

# CALCULATION OF BREAKTHROUGH TIME IN OIL RECOVERY BY THE USE OF FRACTAL DIMENSION IN A PERCOLATION MODEL

C.P.OGBOGBO

Department of Mathematics, University of Ibadan  
Ibadan, Nigeria

E-mail:chisaraogbogbo@yahoo.com

## Abstract

Recent studies indicate that percolation models may be used to predict breakthrough time by the use of the fractal dimension of the shortest path. The paper shows that better predictions can be made about the breakthrough time when the actual fractal dimension of the boundaries of the oil reservoirs are calculated. The values obtained for time of breakthrough are strongly correlated to empirical values. They are also in agreement with expected results, in terms of relationship between fractal dimension and number of years before breakthrough.

## Introduction

Percolation models are often used to make predictions about important parameters in a random medium. This recommends such models for the study of flow in a rock formation (porus medium) containing hydrocarbons. Since it is known that the spatial distribution of rock types is often close to random, the connectivity of sand bodies typifies a percola-

tion cluster. Percolation models have been used (King et al., 2001) to predict breakthrough time ( $t_{br}$ ), employing a value of 1.33 for the critical exponent  $\alpha$ , which is the fractal dimension of the shortest path. The advantage of the percolation model over conventional methods is that  $t_{br}$  is obtained more quickly, in a fraction of a second on a spreadsheet. The conventional approach such as the Buckley-Leverett method is computationally very expensive involving a lot of intricacies in obtaining required quantities and measurements. Much simpler models which can predict the uncertainty in performance is required. The shape of an oil reservoir as represented by its contour map can be quantified using its fractal dimension. Flow physics in an oil field is reservoir specific. Some of these specifics can be captured by calculating the fractal dimension of the oil reservoir. The assumption of universality is relaxed by examining the structural details of the reservoir. By using the actual fractal dimension of the reservoir some of the flow dynamics can be captured. A percolation model is used to calculate  $t_{br}$  using the actual fractal dimensions of boundaries of the oil reservoirs. In reservoir engineering oil at another well (the per-

ducer well). The water injected into the well one method of oil recovery is the displacement method (i.e. water drive mechanism). Water injected at one well (the injected well) is used to “push out” “breaks through” at some time. This time of breakthrough is a very important parameter in oil recovery because of its economic implications for the oil industry. Once the water breaks out not much oil is left in that well. Breakthrough time is also considered a measure of performance. For oil to flow out under water pressure between two wells, a percolation cluster (well connected open channels) must exist. The percolation approach to recovery prediction is a bond percolation model where the oil reservoir is modeled as a percolation cluster. The flow is directed from the injector well  $A$  to the producer well  $B$ , such that a directed percolation process is achieved. The passage of time involved in the flow (before breakthrough time) led to the study of oil displacement process as a directed first passage percolation. The problem at hand is to predict the time of breakthrough.

### Experimental

Data on some selected reservoirs in Nigeria’s Niger Delta were used as real field data. The results obtained for  $t_{br}$  using the fractal dimension of the boundaries of the reservoirs showed a remarkable improvement on the results obtained when a universal value of 1.33 was used for  $\alpha$  in calculating  $t_{br}$ . The fractal dimensions were calculated by covering the boundaries of the reservoirs with square boxes of small side lengths. Basic concepts and preliminary results had been provided. Criti-

cal exponents and scaling laws in predicting  $t_{br}$  had been discussed. The results obtained using the universality assumption were analysed.

### *Basic concept of percolation and preliminary results*

The medium being considered consists of microscopic pores and channels through which the fluid might pass. Each channel will be opened or closed to the passage of the fluid depending on several characteristics of the medium. In the simplest case each channel, independent of others, is opened with probability  $p$  (the single parameter of the model) and closed with probability  $1 - p$ . A basic question is the occurrence or not of percolation i.e. the existence of an infinite path, through open bonds (edges) only and cutting through (spanning) the medium. Let  $d$  denote dimension,  $d \geq 2$  as in Luiz & Sindoravicius, (2002).  $\mathbb{Z} = \{\dots -1, 0, 1, \dots\}$  for the set of all integers and  $\mathbb{Z}^d$  for the set of all vectors  $x = (x_1, x_2, \dots, x_d)$  of integers. For  $x \in \mathbb{Z}^d$ ,  $x_i$  is the  $i$ th co-ordinate of  $x$ . The distance from  $x$  to  $y$ ,  $\delta(x, y)$  is defined by the  $l_1$ -norm so

$$\delta(x, y) = \sum_{i=1}^d |x_i - y_i|. \quad (1)$$

Writing  $|x|$  for the distance from the origin of  $\mathbb{Z}^d$  to  $x$ , then  $|x| = \delta(0, x)$ .  $\mathbb{Z}^d$  is turned into a graph called the  $d$ -dimensional cubic lattice, by adding edges  $\{x, y\}$  between all pairs  $x, y \in \mathbb{Z}^d$  with  $\delta(x, y) = 1$ . This lattice is denoted by  $\mathcal{L}^d = (\mathbb{Z}^d, \mathbb{E}^d)$ , where  $\mathbb{Z}^d$  is the set of sites of the lattice and  $\mathbb{E}^d = \{(x, y) \in \mathbb{Z}^d : \|x - y\|_1 = 1\}$  is the set of nearest neighbour edges (bonds) bracket (Grimmet, 1989). Two vertices  $x$  and  $y$  are said to be adjacent

or neighbours if  $\delta(x, y) = 1$ .  $x \sim y$  if  $x$  and  $y$  are adjacent. The corresponding edge is denoted by  $\{x, y\}$ . Let  $p$  and  $q$  satisfy  $0 \leq p \leq 1$  and  $p + q = 1$ . A probability space  $(\Omega, \mathcal{F}, P_p)$  with sample space  $\Omega = \{0, 1\}^{\mathbb{E}^d}$ . Point of  $\Omega$  are represented as  $w = (w(e) : e \in \mathbb{E}^d)$  and called configurations. The value  $w(e) = 1$  corresponds to  $e$  being open and  $w(e) = 0$  corresponds to  $e$  being closed.  $\mu_e(w(e) = 1) = p$  and  $p$  and  $\mu_e(w(e) = 0) = q$ , where  $\mu_e$  is the Bernoulli measure on  $(0, 1)$

*Critical percolation and critical exponents*

A principal quantity of study in percolation theorem is the percolation probability  $\theta(p)$  which is the probability

$$\theta(p) = \begin{cases} 0 & \text{if } p < p_c \quad \text{subcritical phase} \\ > 0 & \text{if } p > p_c \quad \text{supercritical phase} \end{cases}$$

This critical probability is defined as

$$p_c(d) = \sup\{p : \theta(p) = 0\}. \quad (3)$$

It is believed that percolation probability has a singularity at  $p = p_c$  and that there is a “power law behaviour” at and near this singularity. The nature of the singularity is supposed to be canonical, It is expected to have certain general features in common with phase transitions in other physical systems. These features are referred to as “Scaling limits” and they relate to “critical exponents” as in Grimmet, 1989.;Grimmet 1996.. Near the critical point,  $p$  approaches  $p_c$  from above (or below);  $\theta(p)$  and  $X(p)$  are believed to behave as powers of  $|p - p_c|$ .  $X(p)$  is the mean size of an open cluster which is interpreted as the mean number of vertices in the open cluster at the origin.

that a given vertex belongs to an infinite open cluster. Grimmet (1996) is defined the percolation probability as

$$\theta(p) = p(0 \longleftrightarrow \infty)$$

Because  $\theta(p)$  is the probability that the origin belongs to an infinite cluster, it is more elegantly defined by

$$\theta(p) = p(|c| = \infty) \quad (2)$$

where  $|c|$  is the cardinality of the open cluster of the origin,  $|c|$  is thus, a random variable which can take the discrete values  $1, 2, \dots, \infty$ .

Fundamentally, there exists a critical value  $p_c = p_c(d)$  of  $p$  (called the critical probability) such that

According to Kesten, (2006) the following exist

$$\begin{aligned} \gamma &= -\lim_{p \rightarrow p_c} \frac{\log x(p)}{\log |p - p_c|} \\ \beta &= \lim_{p \rightarrow p_c} \frac{\log \theta(p)}{\log |p - p_c|} \\ \delta^{-1} &= -\lim_{n \rightarrow \infty} \log \frac{P_{p_c}(|c| \geq n)}{\log n}. \end{aligned} \quad (4)$$

The quantities  $\gamma, \beta, \delta$  are called critical exponents. These exponents are expected to be universal, their values may only on the dimension  $d$  of the lattice and not on the structure of the lattice or the model. Such quantities which have peculiar (power) relationships with the critical exponents include  $\theta(p), x(p), x^f(p), K(p)$ . The meaning, behaviour and notation less for the critical exponents are summarized in Table 1 below, as established in Grimmet (1996).

Table 1: *Eight functions and their critical exponents*(Kesteyn, 1987)

<i>Function</i>		<i>Behaviour</i>	<i>Exponents</i>
Percolation Probability	$\theta(p) = P_p( c  = \infty)$	$\theta(p) \approx (p - p_c)^\beta$	$\beta$
Truncated clusters size	$X^f = E_p( c ;  c  < \infty)$	$x^f(p) \approx  p - p_c ^{-\gamma}$	$\gamma$
Number of clusters per vertex	$K(p) = E_p( c ^{-1})$	$K'''(p) \approx  p - p_c ^{-1-\alpha}$	$\alpha$
Cluster moments	$X_k^f = E_p( c ^k;  c  < \infty)$	$\frac{X_{k+1}^f(p)}{X_k^f(p)} \approx  p - p_c ^{-\Delta}$	$\Delta$
Correlation length	$\xi(p)$	$\xi(p) \simeq  p - p_c ^{-\nu}$	$\nu$
Cluster volume		$P_p( c  = n) \approx n^{-1-1/\delta}$	$\delta$
Cluster radius		$p_{pc}(\text{rad}(c) = n) \approx n^{-1-1/\rho}$	$\rho$
Connectivity function		$p_{pc}(0 \longleftrightarrow x) \approx   x  ^{2-d-\nu}$	$\nu$

It is believed that the exponents are not independent variables but rather they satisfy the following scaling relation as shown by Kesteyn (1987 ab).

$$\begin{aligned} 2 - \alpha &= \gamma + 2\beta = \beta(\delta + 1) \\ \Delta &= \delta\beta \\ \gamma &= \nu(2 - \eta).. \end{aligned} \quad (5)$$

*Definition(power law)*

An interesting concept in percolation theorem is the power law relationship. A power law is defined simply as any polynomial relationship that exhibits the property of scale invariance. Most known power laws relate two variables and have the form  $f(x) = ax^k$ , where  $k$  is the scaling exponent. For example,  $p(k) \sim k^r$  and  $p(ak) \sim a^{-r}p(k)$  define a power law, where  $p(k)$  is the probability of occurrence of some event  $k$ . This relationship does not depend on the scale  $k$ , but on the coefficient

$a$ . The conventional method for calculating breakthrough time ( $t_{br}$ ) is the Buckley- Leverett method. The time at which breakthrough occurs is given as

$$t_{bt} = \frac{W_{id_{bt}}}{q_{id}} \quad (6)$$

$W_{id_{bt}}$  is the dimensionless number of pore volumes of water injected at time of breakthrough,  $q_{id}$  is the dimensionless injection rate, and  $t_{bt}$  is the time of breakthrough, which is analogous to  $t_{br}$  being calculated in the paper. To use the Buckley-Leverett method, the relative permeability curves, the Buckley-Leverett equation and the expression for average water saturation are required. To utilize each of these components, one needs to have values for some other important quantities. These include velocity of the plane  $V_{sw}$ , full differential of water saturation  $dS_w$ , fractional flow of water at any point in the reservoir  $f_w$ , current

value of water saturation  $S_{we}$ , flood front saturation  $S_{wf}$ , fractional flow at the producing well  $f_{we}$ , the cumulative water injected  $w_i$  and the injection rate  $q_i$ . Several values which are required for detailed calculations must be obtained first, before the oil recovery calculations can be made. Thus, a lot of intricacies are involved when using the Buckley-Leverett method.

*Result in Percolation Theory: Use of critical exponents and scaling laws of percolation theory in predicting the breakthrough time for Recovery.*

According to Andrade et al. 2000, there is qualitative resemblance between the shortest path and the minimal traveling time of the tracer particle. The shortest path connecting two sites on a percolation cluster is defined as the shortest path or chemical distance. It is denoted by  $l$  and for some particular case, its value may be denoted  $l^*$ . The geometrical distance between the sites on the cluster is denoted  $r$ .  $l^*$  scales with geometrical distance  $r$  as follows:

$$l^* \sim r^{d_{\min}}$$

$$d_{\min} = \begin{cases} 1.13 \pm 0.02 & d = 2 \\ 1.374 \pm 0.005 & d = 3 \end{cases} \quad (7)$$

$d_{\min}$  is the fractal dimension of the shortest path.

*Minimal traveling time (breakthrough time) and fastest path*

Minimal traveling time (or breakthrough time) has to do with the dynamics of the flow on the percolation cluster. Interest is on scaling properties of the distribution of minimal traveling time and length of the path corresponding to the minimal traveling time (fastest path) of the tracer particles.

In the model by King et al. (2001), the simulation is run for the flow tracer particles starting at the injection point  $A$  to the recovery point  $B$ . The minimal traveling time  $t_{\min}$  corresponds to the breakthrough time of the liquid (water) that displaces the oil during recovery (Lee et al., 1999). Exponents  $d_x$  where  $x$  denotes

$$l_{\min}, t_{\min}, \bar{l} \text{ or } \bar{t}$$

are defined by

$$x^* \sim r^{d_x} \quad (8)$$

Here  $x^*$  is the characteristic length or time of the corresponding distribution. Andrade et al. 2000 obtained  $t_{\min}$  which scales with  $l^z$ , i.e.  $t_m \sim l^z$  where  $z \approx 1.17$ . The expression “scales as” denoted by  $\sim$  means “is proportional to in the limit”. For the particle traveling between two points  $A$  and  $B$ ,  $d_{\min} < d_{tm} < d_B$ , where  $d_{tm}$  is the fractal dimension of a subset of the of the system,  $d_{\min}$  is the fractal dimension of the minimal path and  $d_B$  is the fractal dimension of the entire cluster. Since  $l$  scales as it is proposed that  $t_{\min}$  scales as  $r^{d_{tm}}$ , where

$$d_{tm} = z d_{\min} = 1.33$$

and

$$t_{\min} \sim r^{d_{tm}} d_{t_{\min}} = 1.33 \pm 0.005 \quad (9)$$

and

$$t_{\min} \sim r^{z d_{\min}}$$

such that

$$t_{\min} \sim r^{1.33}. \quad (10)$$

*Predicting breakthrough time using percolation model*

Andrade et al. 2000 have shown that the breakthrough time is strongly correlated with the shortest path length

(chemical path). There is a power law relationship

$$t \sim r^\alpha \quad (11)$$

The details about the variables in this expression are given below.  $\alpha$  here is  $d_{t \min}$  (Kesten, 1987. a).

This is a particular case of the general relationship obtained by Andrade et al. 2000 and was given in equation (8). King et al. 2001 considered a single well pair separated by Euclidean distance  $r$ , such that the  $t_{br}$  corresponds to the first passage time for transport between the injector and the producer. Basically equation (10) provides the relationship applied to real field data. When dealing with more practical aspects of reservoir engineering such as well test analysis, it is conventional to switch to what are called practical or field units (Dake, 1998). To apply the above results to real field data, units from the dimensionless scaling form of equation (10) must be converted to real field units. For example, time and distance which are prominent in the scaling law a non-linear relationship. In line with this, King et al. (2001) interpreted equation (11) as follows:

$$t \sim r^\alpha \text{ means } \frac{t}{t_0} = \left( \frac{r}{r_0} \right)^\alpha \quad (12)$$

where  $r_0$  is typical length and is taken as sand body dimension  $r_s$  and  $t_0$  is time taken to transit through one sand body. In a homogeneous region of permeability  $k$  (in Darcies), and for a pair of wells separated by a distance  $r$  (in cm), the transit time (in seconds) is given by Andrade et al. 2000 as

$$t_0 = \frac{4\eta U r_s^2}{3k\Delta P},$$

where  $\eta$  is the velocity of the fluid (in centipoises),  $\Delta P$  is the pressure drop between the wells (in atmospheres).

$$U = \log \left( \frac{r}{r_w} \right) \quad (13)$$

where  $r_w$  is well-bore radius (in cm).  $\Delta P$  is linear in the number of sand bodies between the wells such that the drop for each body is  $\Delta P/(r/r_s)$ . A later simulation Dokholyan et al. 1998, gave an expression for  $t_0$  as

$$t_0 = \frac{12\eta U r_s^2}{15k\Delta P/(r/r_s)}, \quad (14)$$

where  $U$  is the log of the dimensionless sand body size. The results have been summarized in the theorem below. The critical exponent which is applied here is  $\alpha$ , the fractal dimension of the shortest path.

#### Theorem

Let the  $t_{\min}$  corresponds to the  $t_{br}$  of water that displaces oil.  $t_{\min}$  "scales as"  $r^{zd_{\min}}$  is denoted  $t_{\min} \sim r^{zd_{\min}}$   $zd_{\min} = \alpha = 1.33$  is the fractal dimension of the shortest path. There is a power law relationship namely  $t_{\min} \sim r^\alpha$  Let  $\frac{t}{t_0} = \left( \frac{r}{r_0} \right)^\alpha$  denote "  $t_{\min} \sim r^\alpha$ " in real field units. Then  $t_{br} = \left( \frac{r}{r_0} \right)^\alpha t_0$  where  $\alpha$  is the fractal dimension of the path,  $r$  is the distance between a pair of wells, the injector well and the production well.  $r_0$  is typical length  $t_0$  is time taken to transit through one sand body and is given as

$$t_0 = \frac{12\eta \log(r/r_w) r_s^2}{15k\Delta P/(r/r_s)},$$

where  $\eta$  is velocity,  $k$  is permeability,  $\Delta P$  is the pressure drop between the wells and  $r_w$  is well bore radius.

## Results and discussion

Some known characteristics of the Niger-Delta field/fluid such as average field dimensions, viscosity, permeability and gravity values (API) were substituted into the scaling equation (12) to obtain  $t_{br}$  for the reservoir of interest. Some other quantities such as well bore radius, change in pressure, well separation, etc. were used as given in King et al. (2001). An average field in the Niger Delta is of length 8, width 2 and has excellent sand quality (even though the sands are poly-genetic). Porosities and permeabilities are high, up to 40 percent porosity rate and permeability of between 1 and 2 Darcies (Whiteman 1982). Therefore,  $r_s = 8$  and  $k = 1.5 D$ . The API values from Whiteman, (1982) were used to calculate density ( $g/cm^3$ ) used in the conversion of units of viscosity. Both the Ewan and the Opolo fields are offshore Niger Delta fields. Data and other information on these fields were

supplied by Chevron/Texaco Nigeria Limited. On the field, once  $\eta$  which is viscosity for oil is much higher than 1, which is the viscosity for water, water is likely to finger through. This is not good for the water drive mechanism. An alternative recovery method may be applied. According to Dake(1998) consideration could be given to the application of thermal recovery methods with the aim of reducing the viscosity ratio. However, in some cases (as is observed for some Niger Delta Fields) where water flooding seems a feasible option despite the high viscosity of the oil, very high pressure is used. Indeed,  $\Delta P$  is as high as 500 psi. This way a good percentage of recovery is still made.  $\Delta P$  is a reflection of the reservoir force i.e. the force behind the drive. The values of  $t_{br}$  were calculated for reservoirs in both fields using a value of 1.33 for  $\alpha$  which is the fractal dimension of the shortest path. The results are shown in table 2 below.

Table 2:

<i>Field reservoir</i>	<i>Well separation acres km</i>	<i>API Values</i>	<i>Viscosity (CP)</i>	<i>Permeability (md)</i>	<i>Breakthrough time Range (years)</i>	
					<i>Theoretical value with(S.E)</i>	<i>Range(years)</i>
<b>Ewan</b>						
<i>B – 12/EW – 02</i>	2018	17.3	8.9	530–	52.5(one well)	1.5 (one well)
(one well)	2.90			1500		
<i>C – 02/EW – 02</i>	326	21.5	3.49	530–	2.56(0.12)	5.0(1.70)
	1.15			2000		
<i>C – 03?EW – 01</i>	394	20.1	5.32	505–	4.03(0.18)	5.6(1.74)
1.30				2673		
<i>D – 01/EW – 01</i>	204	25.4	2.04	100–	2.8(0.15)	4.5(1.08)
	0.91			2000		
<b>Opolo</b>						
<i>D – 01/OP – 02</i>	130	40	0.2	500–	$4.6 \times 10^{-6}$	1.8(1.08)
	0.72			2000	$(2.30 \times 10^{-7})$	

Standard error for the theoretical and empirical values of breakthrough time is given in bracket.

From table 2, the following remarks have been made.

- (i) Ewan B12 is a one well reservoir. In Evan B12, rock is like shale, oil is glued to the sand, and has a high viscosity of 8.9 cp. In this reservoir/well unless pressure is radically increased recovery will not be meaningful. Field Engineers confirm that this well is not doing well as a result

of low API and high viscosity. It is not surprising, therefore, that the model does not yield any meaningful result for  $t_{br}$  even with  $\Delta P$  of 1000psi. The viscosity is quite high, the reservoir being a one well reservoir has value 1 for  $r/r_s$ . The geology of the field is what could provide explanation for the poor performance of Ewan B12.

- (ii) The Opolo has oil of very low viscosity of 0.2cp, thus the pressure required here need not be high, hence  $\Delta P = 10$  psi is used. If

the actual length of 5 km is used, a value of 2.7 years is obtained, even then Opolo is not selected for further study because of every low viscosity of oil and low value of well separation.  $\Delta P = 150$  psi is also used for  $D-01/EW-01$  since its viscosity is not very high. A value of 2.80 is considered to be a good result because a standard reservoir length of 8km is used in the calculation. The actual length of the  $D-01$  reservoir is 5km. If 5km is used in the calculation, a value of 3.23 is obtained for  $t_{br}$ . (Reservoir length of 5km is used in the calculation of  $t_{br}$  done later).

- (iii) A comparison of the theoretical values of  $t_{br}$  (as calculated from the model) with the empirical values obtained from the field (Chevron Nigeria Limited), reveals that the theoretical values are close to the average value of  $t_{br}$  (empirical value). These values could have approximated better had some of the flow physics been taken into serious cognizance. It has been stated earlier that by some of the assumptions of the model, most of the flow physics is lost.
- (iv) A value of 500psi for  $\Delta P$  is an average value, this means that  $\Delta P$  could be much higher for reservoirs with oil viscosity higher than 5cp.

A major deficiency in this model is the assumption of uniformity of flow pattern for all reservoirs. In reservoir engineering, it is known that  $t_{br}$  is not only reservoir specific, but also well specific. Factors that affect the actual flow

in a reservoir include the size of pore throat, high permeability zones, closeness of the well to a fault, closeness of the well of the water front and particle sorting. These critical real field issues were not taken into consideration by the model. The contouring of the reservoir gives information about the shape of the reservoir. Calculating fractal dimensions will capture some (but not all) of the details about the reservoir which may cater for some of the factors mentioned above. In this way, the paper goes beyond the work of King et al. (2001.), to examine how the fractal dimension of the boundaries of the reservoir could affect results on  $t_{br}$ .

#### *Fractal dimension of Ewan reservoirs*

Let  $A$  be a non-empty compact subset of a metric space  $X$ . For each  $\delta > 0$ , Let  $N(A, \delta)$  denote the minimum number of closed balls of radius  $\delta$  needed to cover  $A$ . The fractal dimension of  $A$  is the number

$$D(A) = \lim_{\delta \rightarrow 0} \frac{\ln N(A, \delta)}{\ln(1/\delta)} \quad (15)$$

The equation

$$Y = \alpha + \beta X \quad (16)$$

is obtained by linearising the equation  $y = ae^{bx}$ , where  $\ln y = Y, \alpha = \ln a, \beta = b$  and  $X = x$ . A non-empty compact subset  $A$ , of a metric space has fractal dimension  $D$ , if

$$N(A, \delta) \equiv c\delta^{-D} \quad (17)$$

where  $\delta$  is the test function and  $C$  a positive real constant as expressed in (Annorize, 2004).

From (17)

$$N(A, \delta) = \lambda c\delta^{-D} \quad (18)$$

where  $\lambda$  is a constant of proportionality such that  $k = \lambda c$ .

A corresponding linear equation to (18) is as follows

$$\ln N(A, \delta) = \ln K - D \ln \delta \quad (19)$$

Comparing (16) and (19) we have the following

$$Y = \ln N(A, \delta) \quad (20)$$

$$\alpha = \ln K \quad (21)$$

$$\beta = -D \quad (22)$$

and

$$X = \ln \delta$$

Equation (22) gives the required fractal dimension.

$$J(\alpha, \beta) = \left| \frac{\sum_{i=1}^n \eta_i}{\sum_{i=1}^n \eta_i X_i} \quad \frac{\sum_{i=1}^n \eta_i X_i}{\sum_{i=1}^n \eta_i X_i^2} \right| \neq 0, n > 1 \text{ and } \eta_i = n \forall i \quad (23)$$

$$\beta = \left| \frac{\sum_{i=1}^n \eta_i}{\sum_{i=1}^n \eta_i X_i} \quad \frac{\sum_{i=1}^n \eta_0 X_i}{\sum_{i=1}^n \eta_i X_i Y_i} \right| / J(\alpha, \beta). \quad (24)$$

The Fig 1 fractal dimension of the boundary of the reservoir contour maps for Ewan C – 02, Ewan C – 03 and Ewan D – 01 were calculated using the box covering method. Each of the maps was covered with square lattices (boxes) of various edge length ( $\delta$ ). By counting the number of boxes needed to cover the required region of a map for each  $\delta$ -value, the number  $N(A, \delta)$  is generated.  $N(A, \delta)$  values for five different values of  $\delta$  ( $n = 5$ ) were tabulated and the  $\delta$ -values were measured both in cm and in km. The contour maps are covered with boxes of small side lengths. Fractal dimensions of 0.94, 1.04 and 0.99 were obtained for Ewan C – 02, Ewan C – 03, Ewan D – 01 reservoir, respectively. These were all less than 1.33 which is the fractal dimension of the shortest path used in earlier calculations.

*Calculation of  $t_{br}$  using actual fractal dimensions*

These values of fractal dimensions obtained for the various reservoirs are not used to calculate the  $t_{br}$  in place of

the universal value of  $1.33 \pm 0.05$  used earlier for  $d_{\min}(\alpha)$ . The shape of an oil reservoir as represented by its contour map can be quantified using its fractal dimension. The flow physics in an oil field is reservoir specific. Some of these specifics can be captured by calculating the fractal dimension of the oil reservoir. The assumption of universality is relaxed by examining the structural details of the reservoir. By using the actual fractal dimension of the reservoir is captured some of the flow dynamics.

*Ewan C-02.*

Using 0.94 for  $d_{\min}(\alpha)$  instead of 1.33 as used in section 3.4.2, for calculating  $t_{br}$ ,

$$t_{br} = \left( \frac{1.15}{8} \right)^{0.94} \times 30.59 = 4.94 \text{ years.}$$

*Ewan C – 03.*

The fractal dimension for Ewan C – 03 is 1.04, such that

$$t_{br} = \left( \frac{1.3}{8} \right)^{1.04} \times 41.13 = 6.22 \text{ years.}$$

*Ewan D-01*

The fractal dimension for Ewan  $D-01$  is 0.99 and the actual length is  $5km$ . Substituting these values into equations (9) and (7), we have

$$\frac{12 \times 2.04 \times 3.65 \times (500000)^2}{15 \times 2 \times 150 / (\frac{0.91}{5})}$$

$$= 2233800000000 / = 9034480000 \text{ sec}$$

which is 28.65 years, so

$$t_{br} = \left(\frac{0.91}{5}\right)^{0.99} \times 28.65 = 5.30 \text{ years.}$$

Table 3 shows the result obtained when the specific fractal dimensions of the boundaries of the reservoirs are used in the calculation of  $t_{br}$ . The values obtained under the universality assumption are displayed against the values obtained using the actual fractal dimension and the actual length of the reservoir.

Table 3:

<i>Field</i>	<i>Reservoir</i>	<i>Fractal dimension</i>	<i>Breakthrough time (years)</i>		
			<i>Theoretical value Initial value</i>	<i>New value</i>	<i>Empirical value</i>
Ewan	$C - 02/EW - 02$	0.94	2.56(0.12)	4.94	5.0(1.7)
	$C - 3/EW - 01$	1.04	4.03(0.18)	6.22	5.6(1.74)
	$D - 01/EW - 01$	0.99	2.80(0.15)	5.30	4.5(1.08)

Standard error for the initial theoretical and empirical values of the  $t_{br}$  is given in bracket beside the values.

From table 3 it can be seen that better values for  $t_{br}$  have been obtained when the actual fractal dimensions of the reservoirs are used in the calculation. The new values of  $t_{br}$  led to predictions of  $t_{br}$  which are closer to empirical values. This confirms the observation that  $t_{br}$  is reservoir specific, from which it is clear that one needs to use more of the flow dynamics to obtain good productive results. As further support, the results obtained are in line with reservoir engineering literature. Sun & Zhaocai (1997) ex-

plained what is to be expected of high fractal dimension with respect to oil flow. Larger fractal dimension implies stronger heterogeneity of pore structure. The stronger the heterogeneity of pore structure, the more irregular the propagation of the water flooding front and so more years are required for breakthrough. Conversely, smaller fractal dimension is linked with better pore structure (it's more homogeneous) leading to shorter  $t_{br}$ .

From the results displayed in Table 3, Ewan  $C - 02$  has the lowest fractal dimension (being 0.94), which corresponds with the lowest  $t_{br}$  of 4.94 years. Ewan  $C - 03$  has the highest

fractal dimension (being 1.04) which corresponds to the highest  $t_{br}$  of 6.22 years.

### Conclusion

The work has used a percolation model to predict the  $t_{br}$  for some Niger Delta reservoirs. King's model was initially used to calculate  $t_{br}$  and predict  $t_{br}$ . The agreement between the theoretical values arrived at by our modification and the empirical values are good enough to interest oil engineers. In modifying King's model to reflect some of the flow dynamics, the effect of using reservoir specific fractal dimensions was investigated. The path through which water and oil flow, i.e. the routes and contours of the reservoirs are not exactly the same. King's model assumed similarity in the routes and used the same fractal dimension for each reservoir. The universality assumption was relaxed and the fractal dimension for each reservoir calculated and used for the calculation of  $t_{br}$ . This yielded better results and confirms the observation that  $t_{br}$  is reservoir specific. The work, therefore, opens new insight into prediction of  $t_{br}$  for oil recovery and could offer a useful guide for decision making on the field.

### Acknowledgement

The assistance of Prof. G.O.S Ekhaguere is gratefully acknowledged. Reservoir maps and other information were supplied by Chevron Nig.Ltd. The study was sponsored by Mac Arthur Foundation and hosted by the Department of Mathematics, Penn State University, U.S.A.

### References

- ANDRADE, J. S. Jr, BULDYREV, S. V, DOKHOLYAN, N. V, HAVLIN, S, KING, P. B, LEE, Y, PAUL, G, STANLEY, H. E (2000). Flow between two sites on percolation cluster. *Phys. Rev E*. 62.
- ANNORZIE, M. N (2004) *Computations of the fractal dimensions of certain boundary sets*. (MPhil.Thesis). Department of Mathematics, University of Ibadan, Nigeria.
- DAKE, L.P.(1998) *Fundamentals of reservoir engineering*. Elsevier Science B.V, Amsterdam, The Netherlands.
- DOKHOLYAN, N. V, LEE, Y, BULDYREV, S. V, HAVLIN, S, KING, P. R & STANLEY, H. E. (1998) Scaling of the distribution of shortest paths in percolation. *J. Statis. Phys.* **93**, 603-613
- GRIMMET, G. (1989) *Percolation*, 2nd edn Springer.
- GRIMMET, G. (1996) *Percolation and disordered systems*. In *Lectures on Probability Theory and Statistics*. Springer, Berlin, *Lectures from 26th Summer School on Probability Theory held in Saint Flour*. 153 – 300
- KESTEIN, H. ( 1987). The critical probability of bond percolation on the square lattice equals 1/2. *Communication in Mathematical Physics* **74**,41-59.
- KESTEIN, H (1987) *Scaling relations for 2D-percolation*.

Communication in Mathematical Physics. **108**, 109-156.

KESTEIN, H. (1987) Percolation theory and first-passage percolation Ann. Prob. **15**,1231-1271.

KESTEIN, H. (1987) What is Percolation? Not. Ams **53** (5), 572 – 572.

KING, P. R, BULDYREV, S.V, DOKHOLYAN, N.V, HAVLIN, S, LEE, Y, PAUL, G, STANLEY, H. E & VANDESTEEG, N. (2001) Predicting of recovery using percolation theory. Petro. Geosci. 7 5105 – 5107.

LEE, Y, ANDRADE, J. S, BULDYREV, S. V, DOKHOLYAN, N. V, HAVLIN, S, KING, P. R, PAUL, G & STANLEY, H.E. (1999) Traveling time and traveling length

in critical percolation clusters. Phys. Rev E 60.

LUIZ, E & SINDORVICIUS, V. (2002) Percolation. *Lecture notes for School and Conferences on Probability Theory, SMR 1407 held at Trieste, Italy.*

SUN, Z & ZHAOICAI, S. (1997) Geology of fossil fuels-oil and gas. *Proceedings of the 30th International Geological Congress*. 18, *part A*. Brill Academic Publishers, Utrecht, Netherlands.

WHITEMAN, A. (1982) Nigeria: Its Petroleum Geology, Resources and Potential, Vol. 2 Graham and Trotman, London.

Received 06 May 2009; revised 20 Oct 2009.