( n \) units and reject the lot, if \( d > 1 \).
iii. If \( d = 1 \), then wait for the next lot.
iv. Accept the current lot with \( d = 1 \) if no defectives are found in the immediately or surrounding preceding \( i \) samples and succeeding \( j \) samples from the same stable and steady state process.

2.4.3 Development of the Operating Characteristics of Isolated Truncated Chain Deferred Sampling Plan

Suppose \( i \) is the event of having ‘0’ defectives in a sample of size ‘\( n \)’ and \( j \) be the event of having ‘1’ defectives in a sample of size ‘\( n \)’. Let \( P(i) \) be the probability of having ‘0’ defectives in a lot with sample size ‘\( n \)’.

Likewise, let \( P(j) \) be probability of having ‘1’ defective in a lot with sample size ‘\( n \)’ with the condition that there are zero defectives in the immediate preceding ‘\( i \)’ lot and succeeding \( j \)’ samples.

That is, \( P(B) = P_{0,n} + (P_0)^jP_{1,n}(P_0)^j \)

Since \( A \) and \( B \) are mutually exclusive events, using the addition theorem of probability,

\[ P(A \cup B) = P_{0,n} + (P_0)^jP_{1,n}(P_0)^j \]

We therefore have the probability of acceptance of lot as:

\[ P_d = P_{0,n} + (P_0)^jP_{1,n}(P_0)^j \]

Assuming Poisson distribution,

\[ P_d = e^{-np} + np e^{-np} \] \( i+j \) times

\[ P_d = e^{-np} + np e^{-np} ((e^{-np}, e^{-np}), i = j = 1) \]

\[ = e^{-np} + np e^{-i+j}np, \text{ when } i = j = 1 \]

On factorising, we have:

\[ = e^{-np}(1 + np)^2 \]

\[ = e^{-np} + np e^{-(i+j)}np, \text{ when } i = j = 1 \]

On factorising, we also have:

\[ = e^{-np}(1 + np e^{-(i+j)}), 1 \neq j \]

Now assuming a Binomial distribution,

\[ P_d = \binom{n}{0}p^0(1 - P)^n + \binom{n}{1}P^1(1 - P)^{n-1} \]

\[ = \binom{n}{0}P^0(1 - P) + \binom{n}{1}P^1(1 - P)^{n-1} \]

Considering \( i = j = 1 \), we also have:

\[ P_d = \binom{n}{0}P^0(1 - P)^n + \binom{n}{1}P^1(1 - P)^{n-1} \]

\[ \text{Product Mean Life Ratio} \left( \frac{\mu}{\mu_0} \right) \]

The product mean life ratio is the ratio of the true unknown life of a product to the specified mean life by the producer when designing his product [5]. These values allow the producer to design his products so that it can be accepted at a high probability. In order to calculate the product life ratio values, the producer’s risk is been considered.

The value of \( \frac{\mu}{\mu_0} \) is the smallest positive number for which the following inequality holds:

\[ \frac{\mu}{\mu_0} = \sum_{i=1}^{n}(\binom{n}{i} P^i(1 - P)^{n-i} \geq 0.95) \]

For a given value of the producer’s risk, for example 0.05, one may be interested in knowing what value of \( \frac{\mu}{\mu_0} \) that will ensure a producer’s risk less than or equal to 0.05 if a sampling plan is adopted. For a given sampling plan \((n, c, \frac{1}{\mu_0})\) and specified confidence level \( P \) the minimum values of \( \frac{1}{\mu_0} \) is said to satisfy the equation below.

\[ \sum_{i=1}^{n}(\binom{n}{i} P^i(1 - P)^{n-i} \]

3.0 Results and Discussion

3.1 Product Failure Rate Analysis

The failure rate function \( h(t) \) is used to obtain the pattern of failure of products that assume log-logistic distribution. This function can be used to describe the performance of an item with time. The result for this analysis is shown in table 1 and figure 1 below.
Table 1: Failure Rates of Log-logistic Distributions

<table>
<thead>
<tr>
<th>$t/\mu_0$</th>
<th>Log-logistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.628</td>
<td>0.3229</td>
</tr>
<tr>
<td>0.942</td>
<td>0.2643</td>
</tr>
<tr>
<td>1.571</td>
<td>0.1318</td>
</tr>
<tr>
<td>2.356</td>
<td>0.0223</td>
</tr>
<tr>
<td>3.141</td>
<td>0.0025</td>
</tr>
<tr>
<td>3.972</td>
<td>0.0013</td>
</tr>
<tr>
<td>4.713</td>
<td>0.0003</td>
</tr>
</tbody>
</table>

Figure 1: Effect of Mean Life on Failure Rate Plot for Studied Distributions

From the Table 1 and Figure 1, as the products’ mean life ratio increases, the failure rate reduces.

3.2 Minimum Sample Size

From literature, such as Aslam and Shabaz (2007), Srinivasa (2011), Aslam, Jun and Ahmad (2009), Aslam and Shabaz (2007) and Aslam and Shabaz (2007), acceptance maximum allowable percent defectives, the acceptance number and test ratio are usually set as follows: acceptance number ($c = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9$ and $10$), ($\beta = 0.75, 0.90, 0.95$ and $0.99$) and ($t/\mu_0 = 0.68, 0.942, 1.257, 1.571, 2.358, 3.141$ and $3.972$). A programme written in R is used to generate the results.

Simulations for the Study

The minimum sample size is obtained by calculating the probability of failure which is cumulative distribution function ($p$), and there after substituting the value into the modified minimum sample size formula (equation 11) with other assumed parameters. A programme is written in R to accomplish this task.

Tables 2 show the simulated values of the developed sample sizes for the studied product life distribution.
## Table 2: Minimum sample size for Log-logistic distribution

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>c</th>
<th>$\frac{t}{\mu_0}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.628</td>
<td>0.942</td>
</tr>
<tr>
<td>0.25</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>12</td>
</tr>
<tr>
<td>0.10</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>12</td>
</tr>
<tr>
<td>0.05</td>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>12</td>
</tr>
<tr>
<td>0.01</td>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>12</td>
</tr>
</tbody>
</table>

From table 2, the behaviour of selection of parameters from is as follows: The minimum sample sizes are smaller for smaller acceptance number compared to a higher acceptance number for any combination of consumers’ risk ($\beta$), and experiment time ratio ($\frac{t}{\mu_0}$).

### 3.3 Operating Characteristics for Two-pronged Truncated Deferred Sampling Plan (TTDSP) for Log-logistic

The generated probabilities of acceptance values for the proposed sampling plan are presented in table 3.
From table 3, the operating characteristics increases as the product mean life ratio increases, which show that; products with increased mean life will be accepted with higher probability compared with products with lower mean life ratio.

### 3.4 Product Mean Life Ratio

The product mean life ratios guide the producer at improving his product quality for acceptability with high probability and minimized producer's risk. For any specified sampling plan and producer's risk (say $\alpha = 0.05$), the minimum value of $\left( \frac{\mu}{\mu_0} \right)$ is obtained. This is accomplished by combining values of acceptance number, sample size, Maximum Allowable Percent Defective and the experimental ratio in the course simulation.
From table 4, as the Test Suspension Ratios (TSR) increases, the minimum product life ratio of true to specified mean life increases. It also decreases as the acceptance number increases with decrease in consumers’ risk.
4.0 CONCLUSION

This article has a new sapling plan, thereby given an insight as regards the failure rate pattern of product that assumes Log-logistic distribution; it has also enriched producers and users of this distribution on information that can enhance decision making when considering this distribution in product quality control. The developed plan might also economically prefer when the test is destructive and costly, in order to save cost and time of testing.

ACKNOWLEDGMENTS

The authors are grateful to the unspecified reviewer and all the editorial team for thoroughly going through the paper and for their helpful comments and submissions that has improve this work.

REFERENCE


