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# PERFORMANCES OF THE FULL INFORMATION ESTIMATORS IN A TWO-EQUATION STRUCTURAL MODEL WITH CORRELATED DISTURBANCES

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**ABSTRACT**

The performances of two full information techniques, Three Stage Least Squares (3SLS) and Full Information Maximum Likelihood (FIML) of simultaneous equation models with correlated disturbance terms are compared with the Ordinary Least Squares (OLS) method in small samples. Comparative performance evaluation of the estimators was done using Average of Estimates, Total Absolute Bias (TAB) of Estimates, Root Mean Squared Error (RMSE) and Sum of Squared Residuals (RSS) of parameter estimates. The results of the Monte Carlo experiment showed that OLS is best with large negative or positive correlation, while 3SLS is best with feebly correlated error terms in the case of replication-based averages. The total absolute biases increase consistently as the sample size increases for OLS while FIML estimates reveal no distinct pattern. The magnitudes of the estimates yielded by two estimators, OLS and 3SLS, exhibited fairly consistent reaction to changes in magnitudes and direction of correlations of error terms.

**KEYWORDS:** Disturbance, Simultaneous Equation, Correlation, Structural Parameters, Bias.

## 1.0 INTRODUCTION

The single equation estimation methods lead to estimates that are consistent but, in general, not asymptotically efficient. The reason for lack of asymptotic efficiency is that single equation estimators do not take into account prior restrictions on the other equations in the model. This deficiency can be overcome by estimating all equations of the system simultaneously.

Many studies have revealed that full information methods such as 3SLS and FIML have an advantage over limited information methods like 2SLS and LIML in large samples. This result obtains because full information methods utilize the information concerning the contemporaneous disturbance, $\varepsilon_t$, and the over identifying restrictions arising from other equations, given that the simultaneous equation model is correctly specified. However, for incorrect specification (improper inclusion or exclusion of variables), it is not clear which estimator, limited information, full information or ordinary least squares, is to be preferred. The choice will depend on the form of the misspecification and which equations are involved. Summers[11] (1965) use Monte Carlo studies to study the performance of these estimators when specification error is present.

**The Model**

The following two-equation simultaneous model with a mixture of exactly identified and over identified equations is assumed;

$$Y_{1t} = \beta_{12}Y_{2t} + \gamma_{11}X_{1t} + U_{1t}$$

$$Y_{2t} = \beta_{2t}Y_{1t} + \gamma_{22}X_{2t} + \gamma_{23}X_{3t} + U_{2t}$$

where the Y's are the endogenous variables, X's are the predetermined variables and U's are the random disturbance terms, $\beta$'s and $\gamma$'s are the parameters.

The following levels of correlation between pairs of random deviates are assumed,

(i) highly negatively correlated $\left(r_{\varepsilon_1,\varepsilon_2} < -0.05\right)$

(ii) feebly negatively or positively correlated $\left(-0.05 < r_{\varepsilon_1,\varepsilon_2} < +0.05\right)$

(iii) highly positively correlated $\left(r_{\varepsilon_1,\varepsilon_2} > +0.05\right)$

Other assumptions about the random error are,

$$\mathrm{E}(U_t) = \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix}$$

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$$\mathrm{E}\left(U_{2N\times 1}U'_{1\times 2N}\right)=\mathrm{E}\left[\begin{bmatrix}u_{11}\\ .\\ .\\ u_{1N}\\ u_{21}\\ .\\ .\\ u_{2N}\end{bmatrix}\begin{bmatrix}u_{11} & . & . & u_{1N} & u_{21} & . & . & u_{2N}\end{bmatrix}\right]$$

$$\mathrm{E}\left(u_{it}\quad u_{i't'}\right)=\begin{cases}\sigma_i^2, & i=i', \quad t=t'\\ 0, & i=i', \quad t\neq t'\\ \sigma_{ii'} & i\neq i', \quad t=t'\\ 0, & i\neq i', \quad t\neq t'\end{cases}$$

$$=\left[\begin{array}{ccc|ccc}\sigma_{11} & 0 & 0 & \sigma_{12} & 0 & 0\\ 0 & \ddots & 0 & 0 & \ddots & 0\\ 0 & 0 & \sigma_{11} & 0 & 0 & \sigma_{12}\\ \hline \sigma_{21} & 0 & 0 & \sigma_{22} & 0 & 0\\ 0 & \ddots & 0 & 0 & \ddots & 0\\ 0 & 0 & \sigma_{21} & 0 & 0 & \sigma_{22}\end{array}\right]$$

$$=\Sigma=\Omega\otimes \mathrm{I}_{2N}$$

where

$$\Omega=\begin{bmatrix}\sigma_{11} & \sigma_{12}\\ \sigma_{21} & \sigma_{22}\end{bmatrix}$$ and

$\mathrm{I}_{2N}$ is the 2Nx2N identity matrix.

The reduced form equations are,

$$y_{1t}=\Pi_{11}X_{1t}+\Pi_{12}X_{2t}+\Pi_{13}X_{3t}+V_{1t} \tag{1.2}$$

$$y_{2t}=\Pi_{21}X_{1t}+\Pi_{22}X_{2t}+\Pi_{23}X_{3t}+V_{2t} \tag{1.3}$$

where,

$$\Pi=\begin{bmatrix}\gamma_{11}\beta^* & \beta_{21}\gamma_{22}\beta^* & \beta_{21}\gamma_{23}\beta^*\\ \beta_{12}\gamma_{11}\beta^* & \gamma_{22}\beta^* & \gamma_{23}\beta^*\end{bmatrix}$$

$$V_{1t}=\left(u_{1t}+\beta_{21}u_{2t}\right)\beta^*,$$

$$V_{2t}=\left(\beta_{12}u_{1t}+u_{2t}\right)\beta^*$$

$$\beta^*=\left(1-\beta_{12}\beta_{21}\right)^{-1}$$

Equations (1.2) and (1.3) are subsequently used in deriving the values of stochastic endogenous variables, $y_{1t}$, $y_{2t}$ from selected values of $X_1$, $X_2$, $X_3$, assumed values of $\beta_{12},\beta_{21},\gamma_{11},\gamma_{22},\gamma_{23}$ and assumed distribution of $u_1$ and $u_2$.

The FIML estimator maximizes the likelihood function of an entire system's current endogenous variables, subject to the restrictions placed on the reduced form of Y, $\left[\beta \quad \Gamma\right]\begin{bmatrix}\Pi\\ \mathrm{I}_k\end{bmatrix}=0$, by the over identification of all equations.

The FIML estimator utilizes all information, hence the term "full", whereas the limited estimator uses only that information particular to the given equation. Naturally, the FIML estimator possesses all the properties of a maximum likelihood estimator, that is, consistency,

asymptotic normality and asymptotic efficiency in that the asymptotic covariance matrix of the FIML estimator achieves the asymptotic Cramer-Rao lower bound.

Where there exists no prior information on the variance-covariance matrix of the structural disturbances, $\Sigma$ (for example, no covariance restrictions of the form $\sigma_{ij} = 0$), the 3SLS and FIML estimators, though numerically distinct in small samples, have the same asymptotic distribution (Schmidt[8] (1976)). It follows that 3SLS, is asymptotically efficient in the presence of normally distributed errors. In contrast, however, when prior information concerning $\Sigma$ is available, FIML estimator is asymptotically more efficient than 3SLS (Rothenberg and Leenders[7] (1964)). Finally, FIML estimator is defined only when all equations in the system are identified.

## 1.0 SIMULATION PROCEDURE

Monte Carlo methods constitute a fascinating, exacting and often indispensable craft with a range of applications that is already very wide yet far from being fully explored. The Monte Carlo method provides heuristic solutions to a variety of mathematical problems by performing statistical sampling experiments on a computer.

In econometrics, while asymptotic properties of estimators obtained by using various econometric methods are deductive in character, an approach which is often described as analytical, small sample properties of such estimators have always been studied from simulated data referred to as the Monte Carlo studies (an experimental approach) which is inductive in nature Nwabueze[6] (2005). The output of the analytical approach in the "finite-sample" area is very limited compared with what has been produced in the way of asymptotic results. The results of the analytical method in the finite-sample area are invariably very complex and exceedingly difficult to interpret which pose a major problem. In the Monte Carlo approach, findings are based on reasoning by inference. The use of this approach is due to the fact that real life observations on economic variables are in most cases plagued by one or all of the problems on multicollinearity, non-spherical disturbances and measurement errors.

The behavior of the system estimators are now available using Monte Carlo studies. The most important of these are Summers[11] (1965), Cragg[2] (1967), and Mosbaek and Wold[5] (1970). Interestingly, this method has been excellently reviewed by many authors, especially Johnston[4] (1972), Smith[9] (1973), Intriligator[3] (1978) and Sowey[10] (1973). This approach is used in this work.

This approach may be described in broad terms as follows. The experimenter sets up an artificial system. Values are generated for the random disturbances for some specified sample size and using these values, values are calculated for the endogenous variables based on the assumptions of this artificial problem at each sample point. Pretending that the parameters are unknown and using only the values of the endogenous and predetermined variables at each sample point, several estimating techniques are applied in turn to obtain associated estimates of the parameters. The process of generating values for the disturbances, obtaining values for the endogenous variables, and calculating estimates of the parameters is repeated, or replicated, a large number of times. The set of estimates of each parameter by each estimator is then used to infer properties of the estimators for the given sample size and for the chosen values of the parameters.

The study uses three sample sizes N=15, 25 and 40 each replicated 50 times. Each set of normal deviates with the different sample sizes and replications is then transformed using the upper ($P_1$) triangular matrix. The procedure was repeated using the lower triangular matrix ($P_1'$), such that in each case, $\Omega = P_1 P_1'$. Using the upper triangular matrix

$$\Omega = P_1 P_1' = \begin{bmatrix} S_{11} & S_{12} \\ 0 & S_{22} \end{bmatrix} \begin{bmatrix} S_{11} & 0 \\ S_{12} & S_{22} \end{bmatrix} = \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix}$$

to obtain a pair of random disturbances and for the lower triangular matrix,

$$\Omega = P_2 P_2'$$

where

$$P_2 = \begin{bmatrix} S_{11} & 0 \\ S_{12} & S_{22} \end{bmatrix}$$

The behaviors of estimators are therefore examined across these triangular matrices. After estimating the parameters, the robustness of each estimator to the inadvertent correlation of the stochastic terms was examined using; average of estimates, absolute bias of estimates, root mean square error and sum of squared residuals of parameter estimates.

**Table 1**: Summary of Estimators using Average R=50, $P_1$

| Estimator | Level of correlation | EQ1 | | | | | |
|---|---|---|---|---|---|---|---|
| | | $\beta_{12}(1.5)$ | | | $\gamma_{11}(1.2)$ | | |
| | | N=15 | N=25 | N=40 | N=15 | N=25 | N=40 |
| OLS | r<-0.05 | 1.067074 | 1.070771 | 1.063577 | -0.40953 | -0.50417 | -0.56386 |
| | -0.05<r<0.05 | 1.033484 | 1.042028 | 1.046355 | -0.49904 | -0.53441 | -0.50779 |
| | r>0.05 | 1.013901 | 1.02698 | 1.030384 | -0.60413 | -0.57562 | -0.58862 |
| 3SLS | r<-0.05 | 1.083419 | 1.296365 | 1.435163 | -0.47362 | 0.029132 | 0.230474 |
| | -0.05<r<0.05 | 1.042502 | 1.355854 | 1.43021 | -0.53324 | 0.159156 | 0.302414 |
| | r>0.05 | 0.989754 | 1.229802 | 1.271803 | -0.40984 | -0.19972 | -0.12357 |
| FIML | r<-0.05 | -1.55879 | -4.60894 | -0.49344 | -2.03921 | -4.91175 | -1.24092 |
| | -0.05<r<0.05 | -1.61711 | -2.37039 | -2.1021 | -2.28008 | -2.03702 | -1.15926 |
| | r>0.05 | -0.8352 | -1.06144 | -0.91255 | -0.41497 | -0.80337 | -0.73766 |

**Table 1:** Summary of Estimators using Average R=50, $P_1$ (continued)

| Estimator | Level of correlation | EQ2 | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | $\beta_{21}(1.8)$ | | | $\gamma_{22}(0.5)$ | | | $\gamma_{23}(2.0)$ | | |
| | | N=15 | N=25 | N=40 | N=15 | N=25 | N=40 | N=15 | N=25 | N=40 |
| OLS | r<-0.05 | 0.895693 | 0.901183 | 0.897693 | -0.00314 | 0.080167 | 0.136797 | 0.50835 | 0.488816 | 0.406874 |
| | -0.05<r<0.05 | 0.93639 | 0.935993 | 0.92698 | 0.032062 | -0.01435 | -0.00525 | 0.61272 | 0.624402 | 0.593017 |
| | r>0.05 | 0.948343 | 0.961693 | 0.94834 | 0.059402 | -0.00299 | 0.261901 | 0.638427 | 0.753 | 0.411395 |
| 3SLS | r<-0.05 | 0.975957 | 1.13154 | 1.244629 | 0.009263 | 0.146842 | 0.496228 | 0.410318 | 1.06033 | 1.084992 |
| | -0.05<r<0.05 | 0.994895 | 0.942695 | 1.12331 | -0.23727 | 0.345512 | 0.107784 | 1.041672 | 0.41698 | 1.01786 |
| | r>0.05 | 1.300566 | 1.038556 | 1.216321 | -0.07144 | -0.00088 | 0.62785 | 1.782975 | 0.872167 | 0.812499 |
| FIML | r<-0.05 | -0.96486 | -2.77071 | -0.16686 | -0.39955 | -1.64419 | -0.26099 | -0.68349 | -1.76555 | -0.02052 |
| | -0.05<r<0.05 | -1.28313 | -1.50835 | -1.46402 | -0.85273 | -0.67438 | -0.22465 | -1.28932 | -0.70913 | -0.81512 |
| | r>0.05 | -0.64733 | -0.72131 | -0.61169 | -0.01846 | -0.09378 | 0.030535 | -0.18251 | -0.11286 | -0.29474 |

**Table 2:** Summary of Estimators using Average R=50, $P_2$

| Estimator | Level of correlation | EQ1 | | | | | |
|---|---|---|---|---|---|---|---|
| | | $\beta_{12}(1.5)$ | | | $\gamma_{11}(1.2)$ | | |
| | | N=15 | N=25 | N=40 | N=15 | N=25 | N=40 |
| OLS | r<-0.05 | 0.992898 | 1.007177 | 1.005596 | -0.58899 | -0.65394 | -0.61268 |
| | -0.05<r<0.05 | 1.03872 | 1.043855 | 1.046446 | -0.50066 | -0.53163 | -0.57261 |
| | r>0.05 | 1.0644 | 1.071153 | 1.077282 | -0.45167 | -0.50358 | -0.47483 |
| 3SLS | r<-0.05 | 1.138964 | 1.257422 | 1.382585 | -0.39606 | -0.12574 | 0.196764 |
| | -0.05<r<0.05 | 1.091362 | 1.272043 | 1.405338 | -0.42857 | -0.08312 | 0.210843 |
| | r>0.05 | 1.248948 | 1.481571 | 1.433504 | -0.04207 | 0.460571 | 0.307667 |
| FIML | r<-0.05 | -2.24675 | -1.56517 | -2.07313 | -2.35591 | -3.29957 | -3.00509 |
| | -0.05<r<0.05 | -3.76624 | -6.44124 | -1.57687 | -3.62188 | -5.85855 | -2.09379 |
| | r>0.05 | -3.80075 | -1.10505 | -1.05307 | -3.17179 | -1.12594 | -0.34167 |

**Table 2**: Summary of Estimators using Average R = 50, $P_2$ (Continued)

| Estimator | Level of correlation | EQ2 | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | $\beta_{21}(1.8)$ | | | $\gamma_{22}(0.5)$ | | | $\gamma_{23}(2.0)$ | | |
| | | N=15 | N=25 | N=40 | N=15 | N=25 | N=40 | N=15 | N=25 | N=40 |
| OLS | r<-0.05 | 0.96792 | 0.96679 | 0.919973 | 0.07633 | 0.071831 | 0.137499 | 0.693704 | 0.701117 | 0.499387 |
| | -0.05<r<0.05 | 0.936266 | 0.938178 | 0.923779 | 0.062311 | 0.011683 | 0.131808 | 0.606942 | 0.67296 | 0.478739 |
| | r>0.05 | 0.907646 | 0.91977 | 0.913413 | -0.02132 | -0.04718 | 0.10778 | 0.571083 | 0.663349 | 0.461893 |
| 3SLS | r<-0.05 | 1.173429 | 1.558212 | 1.068858 | -0.4057 | 0.78732 | 0.233205 | 1.658431 | 1.876514 | 0.689285 |
| | -0.05<r<0.05 | 1.668581 | 1.513603 | 1.206698 | 2.308587 | 0.685876 | 0.491424 | 0.617603 | 1.772122 | 0.955078 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | r>0.05 | 1.127006 | 0.950839 | 0.884521 | 0.016592 | 0.067383 | -0.07385 | 1.183495 | 0.596628 | 0.502641 |
| FIML | r<-0.05 | -1.76366 | -1.49297 | -0.87906 | -0.96703 | -0.49748 | -0.37876 | -1.73279 | -1.03067 | -0.83246 |
| | -0.05<r<0.05 | -2.99361 | -5.18181 | -0.74722 | -1.71173 | -2.08676 | -0.30778 | -3.02344 | -5.33491 | -0.32156 |
| | r>0.05 | -2.98186 | -0.64979 | -0.79036 | -1.02985 | -0.34432 | -0.13676 | -2.84699 | -0.03626 | -0.19601 |

**Table 3:** Summary of Total Absolute Bias R=50, $P_1$

| Level of correlation | OLS | | | 3SLS | | | FIML | | |
|---|---|---|---|---|---|---|---|---|---|
| | N=15 | N=25 | N=40 | N=15 | N=25 | N=40 | N=15 | N=25 | N=40 |
| r<-0.05 | 4.941557 | 4.961139 | 5.058924 | 4.994664 | 3.335791 | 2.508514 | 12.645898 | 22.70113 | 9.182728 |
| -0.05<r<0.05 | 4.88493 | 4.946335 | 4.946682 | 6.691437 | 3.779803 | 3.018422 | 14.322373 | 14.299279 | 12.765143 |
| r>0.05 | 4.944047 | 4.836939 | 4.936597 | 3.407983 | 4.060074 | 3.450798 | 9.098455 | 9.792754 | 9.526034 |

**Table 4**: Summary of Total Absolute Bias R=50, $P_2$

| Level of correlation | OLS | | | 3SLS | | | FIML | | |
|---|---|---|---|---|---|---|---|---|---|
| | N=15 | N=25 | N=40 | N=15 | N=25 | N=40 | N=15 | N=25 | N=40 |
| r<-0.05 | 4.858138 | 4.907023 | 5.050227 | 3.830927 | 2.220908 | 3.429303 | 16.066133 | 14.885862 | 14.168504 |
| -0.05<r<0.05 | 4.856425 | 4.864958 | 4.991839 | 5.359608 | 2.210336 | 2.73062 | 22.617801 | 31.903266 | 12.047216 |
| r>0.05 | 4.929852 | 4.896483 | 4.914466 | 3.466025 | 3.443008 | 3.94552 | 20.831248 | 10.261373 | 9.517872 |

**Table 5**: Summary of Estimators using Root Mean Square Error R=50, $P_1$

| Estimator | Level of correlation | EQ1 | | | | | |
|---|---|---|---|---|---|---|---|
| | | $\beta_{12}(1.5)$ | | | $\gamma_{11}(1.2)$ | | |
| | | N=15 | N=25 | N=40 | N=15 | N=25 | N=40 |
| OLS | r<-0.05 | 0.436156 | 0.431532 | 0.437016 | 1.62725 | 1.717513 | 1.766868 |
| | -0.05<r<0.05 | 0.467685 | 0.458839 | 0.454077 | 1.712024 | 1.742782 | 1.710205 |
| | r>0.05 | 0.491549 | 0.47391 | 0.469965 | 1.8375 | 1.78329 | 1.791164 |
| 3SLS | r<-0.05 | 0.809737 | 0.342553 | 0.546158 | 2.432383 | 1.397053 | 1.632861 |
| | -0.05<r<0.05 | 0.708987 | 0.584134 | 0.478117 | 2.012958 | 1.873058 | 1.422934 |
| | r>0.05 | 0.953527 | 0.609297 | 0.471323 | 2.031971 | 2.068709 | 1.641591 |
| FIML | r<-0.05 | 4.610025 | 9.772582 | 2.398144 | 6.264157 | 11.52345 | 3.128115 |
| | -0.05<r<0.05 | 7.99254 | 6.476522 | 4.782457 | 10.47733 | 5.782321 | 3.21884 |
| | r>0.05 | 2.486464 | 3.111921 | 2.918378 | 1.851861 | 2.693205 | 2.598102 |

**Table 5**: Summary of Estimators using Root Mean Square Error R=50, $P_1$ (continued)

| Estimator | Level of correlation | EQ2 | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | $\beta_{21}(1.8)$ | | | $\gamma_{22}(0.5)$ | | | $\gamma_{23}(2.0)$ | | |
| | | N=15 | N=25 | N=40 | N=15 | N=25 | N=40 | N=15 | N=25 | N=40 |
| OLS | r<-0.05 | 0.905521 | 0.89962 | 0.902529 | 0.701269 | 0.547223 | 0.433591 | 1.576979 | 1.542993 | 1.611507 |
| | -0.05<r<0.05 | 0.863759 | 0.864116 | 0.873127 | 0.622535 | 1.606007 | 0.560126 | 1.462256 | 1.436591 | 1.428301 |
| | r>0.05 | 0.857112 | 0.838529 | 0.85181 | 0.584447 | 0.577407 | 0.326965 | 1.484068 | 1.279606 | 1.60844 |
| 3SLS | r<-0.05 | 1.286775 | 1.04272 | 1.515382 | 1.600382 | 1.521984 | 2.293749 | 3.567529 | 2.646131 | 3.068935 |
| | -0.05<r<0.05 | 1.172237 | 1.8286 | 1.021716 | 1.853415 | 2.020852 | 0.986875 | 3.994243 | 4.557241 | 2.723832 |
| | r>0.05 | 2.006504 | 1.060093 | 1.021405 | 1.99529 | 1.691074 | 2.038096 | 6.528473 | 2.369789 | 2.18759 |
| FIML | r<-0.05 | 3.416047 | 7.189226 | 2.22517 | 1.603807 | 4.618018 | 1.219463 | 3.472061 | 5.627567 | 2.116898 |
| | -0.05<r<0.05 | 7.10255 | 5.454871 | 4.137026 | 4.168936 | 3.573134 | 1.133531 | 9.378221 | 4.669701 | 3.762647 |
| | r>0.05 | 2.57813 | 2.996221 | 2.7794 | 0.599759 | 0.778447 | 0.533762 | 2.505865 | 2.650702 | 2.734525 |

**Table 6**: Summary of Estimators using Root Mean Square Error R=50, $P_2$

| Estimator | Level of correlation | EQ1 | | | | | |
|---|---|---|---|---|---|---|---|
| | | $\beta_{12}(1.5)$ | | | $\gamma_{11}(1.2)$ | | |
| | | N=15 | N=25 | N=40 | N=15 | N=25 | N=40 |
| | r<-0.05 | 0.510122 | 0.494626 | 0.501262 | 1.81861 | 1.862027 | 1.81824 |
| | -0.05<r<0.05 | 0.46242 | 0.456944 | 0.454163 | 1.714671 | 1.738476 | 1.776118 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | r>0.05 | 0.440366 | 0.430319 | 0.423331 | 1.667882 | 1.710873 | 1.678407 |
| 3SLS | r<-0.05 | 0.605028 | 0.675491 | 0.748272 | 2.121374 | 2.186761 | 2.136393 |
| | -0.05<r<0.05 | 1.218544 | 0.61113 | 0.473233 | 3.623711 | 2.062664 | 1.468751 |
| | r>0.05 | 0.57849 | 0.518955 | 0.375886 | 1.814418 | 1.600259 | 1.265556 |
| FIML | r<-0.05 | 6.109593 | 6.743089 | 5.832326 | 5.124391 | 7.467952 | 6.575807 |
| | -0.05<r<0.05 | 10.61125 | 16.28581 | 4.148964 | 11.802 | 17.38281 | 4.989155 |
| | r>0.05 | 9.730886 | 3.049391 | 2.95474 | 8.309207 | 3.412404 | 1.872224 |

**Table 6:** Summary of Estimators using Root Mean Square Error R=50, $P_2$ (continued)

| Estimator | Level of correlation | EQ2 | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | $\beta_{21}$ (1.8) | | | $\gamma_{22}$ (0.5) | | | $\gamma_{23}$ (2.0) | | |
| | | N=15 | N=25 | N=40 | N=15 | N=25 | N=40 | N=15 | N=25 | N=40 |
| OLS | R<-0.05 | 0.832829 | 0.833769 | 0.899442 | 0.637005 | 0.57612 | 0.459278 | 1.373695 | 1.354464 | 1.533501 |
| | -0.05<r<0.05 | 0.86398 | 0.861902 | 0.876361 | 0.57803 | 0.571873 | 0.418968 | 1.442819 | 1.360709 | 1.537516 |
| | r>0.05 | 0.894405 | 0.880426 | 0.886701 | 0.642272 | 0.606852 | 0.435069 | 1.514568 | 1.362601 | 1.555075 |
| 3SLS | R<-0.05 | 1.694989 | 2.379137 | 1.127651 | 3.0698 | 2.197369 | 1.178233 | 4.036605 | 6.737785 | 2.776149 |
| | -0.05<r<0.05 | 5.632721 | 1.295517 | 1.110134 | 11.1051 | 3.852231 | 1.503287 | 13.75275 | 4.833901 | 2.78772 |
| | r>0.05 | 1.225078 | 1.148319 | 1.083567 | 2.633028 | 1.437763 | 1.226975 | 3.262547 | 2.661511 | 1.996823 |
| FIML | r<-0.05 | 5.638482 | 5.638683 | 4.562609 | 3.606066 | 3.417611 | 2333067 | 7.162911 | 5.380956 | 4.560933 |
| | -0.05<r<0.05 | 9.115358 | 14.04307 | 3.39546 | 6.555847 | 7.772889 | 1.629755 | 10.22504 | 16.02082 | 2.778681 |
| | r>0.05 | 8.230567 | 2.807013 | 2.877896 | 3.422276 | 2.124536 | 1.187865 | 9.410859 | 2.50089 | 2.403147 |

**Table 7**: Summary of Sum of Squared Residuals for Three Correlation Levels R=50, $P_1$

| Estimator | Level of correlation | EQ1 | | | EQ2 | | |
|---|---|---|---|---|---|---|---|
| | | N=15 | N=25 | N=40 | N=15 | N=25 | N=40 |
| OLS | r<-0.05 | 8.519252 | 14.19056 | 22.9875 | 5.652294 | 9.582751 | 16.77818 |
| | -0.05<r<0.05 | 7.872591 | 13.48918 | 22.01066 | 5.293851 | 9.039257 | 15.69381 |
| | r>0.05 | 7.976691 | 11.86255 | 19.50978 | 5.506655 | 7.679018 | 14.58404 |
| 3SLS | r<-0.05 | 52.23134 | 31.7618 | 129.0425 | 100.7806 | 133.8592 | 700.6628 |
| | -0.05<r<0.05 | 28.26443 | 89.056 | 134.4355 | 75.10572 | 547.6233 | 216.7182 |
| | r>0.05 | 37.30773 | 81.22743 | 96.78534 | 545.47 | 128.758 | 270.4573 |
| FIML | r<-0.05 | 1648.747 | 15698.51 | 1369.229 | 878.4006 | 9469.272 | 874.2625 |
| | -0.05<r<0.05 | 7087.641 | 8074.355 | 7036.368 | 6076.905 | 5769.992 | 4840.792 |
| | r>0.05 | 685.7844 | 1859.201 | 2800.426 | 555.1461 | 1465.665 | 2029.094 |

**Table 8**: Summary of Sum of Squared Residuals for Three Correlation Levels R=50, $P_2$

| Estimator | Level of correlation | EQ1 | | | EQ2 | | |
|---|---|---|---|---|---|---|---|
| | | N=15 | N=25 | N=40 | N=15 | N=25 | N=40 |
| OLS | r<-0.05 | 8.138315 | 14.0019 | 21.92372 | 5.932354 | 9.971365 | 16.92984 |
| | -0.05<r<0.05 | 7.629978 | 13.73168 | 21.76534 | 5.294499 | 9.180186 | 16.11784 |
| | r>0.05 | 8.45091 | 11.76171 | 19.15493 | 5.629083 | 7.617836 | 13.30717 |
| 3SLS | r<-0.05 | 31.65482 | 83.26091 | 203.0776 | 266.5712 | 983.7893 | 233.0846 |
| | -0.05<r<0.05 | 146.3053 | 79.31666 | 124.7349 | 3638.535 | 353.7829 | 345.611 |
| | r>0.05 | 46.27148 | 100.0351 | 111.6931 | 150.4811 | 144.4524 | 138.6418 |
| FIML | r<-0.05 | 4018.641 | 8884.87 | 10138.66 | 3108.333 | 5480.683 | 6191.815 |
| | -0.05<r<0.05 | 13077.01 | 51373.8 | 5114.238 | 10195.22 | 41057.14 | 3128.517 |
| | r>0.05 | 12450.62 | 1785.939 | 2811.317 | 9531.924 | 1248.937 | 2149.904 |

## 4.0 RESULTS/DISCUSSION

Tables 1-8 are used to summarize the relative performances of the three estimators using, average of estimates, total absolute bias (TAB), root mean square error (RMSE) and sum of squared residuals (RSS). The three levels of correlation coefficients used are given in the tables using three different sample sizes. This experiment is performed using the upper and lower triangular matrices ($P_1$ and $P_2$). The results of the experiment reveal that on the basis of TAB criterion, the 3SLS method shows asymptotic behavior (the total absolute biases decrease with increased sample size) under $P_1$ while the other two estimators, OLS and FIML exhibit no such pattern. For both equations and irrespective of whether $P_1$ or $P_2$ is used, OLS RSS estimates increased consistently as sample size increased making it inferior to the other estimators.

Judging the performance of the estimators by RMSE shows that as the correlation coefficient changes through the three critical levels, OLS RMSE estimates increase consistently for equation 1 and decrease consistently for equation 2 under $P_1$, the reverse is the case for $P_2$. However, under $P_1$, FIML is remarkably best in the open-ended intervals and remarkably poor at the closed interval

## 2.0 CONCLUSION

The magnitudes of the estimates yielded by two estimators, OLS and 3SLS exhibited fairly consistent reaction to changes in magnitudes and direction of correlations of error terms. The results of this study show that the performances of the estimators vary with the correlation interval with OLS ranking best especially in the open-ended intervals. The results also reveal that choice of triangular matrix upper ($P_1$) or lower ($P_2$) as well as the identifiability status of the equations are factors to be reckoned with. Consequently, the interaction of these factors: correlation levels of error term, identifiability status of the equations and the use of $P_1$ or $P_2$ have not facilitated a conclusive ranking of the estimators using our own definition of 'best' estimators.

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